A HANDBOOK OF PHYSICS

W. H. WHITE

TEXT-BOOKS OF SCIENCE

METHUEN'S TEXT-BOOKS OF SCIENCE

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PREFACE

THIS book, written in response to constant appeals from my students, covers the requirements of the first or intermediate examinations of the universities, a stage which demands a very fair general knowledge of the subject and one beyond which it is nowadays undesirable to trust to a single treatise. Many of the examples are drawn, with permission, from the papers of Aberdeen [Ab], Dublin [D], London [L], Manchester [M], and St. Andrews [St. A]. Those reading the book will of course give an occasional eye to their own syllabus; in particular, to make it more accessible to medical students, a colon : is put after the number at the head of each paragraph with which they are not called upon to concern themselves, and to typical questions from their papers a little m is suffixed.

It is idle to suppose that the general reader will find much inducement to peruse this briefly written book, and to the absolute beginner I recommend some little help from friend or teacher.

I have drawn my diagrams very simply, but to scale

wherever practicable. Those working in an ordinary laboratory will easily picture the necessary wooden frameworks, etc.; others may purchase, very cheaply, catalogues from the advertising dealers; therein they will find abundant woodcuts and explanations of all sorts of apparatus, ancient and modern.

To all those friends upon whose patience I have trespassed in the preparation of this book, and especially to J. W. E., I proffer that very inadequate acknowledgment, my best thanks.

W. H. WHITE

TABLE OF CONTENTS

CHAPTER		PAGE
I.	Units of Length, Time, and Mass	1

MECHANICS

II.	Motion and Force	4
III.	Work and Energy	15
IV.	Gravitation	22
V.	Motion in a Curve	27
VI.	Equilibrium of Forces	33
VII.	Rotation	47
VIII.	Fluids	55
IX.	The Measurement of Pressure	66
X.	Specific Gravity	72
XI.	Fluids in Motion	80
XII.	Elasticity	85
XIII.	The Mechanics of Apparatus of Precision	95
XIV.	The Precise Measurement of Length, Time, and Mass	101

HEAT

XV.	The Expansion Caused by Heat	113
XVI.	Thermometry	131
XVII.	Calorimetry	146
XVIII.	Latent-Heat Calorimetry	152
XIX.	Cooling	159
XX.	Convection and Conduction of Heat	164
XXI.	The Mechanical Equivalent of Heat	172
XXII.	Change of State—Melting or Fusion	176

CHAPTER		PAGE
XXIII.	Change of State—Vaporization	186
XXIV.	The Liquefaction of Gases	207
XXV.	Hygrometry	212
XXVI.	On Meteorology	217
XXVII.	Viscosity	225
XXVIII.	The Liquid Surface	231
XXIX.	Diffusion	241

WAVE MOTION

XXX.	Periodic Motion	256
XXXI.	Wave Motion	266
XXXII.	Interference of Waves	273
XXXIII.	Reflection and Refraction of Waves	279

SOUND

XXXIV.	Sound, its Nature and Speed	287
XXXV.	Pitch and Wave Length of Sounds	296
XXXVI.	Resonance. Pipes and Strings	305
XXXVII.	Vibrators Producing More Complex Tones	319

LIGHT

XXXVIII.	Rays and Shadows. Photometry	331
XXXIX.	The Reflection and Refraction of Light	339
XL.	Lenses	353
XLI.	Spherical Mirrors	371
XLII.	Practical Methods for Mirrors and Thin Lenses	378
XLIII.	Combinations of Lenses	381
XLIV.	Colour	386
XLV.	Aberrations of Mirrors and Lenses	401
XLVI.	The Eye	408
XLVII.	Optical Instruments	417
XLVIII.	Aperture in Optical Instruments	439
XLIX.	Speed of Light	448
L.	Polarized Light	452
LI.	Radiation	458

MAGNETISM

CHAPTER		PAGE
LII.	Magnets and Magnetic Materials	474
LIII.	Magnetic Fields	486
LIV.	The Earth's Magnetism	499

ELECTRICITY

LV.	Frictional Electricity	510
LVI.	Electric Field and Potential	527
LVII.	Electric Capacity and Energy	534

MAGNETISM AND ELECTRICITY

LVIII.	Magnetic Fields and Electric Currents — Electro-Magnetic Induction	544
LIX.	The Measurement of Electric Currents	567
LX.	Resistance	574
LXI.	Electro-motive Force	591
LXII.	Energy and Thermal Effects of Current	598
LXIII.	The Passage of Electricity Through Liquids	609
LXIV.	The Passage of Electricity Through Gases	629
LXV.	Electric Signalling to a Distance	641
	Answers	653
	Index	661

INTRODUCTION

PHYSICS or NATURAL PHILOSOPHY is the study of the nature of things inanimate.

In that last word perhaps lies the cause of the unattractiveness of physical science to many minds, the things it deals with so minutely seem so remote from the things that matter most in everyday life. Those who spent their time περι τον φυσεως fell indeed under the lash of Socratic contempt—doubtless the majority of the mysterious theories advanced by the students of nature in the day of that great student of men would still excite our own derision. But since then, men of science have laid very much to heart his injunction to 'leave out the gods,' and denying themselves the subterfuge of referring a difficulty to the intervention of an attendant spirit, have built more surely in the realm of nature.

Nor in Natural Philosophy have we anything to do with that principle of Life which is the special study of the biologist, and dwells or dwelt in all the structures that find description under Natural History. We may from time to time be able to indicate physical processes that are employed in the organism, but no pretence is ever made that the physically explicable part is the whole of the vital process.

Although the little gods are slain, there is a certain *deus ex machina* frequently given credit far beyond due. That is the 'natural Law.' Such and such happens 'because so-and-so's Law says it must' is an 'explanation' that satisfies a great many who would regard the pious poet's perfectly accurate one, that ''tis their nature to,' as a truism beneath contempt.

A Law in Natural Philosophy is an expression of the originator's

belief, founded on the gradually accumulating evidence of observation and experiment (often very indirect), that under prescribed circumstances matter invariably behaves in the manner stated, provided that all disturbing influences are got rid of. As time goes on, reliance comes to be placed on the law according to the way it is supported, or not, by further experiment. Every success increases its probability and every failure diminishes it, but always it remains a probability: 'So far as is known at present' is the unwritten preface to every accepted natural law.

Of existing laws, some gain but partial recognition, as being still very much in the making. Some, like Boyle's law, survive as being wide generalizations of great convenience and sufficient accuracy for most practical purposes, though not perfectly true. Others, like Ohm's law of electrical conduction in metals, can be described as stringently tested to one part in a million. And a few—the Newtonian laws of motion and gravitation, the conservation of energy principle, etc.—have come to form a very creed of the natural philosopher; an exalted position which does but render them more than ever subject to the keenest attack by every new-discovered method of experiment.

Again, as to the meaning of a Theory. Originally a mere suspicion in some active mind, it has been put forward as a 'working hypothesis' and been found to fit in with the results of large numbers of experiments, until it has become the familiar Theory to which men's thoughts are almost unconsciously moulded, and for a while it controls the progress of its branch of science. But when many new facts come to light which it cannot explain, and it is shown that a quite different supposition agrees with them and also with the facts on which the former theory was based, then it may be time to let it pass away—with the honours of war, for maybe it will become a useful ally again as the fuller theory develops.

In most cases the function of a theory is to give us a mental working model, built up of easily realizable notions of things we can see and feel, which in its action shall imitate and help us to

forecast the stranger and more recondite processes going on among entities of which our appreciation is mainly intellectual. We need not entertain the conceit that in it we have hit upon the ultimate truth ; we are but exploring what promises to be the next turning on the way to that far-distant goal.

With these few words of introduction we must turn at once to the consideration of what by common consent are the physical realities around us. Broadly speaking, the properties concerned in their appeal to our muscular sense are dealt with first ; second comes our temperature sense ; third and fourth (after a deferred mechanical digression) the actions that affect ear and eye. Taste and smell go to the chemist, but *per contra* the things with which the book concludes lie in a region in which the human organism has as yet evolved no special sense whatever.

A HANDBOOK OF PHYSICS

Chapter I

UNITS OF LENGTH, TIME, AND MASS

§ 1. Two of the properties of Matter are these: it occupies three-dimensional space, and it possesses mass.

Matter does not, however, always occupy the same portion of space relatively to surrounding matter; it can move, and one portion may travel a given distance many times over while another traverses it only once.

Here is involved the idea of **Time.** A body* may move in space, and move back, and be as it was; but it has moved also in time, and *there* there is no going back.

Motion in space ÷ motion in time = speed or velocity.

§ 2. **Time** is practically measured as a distance on a scale—across a sundial, round the rim of a clock face, or on the smoked drum of a chronograph—which is traversed in a motion we are content to believe unchanging, or having uniform velocity. Often one inverts this mode of reckoning: always, when travelling by train—"it's a four hours' journey."

Of the longer intervals of time the lunar month is the most easily observed, it was the unit of the Chaldæans and doubtless the 'year' of Methuselah. In Egypt, where all prosperity hung on a flood, the solar year was introduced and published by the priesthood as 360 days, the slight inaccuracy enabling them to remain sole holders of the secret of the date of rising of the Nile. In temperate latitudes, with their sharply marked seasons, the solar year must always have been the unit; witness the many temples oriented to either the Mayday or the Midsummer sunrise,

* Coherent portions of matter are termed *bodies.* A body whose dimensions we wish to disregard is called a *particle.*

best known to most of us the circle of doom on Salisbury (*Sol*isbury) Plain.

§ 3. Matter possesses **mass.** All ordinary physics (and chemistry) is based on the assumptions that mass and matter are inseparable —mass being a strict measure of the quantity of matter—and that neither can be created or destroyed by any physical process. Experiments with a number of chemical processes have shown no definite sign of loss or gain of mass as great as 1 part in 100 millions, i.e. within practicable limits of accuracy.

§ 4. **Units.** Thus there are three fundamental quantities, **length, time,** and **mass.** The units as multiples of which these will be measured in this book are the centimetre, second, and gramme respectively.

The **centimetre** is one one-hundredth of the metre, which is the distance, at the temperature of melting ice, between two marks cut in a bar of durable metal, preserved in Paris.

The square centimetre ($cm.^2$ or sq. cm.) is the unit of Area, and the cubic centimetre ($cm.^3$ or c.c.) of Volume.

The **second** is the common mean solar second, 1/86,400 of the mean interval between successive transits of the sun over the meridian of a fixed observatory.

It appears to be really familiar only to those brought up within sound of a grandfather clock.

The **gramme** is one one-thousandth part of the mass of the kilogram, a certain lump of durable metal, preserved in Paris.

The gramme represents (except to the last uncertain degree of refinement) the mass of 1 c.c. of pure water at its temperature of maximum density (4° C.).

The consistent system of reckoning based on these, the **centimetre-gramme-second (c.g.s.) system,** was first devised in England in 1873.

I trust that nowadays these measures are no longer unfamiliar. In daily life few of us need or wish to desert those marvellous tables of weights and measures drilled into us at school, and now as much a part of our patriotism and as defectively remembered as the National Anthem; tables with never a decimal factor among them and with one name doing duty for three or four different things. One regrets that the yard that came from the arm of a king should give place to the overgrown metre, or the inch breadth of a man's thumb to the thinness of a pinched little finger; but the intricately convertible quantities met with

in physics compel us to forgo our ancestor-worship, lest a flood of arithmetic conceal the underlying notions. Do some carpentering in cm. and mix your drinks in c.c. and these things will be familiar in a week.

TABLE OF EQUIVALENTS

1 centimetre	=	·3937 inch	1 inch	=	2·54 cm.
			1 foot	=	30·48 ,,
1 metre	=	1·093633056 yd.	1 yard	=	91·44 ,,
1 kilometre	=	·6214 mile	1 mile	=	1·6093 km.
	=	·54 naut. mile	1 n.m.	=	1·852 ,,
1 sq. cm.	=	·15501 sq. in.	1 sq. in.	=	6·451 sq.cm.
			1 sq. ft.	=	929 sq. cm.
1 sq. metre	=	1·196 sq. yd.	1 sq. yd.	=	8361 ,,
1 c.c.	=	·06103 cu. in.	1 cu. in.	=	16·386 c.c.
1 litre	=	·03532 cu. ft.	1 cu. ft.	=	28,315 c.c.
			1 fl. oz.	=	28·35 c.c.
1 litre	=	1·7608 pint	1 pint	=	567·9 c.c.
	=	·2201 gallon	1 gallon	=	4536 c.c.
1 cu. metre	=	1·3080 cu. yd.	1 cu. yd.	=	·7645 cu.m.
1 gramme	=	15·432 grains	1 grain	=	·0648 grm.
	=	·03527 oz. avdp.	1 oz. avdp.	=	28·35 grm.
1 kilogram	=	2·2046 lb. avdp.	1 lb. avdp.	=	453·6 ,,
1000 kilograms	=	·9842 ton	1 ton	=	1016 kg.

MECHANICS

CHAPTER II

MOTION AND FORCE

§ 5. **Motion.** Three kinds of **motion** are possible to a body—

(*a*) **Deformation :** it alters in size and shape (clay in fingers).

(*b*) **Rotation :** it turns or spins round its centre.

(*c*) **Translation :** it moves from place to place without either (*a*) or (*b*)—(a pen writing ; a ship's compass-card).

Any or all can go on continuously ; or stop and go back periodically as an oscillation (Chapter XXX).

The most general motion consists of all three at once (e.g. a smoke-cloud curling out of a chimney).

In a *rigid* body (*a*) is impossible, and (*b*) and (*c*) combine to the most usual motion (e.g. cricket-ball, or the bat *swung* to meet it). Rotation will have Chapter VII to itself. We are now going to take only (*c*), the linear motion of a rigid body. Since all parts perform equal and parallel paths, it is sufficient to consider only *one particle, negligibly small in size, but supposed endowed with the whole mass of the body.*

§ 6. **Linear motion of a particle.** If it can move in one straight line only, then calling motion one way + and the other way −, the result of its motion or its 'resultant displacement' is the algebraic sum of all its 'component displacements.'

But if successive displacements are in different directions as in Fig. 1 (i) the resultant is the straight line AZ, which joins the last position to the first and completes the *Polygon of Displacements* ABCZ.

AZ is the 'geometrical,' 'directed,' or 'Vector' sum of AB, BC, etc., each of which is a vector, i.e. represents by its length and direction a quantity possessing definite magnitude and direction.

For only two motions the polygon becomes the *Triangle* ABC (ii). The closing side is the resultant of the other two.

By redrawing with the component motions in different succession the reader can assure himself that this ultimately makes no difference, nor does it if they are broken up into small steps and applied alternately, as in Fig. 1 (iii). And this also shows that the diagonal of a *Parallelogram* is the same as the closing side of a triangle.

§ 7. **Velocity** *is the distance travelled in a unit of time in a given direction.* It is a vector quantity. Suppose two blows given to a particle P, one of which would drive it to Q in a second and

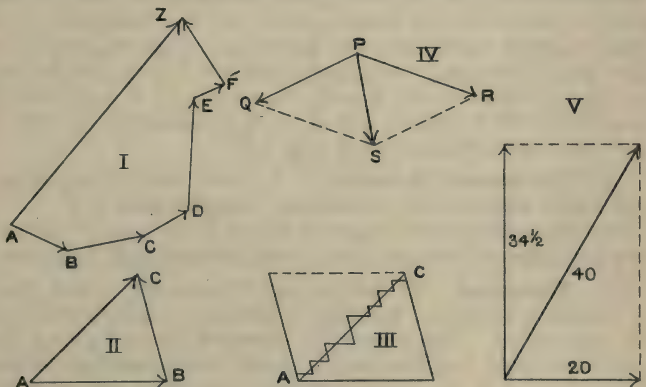


FIG. 1.

the other alone to R, Fig. 1 (iv). The result is that the particle is driven to S, where PS is the diagonal of the parallelogram PQSR or the closing side of the triangle on PQ and a line parallel to PR.

Now the order of the displacements made no difference, nor their going on in any number of alternate steps, i.e. virtually simultaneously. Suppose, therefore, the two blows simultaneous, the velocities combine into one resultant found by the vector parallelogram or triangle exactly as before. And if several blows were struck on the particle at one moment the vector sum of the velocities they produce would again be the closing side of the vector polygon.

Examples of this combination of velocities abound. The fly crossing a moving railway carriage in 2 sec. southwards is

meanwhile carried 160 ft. east and actually moves a little faster than the train in a direction slightly S. of E. relatively to the track. The earth's surface has carried both half a mile nearer the rising moon, and earth and moon have travelled some 30 miles on their journey round the sun. *All motion is relative :* which of two things moves, and the way it moves, is a matter of agreement with the neighbours. Once the fences ran past you, now you regard the earth as fixed, except when thinking astronomically or learning skating.

§ 8. **'Resolution' of vectors.** Since any side of a vector triangle represents the resultant of the other two, the two sides of any triangle that can be built on a given vector as base are possible motions into which the actual motion can be 'resolved.' It is often useful to resolve into two directions at right angles ; i.e. a right-angled triangle is built on the vector as hypotenuse, having its sides parallel to the desired directions, e.g. a ball thrown up at 60° at 40 ft. per sec. has at start a horizontal velocity of 20 ft./sec. and a vertical of $34\frac{1}{2}$ (Fig. 1 (v)).

§ 9. **'Dimensions' of velocity.** Velocity being a length per unit time is said to be of 'dimensions' length÷time, L/T.

The only velocity with a name of its own is the knot, or nautical mile per hour. 'Knots per hour' is an *acceleration*, O landsman !

§ 10. **Momentum.** A massive body is naturally looked upon as containing a greater 'quantity of motion' than a light one at the same speed. This 'quantity,' obtained by multiplying the mass and speed together, **mv**, is called the **momentum** of the body. Like **v**, it is vectorial. A 2-oz. bird flying off south at 32 ft./sec. possesses momentum equal but at right angles to that of a 4-lb. cat ambling west at 1 ft./sec.

§ 11. **Acceleration.** Velocity rarely remains steady, or uniform, for any length of time, but suffers *acceleration* to higher speed or *retardation* (negative acceleration) towards rest. This acceleration is measured as the extra velocity acquired in each unit of time, e.g. a body falls at a speed which exceeds by 32 ft./sec. the speed it had a second before, its speeds at the ends of successive seconds from rest being 32, 64, 96, etc. Thrown upwards it would have upward acceleration $= -32$.

Change of velocity per second is distance per second, per second and is of dimensions L/T^2. It is another vector and is treated by the parallelogram and polygon.

Acceleration can be applied in directions other than the line

of motion, and then alters direction as well as velocity, or in the particular case of *circular motion*, direction *only*.

§ 12. **Distance, time, speed, and acceleration.** A particle with Velocity v passes over v units of length in 1 unit time, therefore in t goes a distance vt.

On a diagram plot t as abscissæ and v as ordinates, then the *distance* vt is represented by the rectangular area in Fig. 2 (A).

If v alters steadily to v' at end of t the average speed is $\frac{1}{2}(v+v')$ and the distance covered is represented by the whole area.

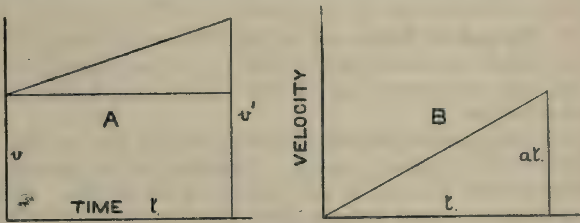


FIG. 2.

In particular, **starting from rest** and steadily acquiring final v

$$\text{distance } s=\tfrac{1}{2}vt=\tfrac{1}{2}(v/t)t^2$$

v is gained in t; v/t, gained in 1, is $\therefore$ *acceleration* a

$$\therefore \text{ distance travelled } \mathbf{s=\tfrac{1}{2}at^2}$$

represented by the triangular area in Fig. 2 (B).

The speed v after t from rest is of course at

$$s=\frac{1}{2a}a^2t^2=\frac{1}{2a}v^2 \text{ or } \mathbf{v^2=2as}.$$

§ 13. **Force. Newtonian Laws of Motion I and II.** Variation of velocity means variation of mv, the momentum of the moving mass; and the product of mass and acceleration (MV/T or ML/T^2) means the extra momentum acquired per second, or the rate of change of momentum.

What does this mean ? What causes the change ? Sir Isaac Newton laid down three *Laws or axioms of Motion*, of which the first two are :—

I. *Every body continues in its state of relative rest or motion in a straight line except when compelled by Force to change it.*

This is a statement of the inertness or Inertia of Matter.

II. *Force is measured by the quantity of motion (Momentum) it produces or destroys per second in its own line of action.*

Our muscular sense informs us that we have to exert force to set ourselves or anything else in motion, or to check its speed, or to persuade it to come round in a curve. We argue that our own experience holds good generally, and that whatever affects the motion of a body is exerting force on it. The second law quantitatively connects force and motion.

The vector polygon and parallelogram evidently apply to forces as well as to accelerations, momenta, velocities, and displacements.

§ 14. *The* **unit of force** *must be that force which acting for one second produces unit momentum*—sets 1 grm. moving at a speed of 1 cm. per sec., producing the unit acceleration in unit mass. This is the **dyne.**

By experiments to be described in Chapter IV it is found that the earth exerts on a gramme mass an attraction which increases its speed about 981 cm./sec. in each second of its motion. That is *the weight of a gramme mass is about* 981 *times the unit of force, the dyne.* The dyne is thus a trifle more than a milligram weight.*

§ 15. **Mass and weight. The dyne and the gramme weight.** Why drag in the dyne? Why not keep to the gramme weight as unit of force? For two reasons:—

(1) *Either* unit force would be 981 times unit increase of momentum, which is its fundamental measure, *or else* unit length must be similarly increased.

(2) The unit quantity of matter, attracted down with 1 grm. weight at sea-level in latitude 45°, weighs 2·5 mg. more at the pole and 2·5 less at the equator, and also less on a mountain-top (§ 29). Masses are comparable by weighing only provided that they are close together on the earth and equidistant from its centre.

Weight is an accident of position. We suffer from it considerably on earth. Why cannot we take a run and a jump and soar up as pleasantly as an aeroplane? Because the earth pulls us down so fiercely that our legs cannot fling us free of its surface for more than a second and a quarter. On the moon a ball would stay up six times as long as here, and flung as a meteorite into

* Its smallness involves big numbers, which the physicist writes in powers of 10; e.g. 981 millions $= 9{\cdot}81 \times 10^8$. And small fractions in negative powers, $\cdot 000033 = 3{\cdot}3 \times 10^{-5}$.

cosmic space its weight would depend on its proximity to an attracting planet. But to get a notion of the speed of fall on the massive sun, shut yourself in a room with a 2-oz. rocket minus its stick.

Mass and momentum are the same everywhere. It would take just as much effort to bowl the cricket-ball at the usual speed on the moon. In water one's small residuum of *weight* would bring one with a very harmless bump to the bottom of the tank, but an inadvertent collision with the bank reminds one, by the violent change of momentum, that one's *mass* remains.

Evidently the number of dynes in the measure of a force will be about 981 times the number of gramme's-weight in it.

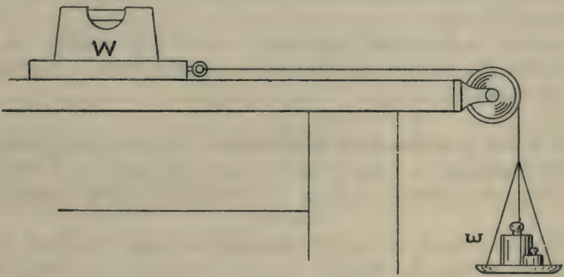


FIG. 3.

§ 16. **Friction.** We never see the first phrase of the laws of motion obeyed. Moving bodies, unaided by applied power or downhill slope, always slow down and stop. We are bidden to look for forces acting *always* to reduce momentum. We know that reducing the roughness of surfaces in contact and their relative speed prolongs the motion, and removal of air enables Mr. Brennan's gyroscopes to spin, undriven, for hours. We have reduced **Friction.**

Friction is a force which always destroys momentum. It breaks down the motion, as in all those contrivances misspelt 'brakes.' It must be subtracted from any force applied to increase speed.

Force applied − friction = increase of momentum per second in direction of force,

or Force applied = friction + ditto.

Friction helps a force applied to decrease active *mv*.

§ 17. **Laws of Friction of Solids. Measurement of coefficient of friction.** The frictional resistance to motion between solid surfaces depends on the material they are formed on and their state of polish. Within limits it is proportional to the force pressing them together. Within wide limits it does not depend on the size of the surfaces nor their speed of rubbing (though always greater just at the start—'stiction').

Liquids and lubricated surfaces follow a different law (§ 240).

The *Coefficient of Friction* is the fraction of the pressing force which must be applied parallel to the surface to cause slipping. Fig. 3 shows how it is measured, say for polished cast-iron on wood; w grm. just drags W slowly along (having been given a start), w/W is the coefficient, e.g. iron on wood ·3, planed pine on hard wood ·22, etc.

The effective frictional resistance to rolling is usually far less than to sliding, for the relative motion of the parts in contact is so much smaller.

§ 18. **Force and change of momentum. Activity and reactivity.** A force F acting on a mass m for t seconds gives it F units of momentum per second, or Ft in all, and if this cause a velocity v

$$Ft = mv.$$

There is no need to specify a movable mass, for there is no such thing as an immovable mass. Pressing your hand horizontally on a rock with force F the whole earth gains momentum F per second in the direction of your pressure. But your feet must be kept from slipping in the opposite direction, in which they press with equal force, giving the earth just as much momentum in the opposite direction. You remain at rest. But push off from the rock and run, you and the earth have equal and oppositely directed momenta, as you run the earth continuously moves back; being massive it does not move fast, $mV + Mv = 0$. To stop running your feet exert backward force for a time just destroying the earth's momentum and the earth pressing on your feet destroys yours.

It is all give and take, controlled by the **Newtonian third law of motion.**—*Activity and reactivity are equal and opposite.*

The horse pulls at one end of the rope and the barge at the other. "The horse pulls the harder and the barge moves. Oh yes! The horse's hoofs push back on the towpath and the path pushes the horse on, i.e. the path pushes harder than the horse. Oh no!" That is popular argument. In truth, the path reacts

harder on the horse, in response to his 'activity,' than the water pushes back on the barge. Therefore he acquires more forward momentum than the barge loses (or acquires backward), and he and the barge store the forward balance of *mv* by increasing in speed, until a speed is reached at which increased frictional resistance of the water just equals towpath reaction and the system moves with unchanging *mv*.

Force applied=friction+increase of momentum.

There has been a perfect equilibrium all the while; it just happens that the invisible 'force' of the horse becomes visible increase of momentum.

The 'friction' is the portion of the produced momentum that the earth acquires. Friction of its surfaces or among its coherent particles prevented the rock you pressed on from moving separately, a greater force would have shifted the rock, having exceeded *the greatest value of the frictional term. It is this that decides whether there is yielding or not.* Compare the starting of a car; while the clutch slips the car, like the earth, acquires only the portion of momentum that the clutch-friction destroys in the fly-wheel, when gripped tighter the engine cannot reach the limiting friction and the car takes all *mv* produced.

§ 19. **Impact. Impulsive forces.** In the collision or **impact** of two bodies the small force exerted between them increases to a very large one as they squeeze each other out of shape. In plastic substances—lead, putty, etc.—or fluids, the force then diminishes as they cease to squeeze closer and the particles become accommodated to their displaced positions. In elastic bodies, it decreases as they move apart again, for their particles have no choice but to move back where they came from.

The whole process takes only a small fraction of a second, but at every instant equal and opposite forces are exerted as the two bodies change their motion, one's gain of forward momentum is equal to the other's loss, and the whole momentum of the system (the two together) remains unchanged.

The **impulse** is measured by the forward momentum imparted to one body: i.e. it is expressed as the average force which acting for 1 sec. would produce the same change as the varying and enormous force which acts for perhaps ·0001 sec. Part of this forward momentum may be used in destroying existing backward, as in a cricket-bat striking and reversing the direction of motion of the ball.

A muscle exerting what we consider a steady force is receiving from 10 to 40 nerve stimuli per second, and can be heard to vibrate. When tired the stimuli are less frequent and the force becomes visibly unsteady, trembling, an obvious sequence of momentum-giving impulses.

On the Kinetic Theory of Matter all substances consist of molecules swarming in rapid motion ; the pressure of a weight on the table becomes the momentum imparted per second by the myriad impacts of one molecular swarm on another. If any one doubts that great solidity can arise in this way, let him spin a bicycle-wheel fast and try to put his finger through it—and that is only a few dozen impacts per second.

Thus the distinction between a steady force and momentum of visible motion can be bridged over in theory, and our way of measuring forces is justified.

§ 20. Notice that all force is exerted between masses. *Every force, whatever its exciting cause, must be 'anchored' on a mass at both ends.* And the old catch about immovable mass and irresistible force is answered, that the non-existence of the one implies the impossibility of the other.

EXAMPLES.—CHAPTER II

1. Express 5 miles per hour in feet per second.

$$5 \text{ miles} = 5 \times 5280 \text{ ft.} \quad 1 \text{ hour} = 3600 \text{ sec.};$$

$$\therefore\ 5\,\frac{\text{miles}}{\text{hour}} = 5 \times \frac{5280 \times \text{ft.}}{3600 \times \text{sec.}} = 5 \times \tfrac{22}{15}\frac{\text{ft.}}{\text{sec.}}$$

(roughly, feet per second $= 1\frac{1}{2}$ times miles per hour).

2. Express miles per hour in centimetres per second.

3. Express 37 knots in miles per hour. 1 nautical mile $= 6081$ ft.

4. Express knots as cm./sec., given 21,600 n.m. $= 40{,}000$ km.

5. Find in how many seconds a train jolts over a number of 30-ft. rails equal to its speed in miles per hour. Also 45-ft. rails.

6. Express an acceleration of 981 cm./sec. per second in ft./sec.^{2}

7. Express an acceleration of 32 ft. per second every second as miles per hour every minute.

Dimensions of acceleration are L/T^2 and every term in the equation must therefore have a length in the numerator and the product of two times in the denominator. *Physical equations should always be scrutinized to see that their dimensions are correct throughout.*

$$32\,\frac{\text{ft.}}{\text{sec.} \times \text{sec.}} = x\,\frac{\text{mile}}{\text{hour} \times \text{min.}} = x\,\frac{\text{ft.} \times 5280}{\text{sec.} \times 3600 \times \text{sec.} \times 60}$$

or $32 = x \times 5280 \div 216{,}000$; $\therefore\ x = \underline{1309 \text{ m.p.h./min.}}$

8. How far should a body fall in 4 sec. ?

By the argument of § 12, $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 981 \times 16 = \underline{7848 \text{ cm.}}$

9. How long will it take an electric train with acceleration $2 \cdot 5$ ft./sec.^{2} to travel 100 ft. from rest? (§ 12.)

10. The splash of a stone is heard $2 \cdot 7$ sec. after dropping it down the well. If sound travels up at 1100 ft./sec., how deep is the well? ($g = 32$.)

11. A ball is thrown up at 40 ft./sec. and 60° to the horizontal. How high does it go, how long is it in the air, and how far away does it strike the level ground?

Resolve the velocity as in § 8 into 20 ft./sec. horizontal and $34 \cdot 5$ vertical. *These are now quite independent of each other.* It will take $34 \cdot 5 \div 32$ sec. for gravity to destroy the vertical component, the ball meanwhile rising at mean speed $17 \cdot 25$ to a height $17 \cdot 25 \times 34 \cdot 5 \div 32 = 18 \cdot 6$ ft. It takes as long again to fall; time of flight $= 2 \cdot 16$ sec., during which it travels $20 \times 2 \cdot 16 = 43$ ft. horizontally.

Horizontal speed does not affect vertical motion at all. There is the common experiment of showing that a ball rolled off the table at any speed falls to the floor just as soon as if dropped vertically from the edge.

12. Define velocity and acceleration. In four successive seconds a body moves 10 cm., 20 cm., 30 cm., and 40 cm., respectively. Calculate its acceleration and its velocity at the end of each second. [L.]

13. A bullet passes in succession through three screens 1000 ft. apart, taking $\cdot 8$ sec. from first to second and $\cdot 86$ from second to third. Find the (negative) acceleration. [L.]

14. Assuming the parallelogram of velocities deduce that of forces, carefully giving authority for each step. [St. A.]

15. Show how the velocity of one moving body relative to another is determined. Two vessels are steaming in opposite directions at 12 and 6 knots. Their smoke-tracks are at right angles to each other and equally inclined to the direction of the wind. Show that the velocity of the latter is nearly $7 \cdot 6$ knots. [L.]

16. What force is required to give an electric train of 150 tons an acceleration of $2 \cdot 5$ ft./sec.^{2}?

Force = gain in momentum per second $= 150 \times 2 \cdot 5$ 'ton' units,
or reducing to English gravitational measure by dividing by $g = 32 \cdot 2$,

$$\text{Force} = 150 \times 2 \cdot 5 \div 32 \cdot 2 = 11\tfrac{2}{3} \text{ tons weight.}$$

Or in dynes, assuming 1 ton = 1,000,000 grms.,

$$\text{Force} = 150 \times 10^6 \times (2 \cdot 5 \times 30 \cdot 5) = 1 \cdot 14 \times 10^{10} \text{ dynes.}$$

17. Express a steam-train acceleration of $20\frac{1}{2}$ m.p.h. in half a minute as ft./sec.^{2}. What force must the engine exert if the whole train weighs 200 tons?

18. What force is required to stop in 3 sec. a 2-ton motor-car travelling at 15 m.p.h. [= loss of mv per second]?

How far does it travel with brakes on? [Average speed $\times$ 3 sec.]

19. Find the pressure on a wall when a hose delivers 100 gal. of water per minute perpendicularly on it at 50 ft./sec.

Force $= mv$ destroyed per second

$$= \frac{100 \times 10}{60} \times 50 \text{ lb.-ft. units} = \frac{50{,}000}{60 \times 32 \cdot 2} = 26 \text{ lb. wt.}$$

$$\text{or } = 100 \times 10 \times 440 \div 60 \times 50 \times 30 \cdot 5 = \underline{1 \cdot 12 \times 10^7 \text{ dynes.}}$$

Note.—Splashing back would increase this, as the wall is imparting backward momentum.

Note.—By the third law of motion it is also the force with which the fireman must hold up the hose : the fluid transmits this to the wall.

20. Find the pressure on a water-wheel struck by 500 kg. of water per second travelling at 400 cm./sec., wheel and water which leaves it moving at half this speed.

21. A 20-grm. bullet moving at 700 metres per second embeds itself in a suspended log of 100 kg. Find joint speed.

Total momentum unchanged. $\therefore$ $20 \times (700 \times 100) = 100{,}020 \times x$.

$\underline{x = 14 \text{ cm./sec.}}$

22. Find (i) the force exerted to give the bullet its speed in ·002 sec., (ii) the average force it exerts in penetrating 17·5 cm. into the wood.

(ii) Average speed during penetration $= \frac{1}{2}(70{,}000 + 0) = 35{,}000$.

$\therefore$ loses 1,400,000 units of momentum in $17\frac{1}{2} \div 35{,}000$ sec.

$\therefore$ loses at rate of $2{\cdot}8 \times 10^9$ per second = dynes force.

23. A mat of 2·5 kg. is struck by a stick of ·5 kg. moving at 500 cm./sec. Find joint speed.

24. A 30-grm. golf-ball is struck by a 400-grm. club at 2000 cm./sec. After impact it moves off twice as fast as the club follows. Find its speed.

25. A ball collides with another of 3 times its mass and bounces back at $\frac{1}{8}$ initial speed. Find speed of other ball.

Total *forward* momentum unchanged.

$\therefore$ $MV = 3Mx + M \times \frac{1}{8}V$. $\therefore$ $x = \frac{3}{8}V$.

Which, of course, is more than if first had stopped dead.

26. An elastic pellet of 1 grm. bounces at 1000 cm./sec. between plates 2 cm. apart. Find pressure on plates.

Strikes each plate $1000 \div (2 \times 2) = 250$ times per second.

At each impact V changes from 1000 up to 1000 down = 2000.

$\therefore$ Momentum given up per second $= 250 \times 1 \times 2000$ dynes = $\underline{{\cdot}5 \text{ kg. (q.p.)}}$.

27. A 25-grm. bullet moving at 300 m./sec. stops after penetrating 3 cm. of bone. Calculate average force it exerted. [L.]

Chapter III

WORK AND ENERGY

§ 21. If both a moving force and a motionless one transmit momentum alike, wherein lies their difference? Is not rolling the grass a more arduous business than leaning on the roller at rest? The difference is that the moving force does **Work** and the motionless does none.

The force must advance.* Sideways motion is inoperative. No work is done by the weight of a rolling ball on the billiard table. The 'work' done by the active 'system' is the product F*s* of the force F it exerts, and the distance *s* it moves forward. This is also called the 'energy the system loses.' It does not follow that the system acted on gains all this energy. And if a would-be active system is driven backward (e.g. back-pedalling) it gains energy, but it does not follow that such can be made use of.

§ 22. A moving mass pushes back a resisting force for some distance before it can be brought to a standstill. Hence it is said to possess **energy of motion** or **Kinetic Energy**.

This can be expressed in terms of its mass and speed. Let all its momentum mv be due to a force F having acted on it t sec. Then $F = mv \div t$. Its speed, having increased steadily from 0 to v, has averaged $\frac{1}{2}v$ for the t sec., i.e. it has been pushed forward a distance $s = \frac{1}{2}vt$. $\therefore$ the work done on it $Fs = \frac{mv}{t} \times \frac{1}{2}vt = \frac{1}{2}mv^2$. And as a matter of experiment, allowing for inevitable friction, as much work can be obtained from it as it is stopped. Hence stored up in mass m moving at speed v is **Kinetic Energy** equal to **half the product of the mass into the square of the speed, $\frac{1}{2}\mathbf{mv}^2$.**

* This is true in more ways than the mere mechanical. Think as hard as you may, you do nothing unless you progress along the line of thought. Sticking *too* long at the hard parts does not pay. Thought without action, speech, or writing, may gain you *Nirvana* perhaps, but neither money nor credit in this world.

§ 23. The 'dimensions' of energy are evidently ML^2/T^2.

Fs shows that it is measured in dynes × centimetres or **ergs.**

An **erg** *of work is done by* 1 *dyne pushing forward* 1 *cm.*

It is a small unit, roughly the work done by a diminutive 1-mg. fly crawling 1 cm. up the window-pane. 981 ergs lift 1 grm. 1 cm., the *gramme-centimetre* of work.

Ten million ergs (10^7) is the **Joule,** a more sizable unit, used in electrical measurements, and about three-quarters (·737) of the *foot-pound.* The latter is the work done in lifting 1 lb. 1 ft., and is the gravitational unit used by all mechanical engineers here and in the States.

§ 24. **Momentum and energy contrasted.**

Recoil. The distinction between momentum and energy is well seen in the recoil of a gun firing a shot, of a swimmer throwing a polo-ball, etc. The third law insists that equal and opposite forces are exerted on projectile and thrower throughout the discharge, i.e. for equal times. They therefore are supplied with *equal and opposite momenta; mv* being numerically the same for both, the speeds they acquire in opposite directions are inversely as their masses.

The distances the two move during discharge are proportional to their speeds, therefore the work done on each (= same force × distance) or *the energy of motion each acquires is inversely proportional to its mass.* As the gun is perhaps 100 times as massive as the projectile its energy of recoil is absorbed without difficulty by a brake. (*Note.*—Holding tight to your shoulder, your own mass adds in with the rifle's.)

Impact.—In impact, which is the sudden inverse of the foregoing process, the total momentum remains unchanged (§ 19). But unless the bodies are perfectly elastic—and none are—there is always loss of energy in crushing, vibration, noise, heat, etc. For take the simple case of a bullet fired into a 99 times heavier log at rest. 100 times the mass moves at ·01 the speed, $\frac{1}{2}mv^2$ reduces to $50m \times \cdot 0001v^2$, the remainder goes in mutual destruction.

§ 25. Every sudden collision reassures us of the reality of Kinetic Energy, yet we never buy energy in that visible form. A mechanism equipped with it was the Howell torpedo, driven by the energy stored in a fly-wheel spun up to 10,000 revs. per min. It was insufficient.

But we will pay to be carried up a hill, to have heavy clock-

weights wound up, for steam, for electric energy, for water under hydraulic pressure, for food, coal, or cartridges. These, whether 'things' or not, we value for the energy of motion of ourselves, of machinery, shot, etc., which we can get from them. For in lifted weights, in steam, in combustibles, etc., is hidden 'what-may-become-energy,' or as we call it, **Potential Energy.**

It is useful to regard this as simply another form of real energy, convertible into or from Kinetic Energy. Let us take some instances :—

The energy $\frac{1}{2}mv^2$ of motion of a *ball thrown vertically up* gradually diminishes to zero at the top of its path. Here the ball is at rest, storing as gravitational potential energy all the work (less air friction) done in lifting it, $\frac{1}{2}mv^2$. Lifted slowly to the same height s against the earth's pull, its weight mg, the work would be $mg \,.\, s$. Equating these leads back to the relation $v^2=2gs$ established in § 12.

We say that its total energy remains unchanged all the while ; (kinetic+potential)=constant.

The energy is all kinetic again by the time the ball strikes the ground, and then is quickly converted into potential energy of elasticity as the ball is squeezed out of shape, to be just as quickly reconverted into kinetic. The diminished rebound shows that the ball has lost *part* of this energy, but this we can account for in air friction and in the heating of the imperfectly elastic rubber, as evidenced in motor-tyres.

A *clock balance-wheel* in vacuo bends or unbends the spring and is thereby stopped at each end of its swing. In this instance of a *Conservative* (energy-preserving) *System* there is a 'flow' of energy from one part to another. If the spring were unhitched from the wheel when most 'wound up,' it would contain all the energy as potential, and the wheel remain at rest. Half a swing later the spring would have remained slack and the wheel gone on spinning with all the energy kinetic.

An *electric tram* gets its energy to climb a hill from the distant engines. On the way down the driver breaks its motion by making its motors return electricity to the wires, thence to be drawn by other trams. The tram itself loses its gravitational potential energy, but the whole connected system retains it, and usefully.

But if you *cycle out against a head wind* and the wind drops, where is the potential energy you fondly hoped you were accumulating to help you home ? Look for it where you invested it ;

that was in the wind, now 50 miles away. The trustee has bolted with the funds in the shape of an increased violence of air motion where you rushed through it, by this time mere frictional heat. The energy has gone to a distant part of the system, it is not destroyed, but you cannot get it.

Investments in potential energy must be made discreetly. Even coal—bottled sunshine—would be useless without the air to burn it.

See also Chapter XXI.

These are a few instances leading up to the enunciation of two principles which we believe to govern all processes, both physical and vital, the principles of the conservation and of the dissipation of energy.

§ 26. **The Principle of the Conservation of Energy states that energy is indestructible.** It may be transformed in all ways into any sort of recognizable kinetic or potential energy—mechanical, luminous, electrical, chemical, thermal—may be scattered broadcast or hidden in ways yet unknown, but cannot be altered in total amount. Fresh supplies may be unexpectedly discovered (e.g. radio-active substances), but they are not fresh creations.

The study of the transformations of indestructible energy occupies the physicist as does the study of those of indestructible matter the chemist.

The Principle of the Dissipation of Energy states that energy, although indestructible, tends in every cycle of changes to become less available for use. No actual transformation of energy can be exactly reversed so as to restore the precise conditions at the start. Always there is more or less irrecoverable loss—friction, noise, electrical disturbance, all ultimately ending in heat of no useful intensity. The engineer is unsparing of efforts to reduce this tax, both in heat engines, when it is inevitably heavy, and in transmission mechanisms, greatly improved of late.

Is therefore the whole universe gradually coming to a tepid standstill? Answer, that while we believe these two principles to apply to all parts of the universe 'visible' to us, we are entitled to argue nothing and imagine anything about what may lie between the limits of human observation and the more distant limits of human imagination.

§ 27. **Power.** *The rate of doing work, i.e. the amount of energy transformed in a unit of time, is called the* Power.

An engine working at the rate of 1 **horse-power** (Watt's liberal estimate of a Cornish mine-horse in warranting his pumping engines) supplies 33,000 foot-pounds per minute, which is $7 \cdot 46 \times 10^9$ ergs or 746 joules per second.

A power of **1 joule per second** is called **1 Watt.**

The **kilowatt** of electrical engineers is thus about $1\frac{1}{3}$ h.p.

Work = force × distance. ∴ **power,** which = work ÷ time = force × distance ÷ time = **force × speed.**

Heavy pulls at slow speeds therefore represent no more *power* than light pulls at high speeds; the resounding puffs of a locomotive leaving a station or the starting effort of an electric train (·1 its weight) mean great *force* but no unusual consumption of steam or 'watts.'

The power transmitted by a rope, etc., is the product of its speed and its pull.

Every prime mover has an optimum speed of working, with a temporary maximum frequently much higher. Its specification must include (1) starting effort, (2) best speed, (3) greatest power at that speed, (4) greatest power obtainable and for how long without injury.

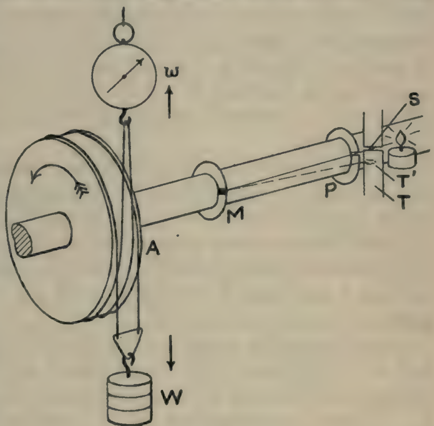


FIG. 4.

Animals are most adaptable in all respects; there is no need for a gear-box on a horse; a man can exert $\frac{1}{2}$ h.p. running upstairs, his usual being $\frac{1}{8}$ h.p.

§ 28. **Measurement of power.** The mechanical measurement of power involves that of a force and the speed at which it moves. Then the total work done is found by multiplying their product by the time of motion.

e.g. the hauling up of a 3300-lb. cage at 1000 ft./min. requires 3300 × 1000 ft.-lb./min. = 100 h.p. The pumping of the same

weight of water the same height per minute likewise calls for 100 h.p. (in this way the earliest engines, which were pumps, were measured).

The **dynamometers** used in engine testing are compact modifications of the first example. The second finds its analogue in the 'pumping' of electricity to a higher 'level' in the electrical engineer's testing-room.

In the ordinary **friction dynamometer** for the 'brake h.p.' of an engine (Fig. 4) a doubled rope takes a turn round the fly-wheel, one end is held by a spring balance and the other carries weights W. When the wheel turns the rope starts lifting, the balance pull decreases and eases the grip of the rope till it slips on the wheel, and soon the machine adjusts itself with the wheel c ft. circumference making N revs. per min. and net frictional drag $W-w$ lb. down at A. As far as the engine is concerned this is equivalent to hoisting $W-w$ lb. up Nc ft. per min. $=(W-w)Nc\div$ 33,000 h.p. Actually, of course, the work is all dissipated as frictional heat.

On the engine-shaft is sketched a **transmission dynamometer** as now used for measuring h.p. transmitted from turbine to propeller by observing how much the shaft is twisted.

P and M are collars on the shaft, in P is a slit and on M a concave mirror. These lie in line straight along the untwisted shaft, so that a ray from a lamp and fixed slit S passing through P is reflected straight back to S.

Once for all the wheel-and-weight was put on the shaft, held fast at the far end. M twisted round more than P, PM became oblique, and the ray SPM was reflected back to form an image at T. A twist measured by ST is therefore equivalent to a pull $(W-w)$ on a circumference c.

In use, once per rev. a flash passes from S through P to M and back to T′, therefore we know that the resistance of the propeller is ST′/ST times $(W-w)$ pulling on c and hence at N revs. per min. the h.p. $=\dfrac{ST'}{ST}\times\dfrac{(W-w)cN}{33,000}$.

This dynamometer measures thousands of horse-power without throwing any work away.

EXAMPLES.—CHAPTER III

1. Define work and kinetic energy. Show that when a force acts on a freely moving mass the work done by the force is equal to the gain in kinetic energy of the mass. [L.]

2. A man climbs a hill. Where does the energy come from and go to? Why does he get hot? [L.]

3. Trace as far back as you can through its various transformations the energy obtained from a water-wheel. [L.]

4. State laws of friction and explain how to find coefficient. How measure practically *efficiency* of any machine? [A.]

5. A man using a certain tackle finds that by exerting a pull of 120 lb. weight and moving steadily at the rate of 3 ft. per second he can raise a weight of 864 lb. through 1 ft. in 6 sec. Find the velocity ratio and the efficiency of the tackle. [L.]

6. State the units of power in the British engineering and c.g.s. systems and indicate the relation between them.

An engine turns a fly-wheel 4 ft. diameter at 550 revolutions per minute. A 10-lb. weight hangs round the rim of the wheel. Find the horse-power. [L.]

7. A cyclist works at $\frac{1}{12}$ h.p., wind and road resistance $=3$ lb. Find speed. [Ab.]

8. Distinguish between Force, Power, and Energy. Where does the energy go to in (i) exhausting the air from a vessel, (ii) 'tacking' a boat upstream, wind being downstream, (iii) the action of the heart? [L.]

9. At what horse-power does a 1-ton car work when climbing 1 in 10 at 15 m.p.h., frictional resistance being 5 lb. per ton? [M.]

15 m.p.h. $=22$ ft./sec. [Ex. 1, c. I.]

$\therefore$ climbs $1/10 \times 22 = 2{\cdot}2$ ft./sec. vertically.

$\therefore$ work done against gravity $=$ weight $\times$ lift $= 2240 \times 2{\cdot}2$ ft.-lb./sec.

Add, 5 lb. overcome in each foot of travel $= 5 \times 22$ ft.-lb. sec.

Total $= 4928 + 110 = 5038$ ft.-lb./sec. $=$ 9·16 h.p. at wheels.

10. Find the horse-power required to draw a cart weighing half a ton up an incline of 1 in 20 at $7\frac{1}{2}$ miles per hour.

Find also the additional horse-power required to overcome a steady frictional resistance equivalent to 20 pounds' weight. [1 h.p. $=550$ ft.-lb. per second.] [L.]

11. Explain the 'efficiency' of a machine. A man, working at $\frac{1}{5}$ h.p., is raising 1000 lb. by pulleys. The mechanical advantage would, in the absence of friction, be load $=25 \times$ pull, and the actual efficiency is $\frac{1}{2}$. Find what pull the man is exerting and at what rate he is drawing the rope in. [L.]

Chapter IV

GRAVITATION

§ 29. **Gravitation** is the mutual attraction of massive bodies.

The theoretical method of measuring forces is to let them act for 1 sec. on a mass and find the momentum they have given it. Using 1 grm. the velocity it acquires in the second (its acceleration) is equal to the force in dynes.

Practically, one *weighs* the force against the gravitational attraction of the earth on a known mass. Now this, the weight of the mass, varies a little from place to place. [For the earth is rotating and the centrifugal tendency reduces weights, having most effect near the equator. Its great result has been to pile material highest where it weighed least in order to preserve internal equilibrium in the quasi-fluid earth (cf. § 82); the thickness of this equatorial bulge still further reduces weights at the equator.] Hence the weight of a gramme cannot be made a primary standard of force, and for accurate scientific purposes we must be ready to find how many dynes it represents locally.

§ 30. This is called g, the force of **Gravity** at a place. Plainly it is the acceleration of a falling gramme, or of every individual gramme in a falling body, and hence these methods of finding it.

1. **Free fall,** Fig. 5. Things fall fast, but measurements may be made with a tuning-fork as timekeeper. A smoked-glass strip drops from the dotted position and the pointer on the fork marks on it one complete wave for each vibration, occupying the very short time P. (How P is found see § 315.) Then distance s, measured from the starting-point, which contains n waves has been fallen in time nP and $s=\frac{1}{2}at^2$ becomes $s=\frac{1}{2}g(n\text{P})^2$, hence g.

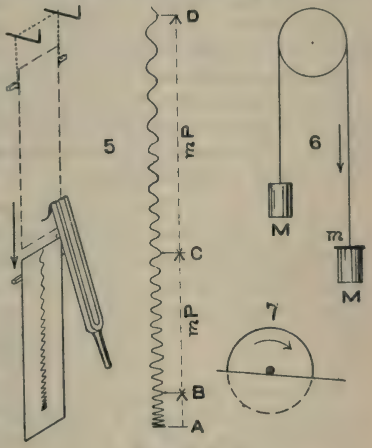
2. **Atwood's machine,** Fig. 6. Atwood (*ca.* 1790) slowed the speed of fall of a weight by making it drag along inactive masses. Equal masses MM balance on a light frictionless pulley. On one m is laid and the force mg dynes pulling it down has now to move the whole lot $m+2\text{M}$, so that the acceleration (force÷mass) is

reduced to $m/(m+2\text{M})$ of g. Time t of fall through s is observed and $s=\frac{1}{2}at^2$ becomes $s=\frac{1}{2}\cdot\frac{m}{m+2\text{M}}gt^2$, hence g.

In practice I use an ordinary aluminium ball-bearing pulley, the finest plaited silk fishing-line, and one M a trifle heavier and readjusted before use till it just feebly crawls down when given a start, indicating that inevitable friction has been neutralized. It is loaded with m, the pulley hauled up the wall till they are at the desired height, and the other M let go from the table as the clock ticks. It works better than elaborate machines.

Note.— Without m Atwood's machine demonstrates the First Law of Motion; with various m's removed after definite intervals, the Second Law, the speed attained measuring the acquired momentum, which is found to be $mg\times$ time of its action.

3. **Roller on inclined plane,** Fig 7. Make the moving pulley massive and omit MM. Let this fly-wheel roll down rails on its axle; a fraction of its own weight acts as m.



FIGS. 5, 6, 7.

e.g. A disc 11·4 cm. diam. had an axle 2 cm. circum. On wooden rails a fall of 1·4 cm. in 60 just kept it moving (friction overcome), an additional 2·9 cm. sent it down in 20 secs., averaging ∴ 1·5 turns per sec. and ∴ ending at double this speed. The mass of a rotating disc can be supposed concentrated in a ring $1/\sqrt{2}$ its diam., which was ∴ $11{\cdot}4\div\sqrt{2}\times\pi=25{\cdot}4$ cm. circum. ∴ at 3 revs. per sec. its kinetic energy was 2×1600 grm.$\times(3\times25{\cdot}4)^2$ and this=its loss of potential energy in the descent $=1600\times g\times2{\cdot}9$ cm. Hence $g=1000$.

4. **Pendulum.** Method 1 is hasty and 2 and 3 grievously affected by friction. The ever-falling pendulum gives by far the most accurate method. From § 38 its time of swing $=2\sqrt{\text{length}\div g}$. ∴ $g=4\pi^2l\div t^2$.

In accurate work it has to be recognized that the simple pendulum does not exist, and two modifications are employed :—

(*a*) *Borda's pendulum*, a heavy ball swung by a fine wire from a knife-edge, and requiring only small measurable corrections.

(*b*) *Kater's reversible pendulum*, § 56, whose length between knife-edges when adjusted to swing in equal times from either end is the simple pendulum length.

With these swinging hour after hour in vacuo the experimental errors are very small.

Some values of g are : Equator 978·1, Lat. 45° 980·6, Greenwich 981·17, Edinburgh 981·54, Pole (calc.) 983·1 (dynes= 1 grm. wt., or acceleration in cm./sec.^{2}.

It was a pendulum with a bob filled with wood, wheat, etc. etc., that Newton used to satisfy himself that the earth attracts all substances proportionally to their masses. In gravitation nothing matters but mass and distance.

§ 31. From astronomical considerations Newton was led further to enunciate the **Law of Universal Gravitation**. *Any two particles of matter attract each other with a force proportional to the product of their masses and inversely proportional to the square of their distance apart.*

$$f=k\frac{mm'}{d^2}$$

The factor of proportionality k, now called the *Newtonian constant of gravitation*, is evidently (by putting $m=m'=1$ and $d=1$) the attraction in dynes between two masses of 1 grm. each, concentrated at points 1 cm. apart.

Fortunately it can be shown that the mass of a sphere attracts as if it were concentrated at its centre, even for points close to its surface.

§ 32 : **Determination of the Newtonian constant, k.** The classic Cavendish experiment, once popularly famous under the title of 'weighing the earth,' is a little complex, and the following method must serve here :—

A kilogram sphere of lead hung by a long wire from a balance high above. A 5-ton sphere of lead was built up with its centre 50 cm. below that of the suspended sphere, and its attraction appeared to increase the weight of the latter by $\frac{1}{6}$ dyne. The counterpoising weights were too far up to be appreciably attracted.

Here ·133 dyne$=k\times 10^3\times(5\times 10^6)\div 50^2$, from § 31.

$\therefore k=6{\cdot}66\times 10^{-8}$, or one fifteen-millionth of a dyne.

§ 33: The gradual firm establishment of the Law from highly technical considerations is well described in Airy's *Popular Astronomy*.

The application of g and k to calculating astronomical masses may be of interest:—

Earth. Using the mathematical expression of the law, f on 1 grm. on earth's surface$=g=981$ dynes

$$=\frac{1}{15{,}000{,}000}\times\frac{1\text{ grm.}\times\text{E grm.}}{(\text{radius}=637{,}000{,}000)^2}$$

$\therefore$ $E=5{\cdot}96\times10^{27}$ grm. or nearly six thousand trillion tons, giving a Mean Density 5·5 (twice the average of its crust).

Sun. Solar Pull on earth keeping it in orbit averaging 92 million miles radius, travelled in 1 year$=$(speed in orbit)$^2\div$radius of orbit (see § 35)$=$ $\therefore$ ·588 dynes per gramme.

$$\therefore\ {\cdot}588=6{\cdot}66\times10^{-8}\times\frac{1\text{ grm.}\times\text{S grm.}}{(\text{radius of orbit})^2}$$

which gives $S=1{\cdot}96\times10^{33}=\frac{1}{3}$ million times E.

Planets with satellites are treated similarly. Planets without satellites occasionally 'perturb' each other's motion when close (e.g. Venus perturbs the earth, and the perturbation of Uranus led to the discovery of Neptune).

Moon. It can be shown, much as in § 532, that the tide-raising power is proportional to (mass of attracting body$\div d^3$). Hence moon's mass in terms of sun's.

Stars. For a few double stars some information has been gleaned by telescope or spectroscope as to their speed and time of revolution. Hence the joint mass as for sun and earth.

§ 34. **Electrical attraction and gravitation.** Wildly swinging pith balls are commonplace, while but few readers will have seen the gravitational attraction of masses actually demonstrated. Why, then, does one ignore electrification as a possible astronomical tie?

Electrical attraction acts on the surface only, gravitation acts on every particle however deeply buried. The surface of a pith ball is dozens of times its mass, but the mass of a core boring through the earth is 7×10^9 the area of its end, off which rubbed sealing-wax might lift a little dust.

EXAMPLES.—CHAPTER IV

1. A smoked plate fell in front of a tuning-fork making 256 vibrations per second and 32 complete waves were counted in 7·7 cm. from the start. Calculate g.

2. A plate fell in guiding grooves past a fork making 540 vibs./sec. and 90 waves were counted in 5 in. from rest. Calculate g and observe the pernicious effect of friction in the grooves.

3. If, as usual, starting-point on plate is blurred, how proceed ?

Mark off two successive sets of m waves, Fig. 5, and measure their lengths BC, CD. The time spent on each is mP, and if t is spent before reaching B—

$AB = \frac{1}{2}gt^2 \quad AC = \frac{1}{2}g(t+mP)^2 \quad AD = \frac{1}{2}g(t+2mP)^2$

Subtracting, $BC = gtmP + \frac{1}{2}gm^2P^2 \quad CD = gtmP + \frac{3}{2}gm^2P^2$

$\therefore CD - BC = gm^2P^2 \quad \therefore \underline{g = (CD - BC) \div m^2P}$

4. The masses on an Atwood's machine each weighed 228 grm. When one was overloaded with 3 grm. it fell 290 cm. in 9·5 sec. Calculate g.

$$s = \tfrac{1}{2}at^2,\ 290 = \tfrac{1}{2}a(9{\cdot}5)^2\ \therefore\ \text{acceleration } 6{\cdot}42$$
$$a = mg/\text{total mass}\ \therefore\ 6{\cdot}42 = 3g/459\ \therefore\ \underline{g = 982}$$

5. Explain what is meant by the conservation of energy and the conservation of momentum.

Two equal masses are attached to the ends of a string passing over a light frictionless pulley. One is supported on a small table, the other raised 10 cm. and let fall freely through that distance. Find velocity of weights after string becomes tight. [L.]

6. Atwood's masses each weighed $5\frac{1}{4}$ lb. and rider 4 oz. ; it falls 6 ft. before being detached and the masses move 9 ft. in the next 3 sec. Calculate g. [St. A.]

7. Atwood weights 470 and 490 grains move 3 ft. in 3 sec. from rest. Find g. [A.]

8. Atwood weights move 100 cm. in 5 sec. starting from rest. One weighs 1 grm. more than the other. Given $g = 981$, calculate mass of each. [L.]

Chapter V

MOTION IN A CURVE

§ 35. It was an ancient doctrine that "circular motion was perfect," but now we hold, with the first Newtonian law, that a body departs from a straight line only because it is given momentum, i.e. force acts on it, in some other direction. Continuous supply of sideways momentum results in continuous change of direction, **Movement in a Curve.** The greater the rate of supply the sharper the curve, but the greater the original momentum the less noticeable is the disturbing effect and the flatter the curve.

To find a relation between initial momentum, transverse supply of momentum, and curvature of path, Fig. 8.

The curvature of any curve, though it gradually varies, is always that of the 'circle of curvature' which just fits it very near the spot under consideration. For an instant the particle is moving in a circle, though it may soon change.

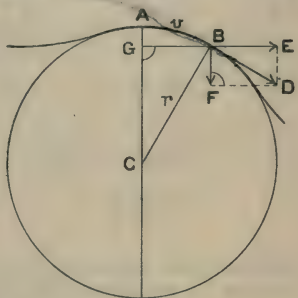


Fig. 8.

It will therefore suffice to study a *circular* path only, which has the constant curvature $1/r$, the reciprocal of its radius. The particle is always moving at right angles to the radius joining it to a fixed point, the centre.

Take a 'particle,' mass m (a train, for instance) moving at constant speed v round a circular curve, radius r. Let it travel AB in 1 sec., AB$=v$. Draw the tangent BD to represent mv at B, resolve this into component momenta BE perpendicular to AC and BF downwards, parallel to AC.

At A the particle had no downward movement, 1 sec. later

at B it has downward momentum BF, $\therefore$ a force has been acting on it$=$BF/BD of mv.

Triangles BFD and BGC are similar, BF/BD$=$BG/BC.

If AB is a *very small arc,** BG becomes$=$arc AB$=v$ and BF points very nearly to C.

$$\therefore \textbf{ force towards centre} = \frac{\mathrm{BF}}{\mathrm{BD}}mv = \frac{\mathrm{BG}}{\mathrm{BC}}mv = \frac{\mathrm{AB}}{\mathrm{BC}}mv = \frac{mv^2}{r}$$

That is, if m at speed v be acted on by a force (i.e. supplied with momentum every second) mv^2/r at right angles to its motion and always directed to a fixed point, it will move round it in a circle of radius r with unchanging speed.

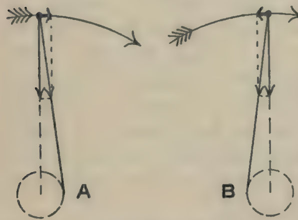


Fig. 9.

Fig. 10.

And to compel a body to move in a circle this force must be continuously applied, say by a string, or by the walls of a cup containing rotating liquid, by the grinding together of rails and wheel-flanges, or by gravitational or any other pull.

From our youth up we know 'Centrifugal Force' and we all say that a body moving round 'exerts centrifugal force.' By all means, but recollect that *a body will not move in a circle unless it is forced.* The 'centrifugal force' is the reaction of the inert mass to the active force which constrains it to move in the curve.

mv^2/r shows that increase of r, as by letting the string slip through your fingers, reduces the necessary constraining force. Letting go altogether the body moves off in a straight line (r infinite) and pays no more heed whatever to the original centre. The yarn of the farmer who crooked his gun and shot round and round the stack is better found than founded.

* For simplicity, AB has been taken as the motion in 1 sec.; by taking it say one-millionth of this the reader will see that the above result is not a mere approximation.

§ 36. If the force is not at right angles to the body's motion it can be resolved into two, one at right angles and the other in the line of motion. The former curves the path, the latter alters the body's speed in it. To swing a weight faster the hand moves in a small leading circle as at A, Fig. 9; slower, in a lagging circle B. Notice how bicycle spokes, which transmit driving or braking effort, are tangent to just such a small circle.

Fig. 10, of the earth's elliptical orbital motion, should be studied in illustration of this. Notice e.g. how autumn (below) is the (accelerated) 'fall' in more senses than one.

§ 37. **Particular case of circular motion. Conical pendulum.** The bob of a 'conical pendulum' goes round in a horizontal circle while the string sweeps out a cone, Fig. 11, elevation and plan.

Resolve, by parallelogram law, the pull of the earth mg into two components, one along the thread, the other horizontally inwards towards C, and, by similar triangles, $= mg \times BC/CA$. This supplies the steady force mv^2/BC necessary to keep the bob moving at v in the circle rds. BC.

$$\therefore\ mg\frac{BC}{CA} = \frac{mv^2}{BC}$$

Cross-multiply, divide out both sides by $v^2 g$, take square root and multiply by 2π.

$$\therefore\ 2\pi\frac{BC}{v} = 2\pi\sqrt{\frac{m\,.\,CA}{m\,.\,g}}$$

Now $2\pi\,.\,BC$ = length of circular path, which divided by v gives the time of 1 revolution,

$$T = 2\pi\sqrt{\frac{m\,.\,CA}{m\,.\,g}}$$

(This shows that if T is diminished by driving round faster, CA must diminish, i.e. the bob rises and opens out as in that familiar example, the steam-engine governor.)

Now if the angle A is very small, CA is very nearly equal to $CB = l$, the length of the pendulum.

$$\therefore \textit{for a small circle}\ T = 2\pi\sqrt{\frac{ml}{mg}}$$

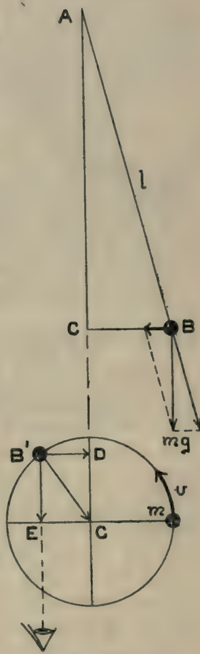


FIG. 11.

§ 38. **Simple pendulum.** Now notice, this holds nearly true for any sized circle provided it is still so small that the vertical rise of the bob is hardly perceptible, i.e. CA is not appreciably less than AB. We should have to watch a long time to detect any difference in the times kept by a metre pendulum swinging in a 5-cm. circle and in a 1-cm. circle.

It should make no difference then if the bob changes from one circle to another, i.e. changes its distance from C, during the swing. This means swinging in a little ellipse. Nor should it matter if the smallest circle touched (the breadth of the ellipse) vanishes altogether, and the bob travels to and fro along a short line.

A small, heavy bob swinging to and fro on a fine thread (of insignificant mass) constitutes a **Simple Pendulum,** and the preceding argument gives its time of complete small swing there and back in seconds,

$$=2\pi\sqrt{\frac{ml}{mg}}=2\pi\sqrt{\frac{\text{mass}\times\text{length}}{\text{weight}}}=2\pi\sqrt{\frac{\text{length}}{\text{gravity}}}$$

both being in foot, or both in cm. units. This is its **period of vibration or oscillation.**

If the swing widens and the bob lifts appreciably this gets farther and farther from the truth :—

Pendulum swinging in whole arc of	5°	10°	20°	40°
Loses per hour, seconds	·43	1·75	7	27

§ 39. **The Simple Harmonic Motion in a straight line.**

Referring to the plan in Fig. 11, the bob moving in the circle does so because there is a force mv^2/r pulling it toward the centre. Take the point at B′ and use B′C itself as the vector to represent this force; resolve it into two components B′D and B′E. Suppose we are looking along EB′, i.e. are out in front of the pendulum on a level with it, its motion controlled by B′E in the line of sight is invisible, while its right and left component motion appears merely as the *motion in a straight line of a particle controlled by a force* (=B′D) *towards the middle point and always proportional to its distance from it. This is a* **simple harmonic motion (S.H.M.).**

This, *the motion to and fro, along a diameter, of a foot of the perpendicular dropped on it from a point moving uniformly round the circle* practically coincides with the motion of a simple pendulum, as is easily seen by arranging two equal pendulums, letting one

fall along the diameter BCE and jerking the other out into a semicircle. Seen from a distance in front the one bob 'covers' the other all the way across their swing.

Evidently the *Period of the S.H.M.=length of circumference of circle÷speed in circle*$=T=2\pi\,.\,BC/v$.

§ 40 : **Tension in a revolving hoop**. 'Centrifugal action' causes in the rim of a revolving wheel or hoop, or in the driving-belt encircling it, a considerable tension. Notice how a boy's hoop, broken at the weld, 'opens out' as it runs faster downhill, or how the belt driving a circular saw, taut enough when at rest, bulges and hardly seems to touch the small pulley at full speed.

Considering a very small piece (say 1 cm. of mass m) of the circle, as in Fig. 12, the force that holds it to its circular path and prevents it flying on straight is the pull exerted on both ends of it by adjoining portions of the rim or rope. The two vectors must have the resultant mv^2/r ($=PR$) towards the centre C. They are tangents at the ends of a 1-cm. arc and therefore inclined to each other at a small angle=arc of 1 cm.÷radius $r=1/r=$the angle PQR in the parallelogram of forces=the small 'arc' PR÷the 'radius' PQ.

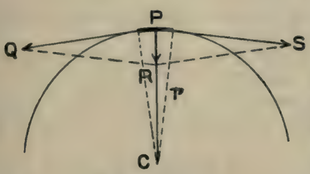


FIG. 12.

Hence the tension PQ is r times mv^2/r, or the *Tension in a rim or a rope travelling at speed v is* mv^2 *or [its mass per cm.×square of its speed in cm./sec.] dynes.*

Beyond a peripheral speed of 2 miles per minute a cast-iron rim is likely to fly to pieces; ropes and belts are never run beyond 1 mile per minute.

EXAMPLES.—CHAPTER V

1. A closed railway carriage moves uniformly along (1) a straight line, (2) a circle. Can an observer inside determine anything as to motion of carriage in either case, and how ? [L.]

2. Investigate the motion of a simple pendulum in a small arc. [L.]

3. Calculate the length of the 'seconds pendulum' (period 2 sec.) at a place where $g=981$.

4. Calculate the period of a pendulum 1 ft. long.

5. A 500-grm. weight is being whirled round once a second on the end of a string 1 m. long ; calculate the average pull on the string.

6. A bucket of water is swung overhead, distance of water surface from shoulder is 1 m. ; what is the least speed to prevent water falling ?

7. Calculate the difference in g at the poles and equator of a sphere 6400 km. radius rotating once in 24 hours.

8. Calculate pull of Sun on 1 grm. of Earth, if the latter revolves in a circle 92,000,000 miles radius once a year.

9. What speed would cause a tension of 2 kg. (say 2 million dynes) in a hoop of wire weighing 2 grm. per metre ?

10. At how many revs. per min. may a steel ring 10 cm. radius (1 cm. cross-section and) weighing 8 grm. per c.c. be spun without bulging if its yield-point is at 5 tons per sq. cm. ?

11. If the Equator were encircled by such a steel ring, which was then slipped off and left rotating in space, what would have to be its strength ?

CHAPTER VI

EQUILIBRIUM OF FORCES

§ 41. According to the Newtonian first law a body unacted on by force remains at rest or else moves uniformly in a straight line. Any application of force upsets this condition. Now we know perfectly well that every body on earth is being affected by at least one force, the gravitational pull of the earth, and every moving thing is also being retarded by a force due to friction. Clearly, to remain at rest a body must be constantly acted on also by some other force which just neutralizes the pull of the earth, and to travel at uniform speed a body, e.g. a train, must in addition be constantly acted on by some force which just neutralizes friction.

Hence when an actual body behaves as if free from forces altogether it is said to be 'in equilibrium' under the action of all the forces actually exerted on it; or all the forces concerned form 'a system in equilibrium.' The study of these constitutes **Statics**.

It has been insisted all along, however, that force is momentum supplied per second, and consequently the forces acting when a body is visibly changing its motion in speed or direction—a falling stone, a stopping train, a piece of a revolving wheel—form just as much a system in equilibrium as when the body is at rest or moving steadily. Only, *one* of the vectors concerned, *one* of the arrows in the diagram, happens to be not a 'feelable' force, but its equivalent a visible change of momentum, once called the *vis inertiæ* of the body, its mass multiplied by [—] its acceleration. The diagram of vectors is perfectly unchanged.

Coming to the simplest possible case, the third law assures us that every single force forms part of a system in equilibrium, for equal and opposite to it is a reactive force. Your weight presses on the ground and the ground presses on your feet, the air drags on the train and the train drags the air forward, you press forward the ball and the ball presses equally back on

your hand, telling you that it is absorbing momentum for flight.

But this individual treatment of forces leads nowhere; they must be grouped. In considering the equilibrium of a body it is convenient to separate all the forces into two groups, viz. those exerted by the body, and the reactive forces on the body; *either of these groups must form a system in equilibrium with itself.* Usually the equilibrium of the forces acting on the body is considered.

The reader must be warned at once that very particular care is necessary to avoid mixing up members of these two groups. Your weight, for instance, is the pull of the earth acting *on you*, but the downward pressure of your feet on the floor is *not on you*, what comes into reckoning here is the reactive upward pressure of the floor. When you jump it is this that lifts you (though of course you call it into being by first of all compressing the elastic floor harder than usual, and you provide all the energy); failing the reaction, as in water, you cannot jump. This increased reaction shows very plainly in jumping off a spring-board or a weighing-machine. It provides the force *ma* acting on the body which is directly opposed by the *vis inertiæ*, mass$\times$(—acceleration) already referred to. It is only during acceleration that the mass of a body comes into account.

§ 42. The **Equilibrium of a particle** may be maintained either by forces all in one line or by forces in different directions.

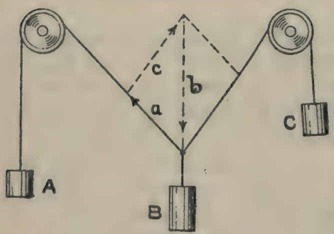


FIG. 13.

With forces in one line their algebraic sum$=0$, any one is equal and opposite to the algebraic sum of the others.

With forces in various directions their vector sum$=0$, any one is equal and opposite to the resultant of the others, to the diagonal of the parallelogram, *or* it must be the closing side of the triangle or polygon drawn the *same* way round as the others. *Three or more forces acting at a point are in equilibrium when they are proportional to the sides, parallel to them and all drawn the same way round, of a closed triangle or polygon.*

For instance, in the apparatus of Fig. 13 the knot settles to

rest when the three weights A, B, C are exerting forces on it parallel and proportional to the sides *a*, *b*, *c*.

§ 43. **Equilibrium of a body.** A rigid body provides a sort of framework to which the forces can be attached before reaching their common point. As with a particle, if they are all in one line their algebraic sum must vanish. If not in a line, then when prolonged to meet one another all three must meet in one point and there obey the triangle law. But there is now a third case, the common point may be 'at infinity,' the forces being parallel to one another. Evidently the algebraic sum must be zero, but this is not now a sufficient condition of equilibrium and the Principle of Moments must be introduced.

Principle of Moments.

In Fig. 14 let $OA=a$ and $OB=b$ be two forces with resultant $OC=c$. The triangles OAC and OBC are equal in area; the area of a triangle$=\frac{1}{2}$ base$\times$perpendicular height; hence $a\times CD=b\times CE=$twice the area of either triangle.

The product of a force and its perpendicular distance from a point is called the Turning Moment of the force about the point.

Hence if two forces are acting at a point their turning moments about a point in the line of their resultant are equal and in opposition. (*Any* point in the resultant, for $C'D' : C'E'=CD : CE$.)

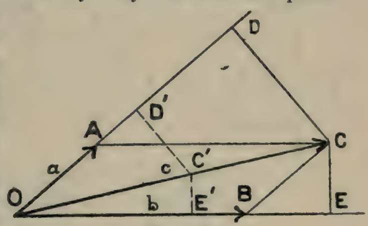


FIG. 14.

This resultant reversed keeps the point O and the whole system in equilibrium, i.e. a rigid body DCE acted on by OA, $-OC$ and OB would be kept in equilibrium.

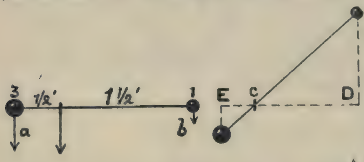
As the angles between the forces diminish till finally they become parallel DCE straightens out and the equilibrating force becomes $-$(the sum of the others) and acts at a point in the perpendicular distance between them such that their turning moments about the point are equal and opposite.

Thus the condition for the equilibrium of a body under the action of parallel forces is that their algebraic sum is zero and that the algebraic sum of their turning moments about any point is zero.

§ 44. **Centre of Mass.** As an instance, let *a* and *b* be the weights of masses of 3 lb. and 1 lb. attached to the ends of a 2-ft. light bar, Fig. 15. Their resultant will act vertically through a point $\frac{1}{2}$ ft. from the 3 lb., since the moments about this point are $3\times\frac{1}{2}$ and $1\times1\frac{1}{2}$ opposite ways. At this point the bar must be supported, the whole weight of 4 lb. appears to act there whatever the tilt of the bar, for 3 lb. $\times$ CE still $=-1$ lb. $\times$ CD.

This point is the **Centre of Gravity (c.g.)** or **Centre of Mass (c. of m.)** of the rigid body. At rest, or moving in a straight line, the whole mass acts as if it were concentrated at this centre; supported there the body rests indifferently in any position, struck there it moves straight off without turning.

Hence to calculate the position of the mass-centre of any number of masses in line, take the sum of the moments of their weights about any point in the line, and equate this to the moment of the total weight acting as at the c. of m. Thus masses arranged on a bar, 1 at 0, 2 at 1 ft., 3 at 2 ft., and 4 at 3 ft., have a total moment $1\times0+2\times1+3\times2+4\times3=20$ lb. $\times$ ft., which $=$ (total 10 lb.) $\times$ 2 ft., $\therefore$ c. of m. is 2 ft. along bar from the 1 lb.



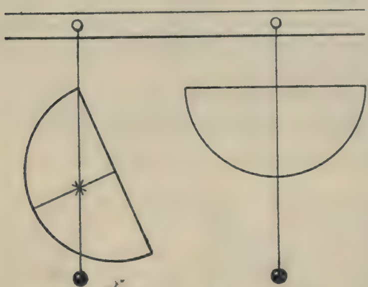


FIG. 15.
FIG. 16.

In practice the body (e.g. semicircle of Fig. 16) can be hung by a thread which supplies a vertical force passing of course through the point of support and the c. of m. The sum total of the moments of all the particles in the left-hand half about any point in the plumb-line = ditto of right-hand half. Then hanging from another point the new plumb-line cuts the first in the c.g.

Sometimes, of course, symmetry points out the c.g. It is at the geometrical centre of *uniform* bars, rectangular blocks, rings, etc., and, as in the last case, is often not situate in the solid material at all.

§ 45. From the way it has been derived it is obvious that the principle of moments is not confined to parallel forces, and it is often convenient and sufficient to use it with forces at angles rather than to draw their parallelogram diagrams. Levers, etc., are treated in both ways below.

Levers. The typical lever of theory is a straight, rigid bar (crowbar) on which act three parallel forces usually called the ' weight, w,' the ' reaction of the fulcrum, f,' and the (power, or better) ' pull, p.' More or less disguised levers build up the greater part of machinery.

CROWBAR, Fig. 17, **A**. Drawing XYZ perpendicular to the forces

$$w \times \mathrm{YX} = \text{and opposes } p \times \mathrm{XZ}$$
$$(\text{and } f = \text{and opposes } w + p)$$

XY being short w lifted may be large.

In practice the forces are rarely parallel, then :—
Either, Fig. **B**, draw XY, XZ perpendicular to the two forces

$$w \times \mathrm{XY} = \text{and opposes } p \times \mathrm{XZ},$$

or, Fig. **C**, producing the forces, the fulcrum reaction must meet them both in one point, hence its magnitude and direction by the parallelogram law. This gives the fuller information that f is not simply vertical, but can be resolved into vertical and horizontal components, the latter of which must be supplied by friction of the fulcrum-block on the ground, or by pressing your toe against it.

BENT LEVER, *Hammer drawing nails*, Fig. **D**. Draw XY, XZ perpendicular to resistance of nail and pull of hand,

$$w \times \mathrm{XY} = \text{opp. } p \times \mathrm{XZ}$$

Or the dotted parallelogram gives the same result and the further information that the reaction f is its (oblique) diagonal.

In a **second way of using** both the straight and bent levers the fulcrum is at the end and the ' weight ' in the middle, producing what are sometimes called ' levers of the second order.'

In Figs. **E** and **F**, $w \times \mathrm{XY} = \text{opp. } p \times \mathrm{XZ}$,

$$\text{and } w = \text{opp. } f + p.$$

These two uses increase force ; the **third way of using** levers diminishes force and increases motion, *p and w change places*, see Figs. **G** and **H**. **H** is sometimes called a ' lever of the third order.' Of these types are the levers which convert the small

movements of strong muscles into the rapid movements of our extremities, see the remaining figures.

Example 1. What force applied to a locomotive crank 13 in. long produces a tractive force of 10,000 lb. when the driving-wheels are 6 ft. 6 in. diameter ?

This is virtually a lever with arms 13 in. and 39 in. long, ∴ force = 30,000 lb. wt.

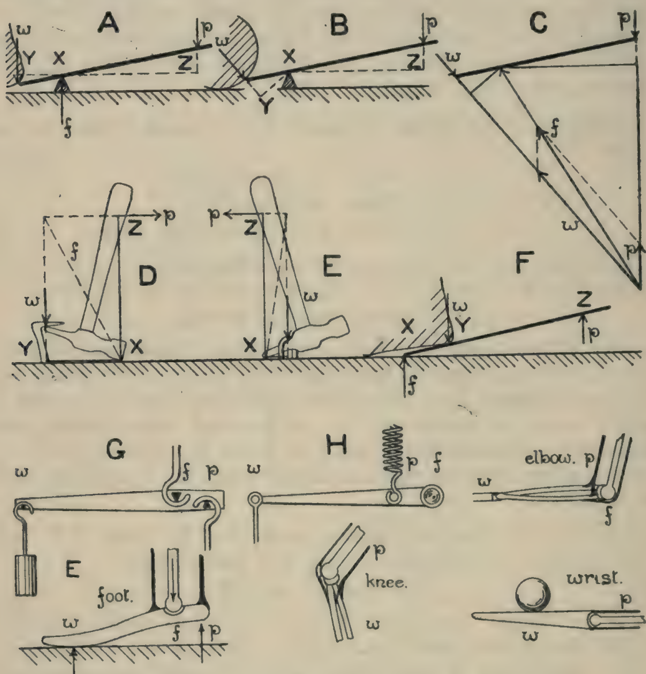


Fig. 17.

Ex. 2. A 10-stone man stands 'on tiptoe' on one foot. From 'toe' to ankle-joint is 6 in., thence to attachment of tendon of Achilles 2 in. Find pressure at joint and pull in tendon.

The forces acting on the foot are shown in Fig. 17. w presses up at toe, f down at joint, p up at heel. Taking moments about the fixed point (on floor)

$$6'' \times f = \text{opp. } 8'' \times p. \quad \therefore\ p = \tfrac{3}{4} f.$$

$$\text{Also we have } p + w = \text{opp. } f. \qquad \therefore\ w = \tfrac{1}{4} f.$$

$$\text{Hence } f = 40 \text{ stone}, \ p = 30 \text{ stone}.$$

[This is a tricky problem, because, standing flat, f is the man's weight plus any little tensions in the front and back muscles of the leg, and one is apt to forget that contraction of these greatly increases f. If f were vertical and muscles relaxed heel and toe would carry $\frac{3}{4}$ and $\frac{1}{4}w$, but before one can safely rise on tiptoe the weight has been transferred forward and the calf muscle is already pulling hard.]

§ 46. **'Virtual Work.'** There is in connection with the other levers of our anatomy a great difficulty in saying just where and in what direction the muscle pulls them. In more complex mechanisms too the construction of force diagrams becomes tedious. The difficulty can be escaped by using the so-called **Principle of Virtual Work.** This is merely a way of applying the principle of the Conservation of Energy to purely mechanical problems.

Let the mechanism, however simple or complicated it is, make a *small* movement, so that one of the forces presses forward and does work on it. Then the machine gives out *an equal amount of work* at the other end by pushing back the force there through a distance obtained from the geometry of the machine, hence

last force × *distance moved against it*
= *first force* × *distance it pushed forward,*

or the forces are in the inverse ratio of the distances they move *in their own lines of action.*

No deduction is made for frictional loss until the calculation is ended, then according to the nature of the machine a percentage correction based on experience is subtracted. The remaining output of work ÷ work put in = the Efficiency of the machine.

[It is important to note that a *very* small movement will not disturb the equilibrium of a system, even if unstable, enough to bring kinetic energy into calculation. In most machines the equilibrium is 'neutral,' § 51.]

Of these three methods — vector diagram, moments, virtual work—sometimes one sometimes another happens to fit the particular problem easiest. Two instances of virtual work follow, the first equally easy to solve by moments, but the second troublesome to tackle any other way.

§ 47. Examples continued :—

Ex. 3. A man pulls an oar with a force of 40 lb. wt., rowlock is 2 ft. away and middle of blade 5 ft. beyond it. Find forces acting on oar.

In Fig. 18, Ex. 3, let the oar rotate a very little about the rowlock.

'Virtual' Work $p \times ZZ' = f \times XX'$.

By similar triangles $ZZ' = \cdot 4XX'$.

$\therefore f = \cdot 4p = 16$ lb. wt. and $w = f + p = 56$ lb. wt.

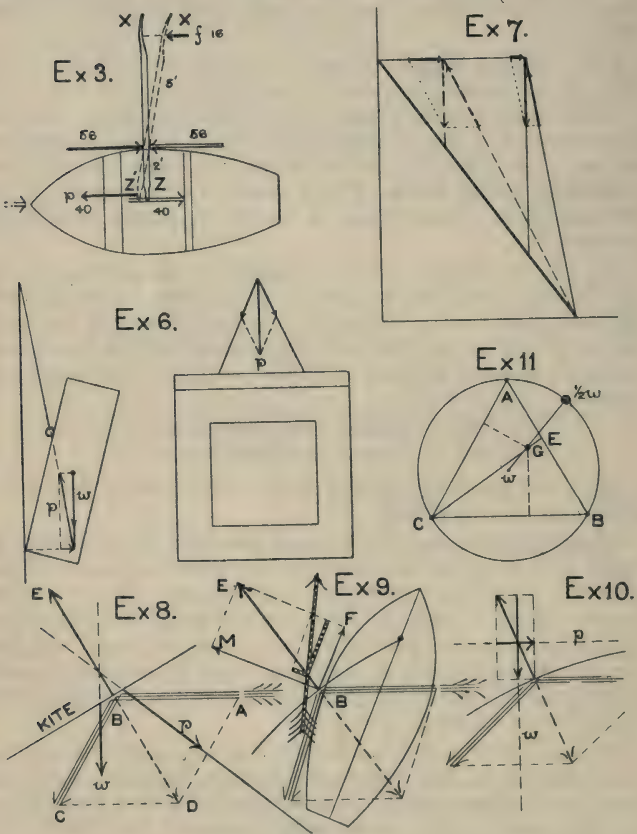


FIG. 18.

Ex. 4. A truck is shifted by applying a crowbar to the wheel, bar is pushed down 2 ft. with average force 60 lb. wt. and truck moves 6 in. What is its frictional resistance?

$2 \times 60 = \frac{1}{2} \times 240$, 240 lb. wt.

Ex. 5. In Ex. 3 how are the forces used in the boat ?

In Fig. 18, Ex. 3, forces acting *on boat* are shown in double line. 56 lb. forward action of oar on rowlock and 40 lb. backward push of rower on stretcher and seat leave 16 lb. propulsive force (really the forward reaction of water on oar). The 16 lb. is not immediately applied to the water resisting the boat, but is temporarily partly used in increasing momentum of boat and rower. During the return stroke this momentum is being dissipated. The rower, who took more than his share in swinging bowward, i.e. faster than the boat, now gives it up by moving sternward.

Ex. 6. Why do pictures hung in the usual way lean forward ?

In Fig. 18, Ex. 6, the c.g. of the thick frame cannot lie in the wall whence spring the supporting tension and a small nearly horizontal reaction. Taking moments about the corner in contact, the moment of the weight towards the right must be compensated by a moment of the string to the left, and to get an ' arm ' the string must lie out from the wall. The three forces meet in one point and are there proportional to the sides, drawn parallel to them, of a triangle of forces. The front view shows the division of p between the two strings.

Ex. 7. A light ladder stands on a rough pavement and leans against a smooth wall. A man climbs the ladder. Prove that its tendency to slip increases as he ascends.

In Fig. 18, Ex. 7, the *smooth* wall can exert only a reaction perpendicular to itself (having no component capable of resisting slip). The man's weight w presses on the ladder vertically ; the reaction at the foot must pass through the common point. It therefore slants more and more as the man ascends and may presently require a larger horizontal component than friction on the ground can supply. (If weight of ladder is taken into account the vertical force acts through c.g. of man and ladder, and moves slower than he does.)

Ex. 8. What are the forces acting on a kite ?

Wind brings per second momentum AB, Fig. 18, Ex. 8, up to the kite horizontally, the air must go away somewhere, suppose it is reflected off carrying momentum BC with it. The force that converted AB into BC is BD (ABD is a triangle of forces). This is the force applied to the wind by the resisting kite, therefore the equal and opposite BE is applied to kite by wind. This force, the weight of kite w and pull of string p act through one point. Evidently string and vertical are on opposite sides of perpendicular to kite. Tail acts as anchor to check oscillations.

Ex. 9. How can a boat sail more or less up-wind ?

By the same argument of wind coming and going we get a force BE acting on the sail, Fig. 18, Ex. 9. Resolving this into two, the forward component BF propels the boat along her keel, the beam component BM causes leeway. If, as in a tub, sideways motion through the water is as easy as forward, the whole drifts down-wind ; but if length, leeboards, keel, etc., make sideways motion more difficult, the beam force produces only a disproportionately small velocity (dotted vectors) and the boat makes a course only a little to leeward of her nose.

Ex. 10. What forces act on an aeroplane?

Thrust p from propeller, weight w, and reaction from the air that the plane meets and drives down. For the whole machine in equilibrium their three resultants pass through a point, Fig. 18, Ex. 10.

Ex. 11. Fig. 18, Ex. 11, is the plan of a three-legged table on whose edge is a weight say half that of table. The c.g. is obviously at G, $\frac{1}{3}$ radius from centre. To find the pressure on each leg :—

Either by levers—draw CG to meet AB in E, then EG/CE of weight presses on C and CG/CE at E. The latter again divides between A and B in the inverse ratio of the distance of E from them.

Or by virtual work—lift each leg in turn and find out what fraction of this distance the centre of mass lifts. (Simply scale perpendicularly to line joining fixed feet.) Pressure on leg = this fraction of whole.

With a four-legged table the problem cannot be solved. A stiff table probably does not stand on all four until warped by heavy loading.

§ 48. The lever reappears in the 'wheel and axle' and throughout all 'gearing,' as in Fig. 19. Problems relating to **Machines** are all most easily solved by 'virtual work.' Pull : weight = distance weight moves : distance pull moves. With forces tangential to circles (real wheels, or circles of motion of capstan-bars or cranks) the distances are obtained from circumferences and speeds of rotation; with linkwork, toggles, cams, etc., graphical constructions or card-board models may be necessary. Calculations of this sort are easy; neglect of them, and the use instead of geometrical force diagrams too difficult for the inventor, has resulted in all sorts of mechanisms promising energy for nothing, from the many ingenious 'Perpetual Motions,' whose invariable failure was part of the foundation of the Principle of the Conservation of Energy, to such modern quackeries as crooked cranks, 'patent lever chains,' etc. Bold curves and complicated movements are so very impressive, but it is

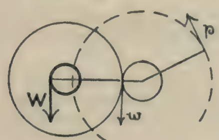


FIG. 19.

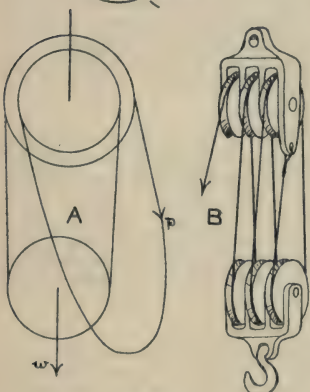


FIG. 20.

only the short, straight progression of the force in its own line that counts.

The **differential pulley-block**, Fig. 20 (A), is a useful sort of 'Wheel and Axle.' When the double wheel goes round once it lets down one side of the chain a distance equal to its smaller circumference and hoists the other a distance equal to its larger, the net shortening of the chain is their small difference and the pulley in the bight of the chain rises half this much, hence w is much greater than the hand pull p. In this machine as usually made the chain friction exceeds 50 % and the load cannot run down of itself.

Ordinary **multiple pulley-blocks** contain sets of two or three *independent* wheels. A rope is rove through and through each block alternately in a way familiar to everybody. It is only necessary to count up how many (n) portions of the cord are pulling on the movable block, each pulls with the same p (barring friction), then $w = np (= 6p$ in Fig. 20 (B)).

§ 49. **The inclined plane**, Fig. 21. On the slope of a hill a body is held in equilibrium by its weight, the reaction of the plane perpendicular to its surface, and a pull up the slope (which may often be merely friction). These form a triangle of forces DEF similar (entirely at right angles) to the large triangle ABC. Hence pull along slope : weight = vertical rise : actual length of slope. Consideration of work done against gravity, the same whether dragged up slope or lifted vertically, also gives this result.

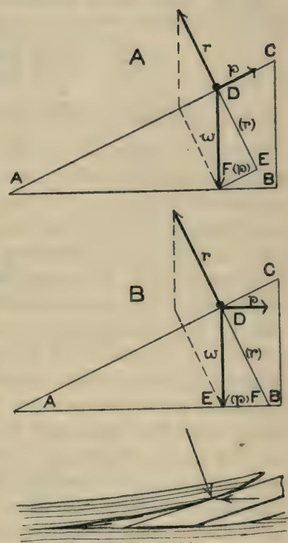


FIG. 21.

Ex. 12. Find the force required to drag a $\frac{1}{2}$-ton wagon up an incline of 1 in 20, and the horse-power at $7\frac{1}{2}$ miles per hour.

$$\text{Force} = \tfrac{1}{20} \times 1120 = 56 \text{ lb.}$$
$$\text{Power} = \text{speed} \times \text{force} = 11 \times 56 = 616 \text{ ft.-lb./sec.} = 1{\cdot}12 \text{ h.p.}$$

If the inclined plane is driven under the 'weight' by a force parallel to its base the force triangle becomes that of Fig. 21 (B)

and horizontal force : vertical force = height : horizontal length of plane.

This is the action of a *wedge*, type of all nails, knives, axes, and chisels (lower diagram).

Wrapping the slope round a cylinder gives the *Screw*. The pull at the end of a long handle moves tangentially round in a circle a distance $2\pi r$ while the screw advances its 'pitch'; hence $w=(p \times 2\pi r) \div$ pitch.

Ex. 13. A copying-press has a $\frac{1}{2}$-in. pitch screw and two handles 5 in. long, 14 lb. is applied at right angles to each. What is the pressure on the book ?

$$w = 14 \times 2 \times 2\pi \times 5 \div \tfrac{1}{2} = 1760 \text{ lb. wt.}$$

Ex. 14. What is the thrust of a steamship's propeller which absorbs 5000 h.p., has an effective pitch of 20 ft., and makes 70 revs. per min.?

$$w \times 20 \times 70 = \text{ft.-lb. per min.} = 5000 \times 33{,}000$$
$$w = 118{,}000 \text{ lb. wt.}$$

§ 50. Internal actions do not affect external equilibrium.

It is a theorem not always easy to credit that the equilibrium of a system relative to the exterior is quite unaffected by disturbances inside it. Yet since activity and reactivity are equal and opposite, if there is no chance of reaction from the exterior there can be no outward action, and for every internal action there must be a balancing internal reaction.

Leaning forward in a boat causes the boat to move backward, both actions stop simultaneously, the centre of mass has not moved. Once balanced on one foot your own exertions will not topple you over, unless you press on the wall or the wind, or on the earth with some other part of your foot. The high-diver, once he has left his board, is as powerless to alter his rotary head-over-heels momentum as to modify his speed of fall. If you lean over sharply on a bicycle the machine merely tilts the other way, steering is unaffected. The natural swaying of a bicycle from side to side both necessitates and makes possible its steering, if compelled to suddenly steer away from the side it happens to be falling you have to pay for it directly after by a hasty swerve towards that side. All you do is to maintain its natural unstable equilibrium; failing that, gravity compels 'the system' to seek other points of support.

Again, contrary to general belief, sudden pressure of the rider's foot on the bicycle pedal cannot cause side-slip unless the pressure arises from actual *motion* of the rider. Steady force is the rate of

change of momentum of a mass so great that its speed is imperceptible. (See § 18, of which the present is really a continuation.) There is no such mass on the bicycle. The pressure on the right pedal is either balanced by pull on the right handle-bar or saddle-grip, so that on the whole no momentum passes into the machine, or else there is left-handed roll of the rider and right-handed roll of the machine, the wheel slips to the left unless, as usual, there suddenly arises frictional resistance enough to transfer all this momentum to the great earth (§ 18).

In all the many apparent contradictions of this principle which may come to mind, careful scrutiny will show that a reaction in some unsuspected direction, often due to friction, is the hidden key.

§ 51. Stable, neutral, and unstable equilibrium.

If the equilibrium of any 'system' is momentarily disturbed by a *small* force what happens? Does it just oscillate a little and go back to its original condition—it is **stable**. Does it move continuously till friction quietly stops it—**neutral**. Does it upset altogether into a different condition—it was in equilibrium, but **unstable**.

The applied small force does work on it, giving it kinetic energy: if this is absorbed into potential energy, the motion tends to stop and then go back—stable; if it is just spent in friction—neutral; but if the system can lose potential energy by continuing the motion (producing more kinetic) the motion increases—unstable.

The difference is usually illustrated by solids under *Gravity*. To upset a flat-bottomed object, or a 'biassed' ball, or an egg lengthways, its centre of gravity has first to be raised (i.e. energy is stored as gravitational potential); these rock more or less and settle back—stable. An egg sideways rolls on with neither loss nor gain of potential energy—neutral; set on end the least displacement lowers its c.g., it expends potential energy, therefore moves faster and upsets.

[In common parlance many things are 'unstable' which are really stable, which have and hold a position of their own, but hold it feebly.

A case in point is the seismologist's 'horizontal pendulum,' which responds to the uprising of the earth's surface in the meadow when relieved of the weight of the morning dew. Another is the 'divining-rod,' the forked hazel twig's elastic tails are

pressing across the shortest distances between two parts of the hands, and a very little yielding of the underlying muscles may give them the chance of escaping sideways and agitating the twig. Again, explosives bear handling, but not much.]

The theorem, however, is of widespread application. The energy can be stored in any form whatever—spin, capillary, elastic, electric, chemical, etc.—and the test of what happens when disturbed is: If a system once started continues to move it is *always in such a manner as to reduce its potential energy.* The Principle of the Dissipation of Energy not only takes tax of its motions, it actually decides in what direction the motion shall go. Energy of motion is less 'available' than energy in store.

EXAMPLES.—CHAPTER VI

15. What are the conditions for three parallel forces to be in equilibrium? Explain how the necessity of each condition can be tested experimentally. [L.]

16. A cube of weight W rests on a rough floor; find force which, acting along upper surface, will tilt it. [L.]

17. A sphere of diameter 10 in. and weight $6\frac{1}{4}$ lb. is hung by an 8-in. string fastened to a smooth wall. Find tension in string and pressure on wall. [A.]

18. Draw a diagram to represent the forces in equilibrium on a tailless kite. [M.]

CHAPTER VII

ROTATION

§ 52: In the Rotation of a body it is evident that different portions of the mass contribute very differently to the total momentum and total energy of the motion, for those near the fixed *axis of rotation* move much less than the outer parts. The totals have to be got by adding together the mv or the $\frac{1}{2}mv^2$ of all the individual particles, a process called Integration and effected geometrically or by the devices of 'the calculus.'

The speed always quoted in rotation is the *angular speed*, q, with which any radius projecting at right angles to the axis changes its direction of pointing. This of course is the same throughout the body. Then $v=qr$; the linear velocity of a particle = angular velocity of body $\times$ distance r of particle from axis of rotation. [Putting v and $r=1$ the unit q is that which causes the end of a 1-cm. radius to move 1 cm. per sec. (1 radian per sec.). The distance round being $2\pi\times$radius, $q=2\pi\times$revs. per sec., e.g. q of the minute hand$=2\pi\times 1/3600$.]

The whole kinetic energy of a rotating body, the sum of $\frac{1}{2}mv^2$, $\therefore$ $=$sum of $\frac{1}{2}(mr^2)q^2=\frac{1}{2}$(sum of $mr^2)q^2=\frac{1}{2}\mathrm{I}q^2$, where I, the 'integral' of mr^2 throughout the body, is called its **Moment of Inertia**.

[Writing $\mathrm{I}=\mathrm{MR}^2$, M is total mass and R the '*Radius of Gyration*.']

§ 53: **Couples.** Inquiring how the body obtained this energy, it was by work done on it by forces. Not a single force, for that would have set it moving forward as a whole, but a pair of forces, equal and opposite though not neutralizing, i.e. not in the same line. Such a pair is called a **couple,** the perpendicular joining them is its *arm*, the product *one force* $\times$ *whole arm* = *turning Moment or Torque of the couple* (measured in dynes $\times$ centimetres at right angles; contrast ergs).

The two forces are felt by finger and thumb when turning a tap, key, or knob. With a crank, one seems to have vanished, but the necessity of holding light machinery firmly down shows that it is

present as a reaction in the bearings, merely causing friction, perhaps so much that a tap may hardly be turned by pressing on one end of its T handle only. A long crank enables the same turning moment to be applied with less pressure and less friction.

The fixed point in the arm of the couple may be between the forces (knob) or on one of them (crank) or outside them (gripping one end of tap-handle with thumb and finger and turning by pressing them opposite ways), it makes no difference to its moment. Every force exerting turning moment is one of a couple, e.g. the fulcrum reaction in a lever consists of the two forces which complete pull couple and weight couple. [Revise the definition of Moment of a force in § 43.]

Work done by a couple = torque × angle turned through (radians), or = either force × sum of distances both travel forward.

§ 54: **Moments of Inertia,** Fig. 22.

Some values of the integral I are, for bodies of mass M rotating

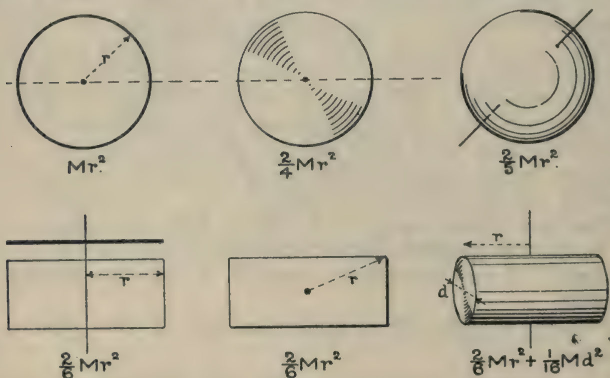


FIG. 22.

about a fixed axis through their centre of mass, and having r as extreme radius from it :—

Thin hoop * or hollow cylinder about usual axis perp. to circle	Mr^2
Disc * or solid cylinder ,, ,,	$\frac{2}{4}Mr^2$
Sphere	$\frac{2}{5}Mr^2$
Thin rod about centre, rectangle or rectangular block about central axis parallel to edge	$\frac{2}{6}Mr^2$

[** half as much when rotating about a diameter.]

Lengths along the axis do not come into calculation.

Rotation about an axis not through the centre of mass means that the body as a whole moves forward, turning as it goes. For instance, a flung stick has moved about your shoulder as centre; quitting your hand it analyses the motion into forward flight as a whole and rotation about its own middle. And in a month the moon travels about $1\frac{1}{2}$ million miles round us and rotates once on its own axis, as evidenced by the travel of sunshine round it.

The whole energy of such motion can be written in two parts, $\frac{1}{2}Mv^2$ due to the linear speed v of the centre of mass, plus $\frac{1}{2}Iq^2$ due to the rotation about it.

$$\text{e.g. a Rolling Ball has } q=v\div r, \text{ hence its Energy}$$
$$=\tfrac{1}{2}Mv^2+\tfrac{1}{2}Iq^2=\tfrac{1}{2}Mv^2+\tfrac{1}{2}(\tfrac{2}{5}Mr^2)v^2/r^2=\cdot 7Mv^2.$$

Or it can be written in one, recollecting that $v=q\times$ distance h of centre of mass from axis of rotation.

$$\begin{aligned}\text{Energy } \tfrac{1}{2}Mv^2+\tfrac{1}{2}Iq^2&=\tfrac{1}{2}M\,.\,h^2q^2+\tfrac{1}{2}M\,.\,R^2\,.\,q^2\\&=\tfrac{1}{2}M(h^2+R^2)q^2=\tfrac{1}{2}I'q^2\end{aligned}$$

where I' is a new moment of inertia about the new axis, and is evidently equal to the moment of inertia about the central axis + (mass $\times$ square of distance h between central axis and the new axis).

§ 55: **The Compound Pendulum,** Fig. 23. [The following

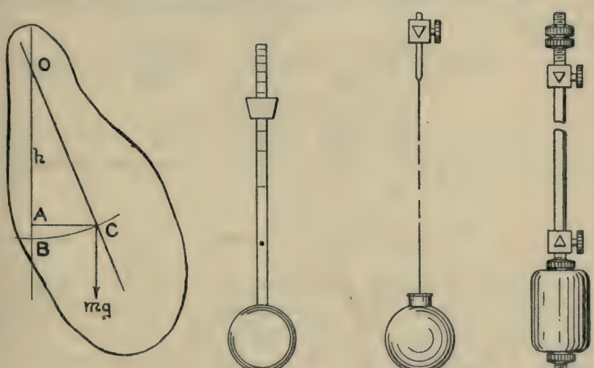


FIG. 23.

investigation is easily applied to any solid whatever oscillating under the control of gravity, magnetic or electric force, etc.]

A body of any shape makes *small* swings about a point O, h above its centre of mass C. About the point it has $I'=M(R^2+h^2)$.

At end of swing its mass-centre has been lifted $AB=(AC)^2/2h$ (§ 113) and its potential energy $\therefore = Mg\times(AC)^2/2h$.

At mid-swing this is converted into kinetic energy

$$\tfrac{1}{2}M(R^2+h^2)q^2.$$

These are equal, hence $\dfrac{AC}{q}=\sqrt{\dfrac{M(R^2+h^2)h}{Mg}}$

At mid-swing C has velocity qh and we have seen (§ 37) that this, maintained constant, would carry it round a circle of radius AC in the time of swing (straight or circular) $T=2\pi(AC)/qh$.

Hence $T=2\pi\sqrt{\dfrac{M(R^2+h^2)}{Mgh}}=2\pi\sqrt{\dfrac{I'}{Mgh}}$ where I' is about the actual axis of rotation and $Mg\,.\,h$ evidently = moment of controlling force about axis if body is held at right angles to force.

§ 56: **Kater's reversible pendulum** (Fig. 23, right) is a bar, with a bob at *one* end, which can be swung from axes (knife-edges), near either end in turn, distant h and k from its centre of mass. Its times of swing either end up are adjusted to be the same. Now *to prove the distance between the knife-edges, $h+k$, is the length of the theoretical simple pendulum having the same time of swing*:—

$$T=2\pi\sqrt{\frac{M(R^2+h^2)}{Mgh}}=2\pi\sqrt{\frac{M(R^2+k^2)}{Mgk}}$$

$$\therefore\ (R^2+h^2)/h=(R^2+k^2)/k.$$

$$\therefore\ (k-h)R^2\ =hk(k-h).$$

$\therefore$ either $k=h$, a symmetrical bar which gives no information,

or $\quad R^2=hk.$

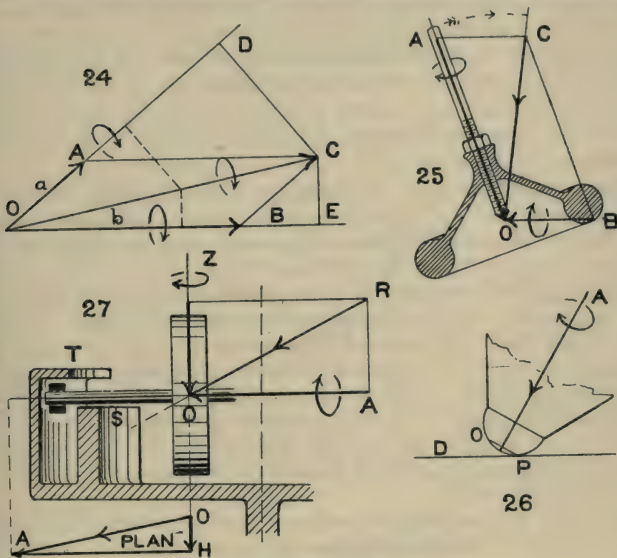
$$\therefore\ T=2\pi\sqrt{\frac{M(hk+h^2)}{Mgh}}=2\pi\sqrt{\frac{M(k+h)}{Mg}}$$

=period of Simple Pendulum of length $k+h$ [§ 38].

This is the instrument for determining g referred to in § 30. Half its length or more is omitted in Fig. 23. Beside it is Borda's pendulum, § 30, and to the left of this a common metronome pendulum which oscillates on the axis marked by a dot and has its period lengthened by raising the sliding weight, thus raising its centre of mass, shortening h, and so reducing the controlling moment.

§ 57 : **Rotors.** Rotations, angular velocities and momenta, and couples, can be represented like their rectilinear analogues by vectors, now called **rotors.**

The rotor is drawn along the axis of rotation, has a length



FIGS. 24, 25, 26, 27.

proportional to the quantity it represents, and looking in the direction in which it points the rotation is clockwise round it.

To combine rotors. The product of a rotor representing angular velocity into a line at right angles to it is evidently a rectilinear motion perpendicular to the plane of both, since angle $\times$radius$=$arc; $qr=v$. In Fig. 24 a and b are two rotors which cause the body to perform *small* rotations on the axes OA and OB, consequently the point C goes down through the paper a small distance $a\times$CD and comes up $b\times$CE. If these are equal C does not move, i.e. it is a point on the fixed axis of the resultant rotation. The reader will see that as in § 43 $a\times$CD$=b\times$CE, the diagram is a replica of Fig. 14, and the argument will show that OC, the diagonal of the parallelogram, is the resultant rotor of OA and OB. *Rotors, whatever they represent, combine according*

to the familiar parallelogram law. [Unlike linear motions, both immediately begin to change their directions, and any particular diagram is only ephemeral, but it shows the immediate direction of motion and that is enough for our purpose.]

Anyone who has ‘laid up’ a piece of electric flexible cord, or of rope, will understand how two or three rotors can combine into one, and an excellent instance of one resolving into three is afforded by a camera on a rather rickety tripod. Turning it about a vertical axis to alter the view causes all three legs to twist on their own axes (and is the severest test of a tripod's rigidity).

§ 58: **Spinning tops.** The foregoing will help us to understand the strange motions of a spinning top. In the first place, to get rid of the disturbing influence of gravity, consider a special top with its toe at its centre of mass. It rests indifferently at any tilt, it will be found to spin in any position without attempting to change it; when extremely well made it continues to point to a fixed star while the earth turns under it. Rotor AO, Fig. 25, represents its large angular momentum of spin (spinning right-handed as usual). Now attempt to push the stem away to the back of the paper; it resists strongly, and escapes by running round your finger so as to go off to the right. Your attempt was to turn the whole over about the axis (in the paper) BO, you acted on it with rotor BO.

AO and BO combine to produce a resultant rotor CO, that is, *the top tilts to the right and places its axis in this line.* Since AO is large BO must also be a fairly large couple. The stem presses back hard on your finger. If you continue touching, it runs round and round pressing all the while.

An ordinary top is supported below its centre of mass, and as soon as it tilts a rotor tending to topple it over comes into existence, just like that you supplied above. Its stem moves as described, at right angles to the expected line of fall. It now slants in a different direction, and moves at right angles to that, and so on, with the result that it sways round or *precesses* in a circle, the same way round as it is spinning. As the top slows down OA diminishes and the angle AOC resulting from the action of OB (constant for a constant angle of slant) increases, the top precesses faster.

The special top, Fig. 25, can also be supported above its centre of mass, so as to tend to stand up instead of fall down; it then precesses the opposite way.

The going to *sleep* of a top is caused by friction on the floor. Fig. 26 shows the toe of the leaning top; its point of support is not in the axis, but lies in the small circle OP which grinds round on the floor. Friction provides a resisting force pointing back through the paper and tending to trip up the top so that it should fall forward (out of the paper), i.e. produces a rotor such as DP in the paper. This compounds with the axial rotor and raises the top to a more erect position. The stubbier the toe the larger the circle OP and the quicker the top goes to sleep.

§ 59 : **The gyroscope** is simply a spinning top enclosed in a frame so that it can be handled in a greater variety of ways. The hind wheel removed from a bicycle, with the back step acting as handle, is an excellent gyroscope. When spun up it is easily carried by the finger crooked round the end of the step. Its weight causes it to slowly precess. If you turn round at the speed it suggests the axle remains horizontal, if you grip the step and resist precession it slowly sinks till the axle hangs vertically from your hand—without precession it cannot save itself from fall—if you attempt to accelerate precession it rises and ultimately stands sleeping on your hand.

The gyroscope of the Brennan mono-rail car has a pair of heavy fly-wheels spinning in vacuo about a horizontal axis across the car (rotor AO, Fig. 27) and supported in gimbals. When the car tilts to the right it lifts S (a shelf curved in plan to centre O) and presses it under the axle, which gets a frictional grip and runs forward along it, i.e. rotor ZO arises. The resultant rotor is RO, and the wheel makes strenuous exertions to set its axle in this direction, i.e. it presses very hard on S and tips the car back again.

The wheel is now slewed round, and to restore it to its normal position the shelf T on the car presently comes in contact with the top of the axle. But here a loose collar intervenes and permits free slipping, so that there is no horizontal push on the axle, only a small direct downward pressure. This causes the horizontal rotor HO seen in the PLAN underneath, and this combines with OA to slew the axle round as along HA, i.e. to bring the displaced axle back into the plane of the elevation.

A twin wheel, etc., on the right deals with tilts to the left. The wheels' objections to curves on the line are made to neutralize each other by suitably linking together their gimbal frames.

Many other things kept in position by spin will occur to everyone—hoops, rifle bullets, 'diabolo' spools, etc. It is noteworthy that if the moment of inertia is the same about all directions the

axis of rotation can wander anyhow ; a ball is always changing its axis of spin, and on a uniform spherical earth one would stay at home and wait for the north pole to come past.

The Gyro-compass, controlled by a gyroscope instead of a magnetic needle, is a new and valuable navigating instrument.

EXAMPLES.—CHAPTER VII

1. Calculate the moment of inertia of an iron ring 95 cm. radius and weighing 2000 kg.

2. Calculate the moment of inertia of an iron disc 90 cm. radius and weighing 600 kg.

3. The foregoing together form a fly-wheel making 240 revs. per min. Calculate its energy of rotation.

4. Explain angular acceleration and radius of gyration. Prove kinetic energy of rotating body $=\frac{1}{2}Iw^2$. [St. A.]

5. A top is spun by a string. When the length unwound is 60 cm. and the string has been pulled with a steady force of 2 kg., points on the rim of the top (3 in. radius) have acquired a velocity of 28·8 in. per sec. What is the moment of inertia of the top about its axis ?

6. Compare the energies of forward motion and of rotation in a hoop rolling on the ground.

7. Two spheres of equal weight, one of wood s.g. ·78 and the other of steel s.g. 7·8, roll down the same plane inclined 1 in 60. Compare their speeds after rolling 25 ft., and compare also their energies of translation and rotation respectively.

8. Calculate the time of swing of a thin rod 1 m. long about a point 10 cm. from one end. Show that is less than if swinging from one end, but becomes large near middle.

9. Calculate the moments of inertia of a bar magnet 10 cm. $\times$ 1 cm. $\times$ ·5 cm., weighing 40 grm., about the 3 axes through its centre parallel to its edges.

10. This magnet, broad face horizontal, swings as a compass needle under the magnetic control of the earth, with 10 sec. period. A brass rod of equal length, 1 cm. diameter and weighing 65 grm., is laid on it. This does not affect the controlling force. Calculate the new time of oscillation.

11. Describe the motion of the axis of a spinning top and show how it may be accounted for theoretically.

12. Prove that the steering-wheel of a bicycle running fast gyrostatically turns in the direction necessary to prevent falling, when the machine tilts.

13. The equatorial bulge of the earth forms virtually a heavy ring inclined to the plane joining earth and sun. Solar gravitation would pull the ring at rest into this plane ; the ring is spinning, show that the result is that the earth's axis precesses so that the pole moves in a circle [of $23\frac{1}{2}°$ radius] among the stars in the opposite direction to the earth's rotation.

CHAPTER VIII

FLUIDS

§ 60. Matter that can flow is fluid. This broad definition includes not only liquids (to which the name of fluids is popularly confined), but gases, streaming masses of sand, grain, etc., crowds of people, pitch and candle-wax in summer, even glacier ice and metals plastically yielding to excessive stresses.

Every particle, every 'drop,' say, of a fluid of course obeys the mechanical laws already described, but its individual motion can rarely be followed; it is lost in the crowd. Fluids are therefore studied collectively, their special Laws are laws governing the motion and equilibrium of multitudes of particles in close contact.

The sand, ductile metals, etc., referred to above differ from typical fluids in one most important respect. For they act as solids, and do not appreciably respond to stress until it reaches a certain limiting value. For instance, at this 'yield point' metals change from springy solids and behave like very viscous fluids, drawing out into thin threads (wire). Again, sand freely trickles down, but stands solid at a moderate slope. The reason is evident, solid friction among the particles—whether held in close contact by molecular cohesion, or merely by their own weight, has its usual effect of quite preventing slipping under small forces.

But take well-rounded sand grains lubricated with plenty of water and this 'quicksand' notoriously gives way even to light weights.

The typical fluid yields continuously, though it may be slowly, to any force however small.

A very viscous liquid like treacle or pitch yields but slowly to the weight of a pebble; a feather gives evidence of frictional resistance to its fall through a gas. The non-existent theoretically perfect fluid would be perfectly mobile, its particles would glide past one another without friction. Fortunately the presence of viscosity makes no difference to the study of fluids at

rest (or in comparatively slow motion), for in the internal friction among the particles there is never any preliminary 'stiction' stage.

§ 61. Noticing that *Pressure is defined as the force exerted on each unit of area—e.g. lb. per sq. in., dynes per sq. cm.*—there flow from the foregoing these **Laws of Fluids** :—

I. *The pressure of a fluid at rest on any surface bounding it is perpendicular to that surface.* For whatever it may be, the reaction of the surface is equal and opposite to it ; resolving this into two components perpendicular and parallel to the surface, the latter component would urge the superficial layers sideways, and as they are quite incapable of making any stand against it they would move till this component had been reduced to zero.

This principle is familiar to everyone in the resistance felt when a broad surface is slowly* moved flatwise against wind or water, but not when edgewise.

The free surface of a liquid must consequently set itself at right angles to the resultant force acting on it at the point. Usually this is weight, vertically downwards, and hence the surface is a horizontal plane (or rather, conforms to the shape of the earth). But if the liquid is in rotation for instance, centrifugal force comes in and the surface banks up into a wave, or a whirlpool cone.

II. *The pressure at a point in a fluid is the same in all directions.*

For consider a minute equilateral triangular volume in the fluid, a prismatic block so small that its weight is negligible compared with the pressures on its faces. Then if this remains at rest there can be no resultant force acting on it, i.e. by the triangle of forces the three pressures perpendicular to its three faces must be all equal. It can be tilted about anyhow, and we may infer that the pressures are the same in all directions at the point.

The spirting of water with equal violence in every direction from holes in a leaky fire-hose illustrates this principle. But by far the best experimental proof of it is that a very well made† Aneroid Barometer reads the same however it is turned over and about in the hand. In this instrument (§ 74) the heavy pressure

* The law holds very approximately for slow motions, but rapid motion involves viscosity and the pressure inclines a trifle. There is a surface drag or 'skin friction' on a blade swung swiftly edgewise; air blowing rapidly over still water drags along the superficial layers until somewhere a trifling heaping-up is produced, the beginning of a ripple.

† In common aneroids the weight of unbalanced levers causes discrepancies.

of the great ocean of air, in the depths of which we live, is being balanced against the elastic strength of a spring. This of course is unaltered by merely tilting the whole instrument about, hence the constant reading means that the air pressure on the lid of the flat vacuum-box is the same in all directions.

III. *The pressure in a fluid at rest whose weight can be neglected is the same throughout.*

For if different pressures acted on opposite faces of a cubical volume in the fluid it would begin to move and continue till the pressures were equalized.

Of course this law is approximate only: no material fluid is weightless. Still, it takes a good aneroid to measure the difference of atmospheric pressures on the chair and on the table; and the engineer utterly disregards any variations of pressure, due to weight of water, in a hydraulic cylinder where the average pressure is perhaps a ton to the square inch.

[The next paragraph, § 62, is the supplement of this, it deals with the pressures in a heavy fluid due to its weight only.]

It is on this transmissibility of fluid pressure to all parts that steam, compressed air, or hydraulic power distributing systems depend. The **hydraulic press** affords a good instance of its adaptability. In Fig. 28 a force exerted on the small plunger P is transmitted by the water and applied a hundred-fold on the plunger or 'ram' R of 100 times greater circular area. Conversely of course P moves 100 times as fast as R, hence it is necessary to fit valves and make it a reciprocating force pump.

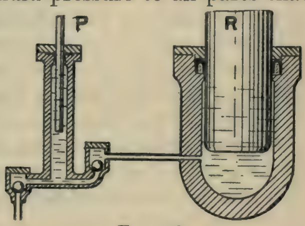


FIG. 28.

The efficiency of the Bramah collar surrounding the ram again depends on this principle. It is a leather ring pressed to a ∩ section and stuffed with tow. The water pressure among the tow presses out the collar tightly against both cylinder and ram and thus prevents leakage.

§ 62. **Pressure due to weight of fluid.** The pressure due to gravity at a point in a heavy fluid at rest is evidently equal to the weight of a 1 sq. cm. vertical column standing on a sq. cm. horizontal area drawn round the point, Fig. 29 (left). For there is no 'stiction' over the vertical sides to support the column and prevent it resting its whole weight on the base.

Weight of column = no. of c.c. it contains × wt. of each.
= depth of pt. below surface × density* of fluid.

$\therefore$ Pressure $p = hd$ grm.-wt. per sq. cm.
$= hdg$ dynes per sq. cm.

If there are several fluids on top of one another $p = h_1d_1 + h_2d_2 + h_3d_3$, etc. If, as in the atmosphere, the lower layers are much compressed by the weight of those above, so that the fluid gradually increases in density downwards, Fig. 29 (right), p is reckoned by dividing the total height into small fractions, assigning an average density to each, and summing the product hd throughout; a process of integration.

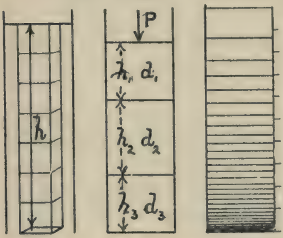


FIG. 29.

Again, unless a vacuum has been created over a liquid surface, there is an air or steam pressure there which has to be added to all gravity pressures throughout (Law III) the liquid to get the total pressure ('absolute' pressure of engineers).

These principles are applied also in Sphygmometers for measuring blood pressure. A small flexible rubber bag containing mercury is strapped over the artery, in the arm say. The mercury is continued through a rubber tube to a glass reservoir and the latter is raised above the bag till the pulse beyond ceases, indicating the collapse of the artery beneath the bag. Then height above bag = blood pressure in cm. of mercury.

§ 63. The pressure will be the same everywhere at the same depth below the level surface. For horizontal motion does not involve the vertical force of gravity at all, hence Law III holds throughout any horizontal plane in any fluid at rest. The pressure at the lower level in Fig. 30 (i to vi) is the same for all (and in (i, ii, and iii) the total forces on the equal bases are the same). Reciprocally, of course, if a number of vessels communicate at one point, the liquid will 'find its own level'—i.e. same height above the common point—in all, whatever their size and shape, and will there remain at rest. Thus the U bends in the figure show the same pressure at the same level on both sides, and the greater or

* 'Density' = mass of the unit of volume.

lesser pressures passed through on the way round the bends need not be reckoned out.

Curiously, in (iii) we see that the pressure on the bottom of a necked bottle may exceed the pressure of the whole bottle on a scale-pan. The explanation is that the pressure of the liquid, everywhere normal to the glass, has a compensating upward component round the shoulder; if the bottle were cracked round, the upper half would be actually lifted till the liquid from the shoulder had run out.

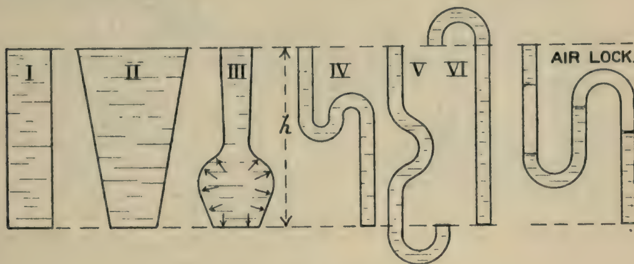


FIG. 30.

§ 64. **The Siphon.** If the limbs of a U tube are filled to different levels there is an unbalanced pressure on the liquid in the bend, forcing it towards the low side; the liquid oscillates, and comes to rest stably at the same level both sides.

But in an inverted ∩ tube equilibrium is unstable. At the vertex, Fig. 31 (left)—

Pressure from left $=$atmospheric $-h_1d$.
,, ,, right$=$,, $-h_2d$, a smaller total;

the resultant $(h_2-h_1)d$ forces the fluid over towards the right. If now the fluid is continuously supplied as in the second figure a steady outflow goes on. This is the **Siphon.** It may be compared to an Atwood's machine, the masses of fluid h_1 h_1 being the inert balancing masses and the dependent weight of fluid in h_2-h_1 supplying the driving force. Consequently the shorter the arch and the longer the long limb the faster the outflow.

But there is a difference, the Atwood cord is under tension, and a liquid cannot be relied upon to endure a tension (a gas cannot possibly). If the atmospheric pressure were removed there would be a *minus* pressure at the vertex, the liquid would break

there and drop back. A siphon cannot act in vacuo, nor if its arch is higher than the barometer filled with that liquid.

The siphon is commonly started by 'sucking the air out.' Fig. 31 (iii) shows another way employed in flushing cisterns, a piston forced down drives a jet of water up the short limb, and this rushes enough water over the arch to act as described below.

Siphons arranged as in Fig. 31 (iv and v) start spontaneously if

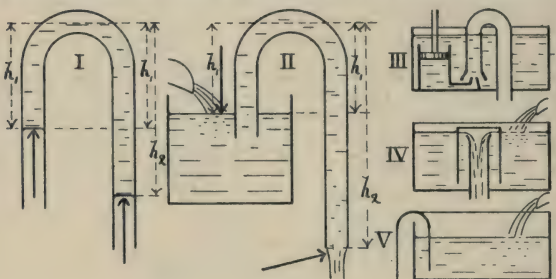


FIG. 31.

the cistern is filled above their arch. Beginning to overflow down the pipe (as in (iv)) the waterfall entangles and carries down the air, and soon the siphon is running full bore until it has nearly emptied the tank. Then air gets in and stops it. As soon as the tank is full again the same automatic flush is repeated. The philosophical toy, the Vase of Tantalus, is an old-fashioned instance of this **Intermittent Siphon.**

§ 65. A fluid can be put under pressure in various ways. There is the obvious way of pouring more liquid into a tall cylinder to increase the pressure at the bottom. Or a piston can be fitted to the cylinder and weights piled on it. Or a flexible containing vessel can be forcibly reduced in size, as happens when a tennis-ball is struck, or the bulb of a rubber syringe or a camera-shutter is squeezed. These ways are not widely useful.

A theoretically simple way is to set the fluid in rapid motion and then to check its speed, when the momentum (grm. × cm./sec.) destroyed per sq. cm. per sec. = the pressure produced (dynes/cm.^{2}). [§ 13.]

This is done in **centrifugal pumps and blowers**. The fluid, admitted along the axle of a rapidly rotating paddle-wheel,

becomes entangled among the blades and is flung off tangentially at a great speed, which it loses in the roomy casing and pipes, where most of its momentum is converted into pressure. These pumps are mostly used for low pressures, below $\frac{1}{4}$ lb. (air) or 50 lb. (water) per sq. in.

Again, when water flowing in long pipes is abruptly checked, as by too quickly closing a tap, the sudden destruction of momentum produces a transient pressure, the blow of a noisy 'water-hammer' in the pipes. Water from the pond flows down a few yards of sloping pipe to the **hydraulic ram,** Fig. 32, and out at the large open valve as shown. The outrush gaining in speed presently lifts the iron plate and shuts the valve. The stoppage causes great pressure, which forces open the little valve on the right and drives a small quantity of water up to the tanks of the country house perhaps a hundred feet above. The transient squeeze relieved, the plate drops and the outrush begins again, and so on.

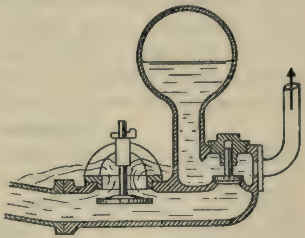


FIG. 32. *Scale* $\frac{1}{10}$.

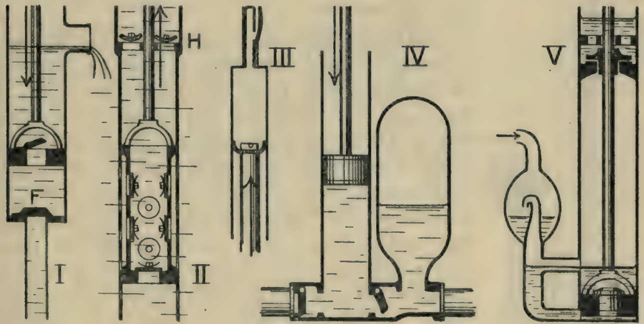


FIG. 33.

§ 66. **Pumps.** Pressure is most commonly worked up or down by **reciprocating pumps.** In Fig. 33 (which should be intelligible to anyone who understands a valve) are diagrams of the common

'lift pump' in one form at (i), and with head-valves H instead of foot-valves F at (ii) (mine pump with several rubber disc valves in 'bucket'), and (iii) bicycle pump. The **'force pump'** (iv) when dealing with liquids should have an air-vessel on its discharge-pipe, the compressed air acts as a spring, steadying the outflow and obviating water-hammer (see also Fig. 32). With high pressures (Fig. 28) the slightly compressible water itself affords elasticity enough.

In the **heart** the auricles dilate as they receive the continuous influx from the veins, then contracting, pass it through valves into the ventricles, which in sudden systole force it through other valves into the arteries. The elastic arteries dilate in a 'pulse' so that normally no shock is felt.

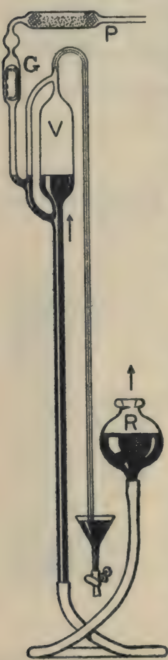


FIG. 34. *Scale* $\frac{1}{18}$.

Air-pumps, for exhausting air, are perhaps of most interest here. The commonest is a 'lift pump' with oiled silk valves, and the clearance spaces between piston and end of cylinder made small. A pair of these, driven opposite ways, for ease against the atmospheric pressure, forms the time-honoured machine for producing a 'vacuum'—rarely containing less than 5 % of its original air. For better results one must relieve the enfeebled air of having to lift a foot-valve and must do away with all clearance and all leakage.

§ 67: **Modern air-pumps.** The modern 'Geryk' pump (commemorating the inventor of the air-pump, Otto von Guericke, 1654) is sectioned in Fig. 33 (v). The descending piston draws out a vacuum above, the air below lifts its valve and passes in. On the up-stroke this is expelled through the head-valve H. Presently the remaining air can no longer lift the piston-valve, the piston is pushed right down and the vacuous barrel put in free communication with the pipes as shown. The piston rises, imprisons any contained air, finally strikes and lifts the head-valve, and the bubble of air escapes through the oil. This oil, very non-volatile, moistureless and non-solvent of air, fills all clearance spaces and quite seals the valves.

The still more perfect Toepler pump, Fig. 34, works very similarly. The highly rarefied gas fills the vessel V, the reservoir R is raised, and its mercury (normally at nearly the barometric height above it) is in the diagram flowing rapidly into V, which it has just shut off from the pipes, and ultimately drives any small air-bubble over the bend, down a fine capillary, and out under mercury. The rough glass valve G prevents mercury getting back into the apparatus being exhausted. P is a drying tube containing phosphoric anhydride.

A recent rotary mercury-pump for high vacua works like a wet gas-meter driven backwards. The compartments of a hollow drum successively fill with the rarefied air through an axial pipe and then plunge under mercury, which expels their contents into the partial vacuum of any ordinary pump.

With due precautions, mercury-pumps have reduced the pressure to only ·000025 mm. of mercury.

§ 68: **Rate of exhaustion by air-pump.** If V be volume of air filling vessel to be exhausted and v that of pump-barrel the first out-stroke increases V to $V+v$ and therefore by § 101 reduces the pressure to $V/(V+v)$ the original. The next stroke reduces it again to the same fraction of this, and in general n strokes would reduce the pressure to $[V/(V+v)]^n$ times the original pressure, provided there were no leakage, clearance spaces, or evolution of gas in the vessel.

In **compressing**, starting at atmospheric pressure, the first in-stroke reduces $V+v$ to V. The next again puts in a barrelful of air v at the atmospheric pressure, and n strokes would put in nv and therefore raise the pressure to $(V+nv)/V$ the original pressure, provided there were no leakage, clearance spaces, or condensation of gas, and that the temperature were kept constant (involving very slow working ; in practice the gas gets hot). The delivery valve of course opens later and later in every stroke as the pressure rises.

§ 69. **Work done by a pump.** The work done by a water lift pump is, in gravity measure, simply the total weight of water passed through × the height it is lifted.

A pump which delivers volume v against pressure p does work pv (ergs, if v c.c. and p dynes per sq. cm.). For if the discharge-pipe were 1 sq. cm. area v c.c. forced into it would drive back resisting pressure p through v cm. = work pv, § 21 ; but as fluid pressure is

the same in all directions there is no need for this restriction on shape.

But in addition, if the liquid in the pipe where p is measured is flowing at speed s cm. per sec. the pump has given it kinetic energy $\frac{1}{2}ms^2=\frac{1}{2}(\text{volume}\times\text{density})s^2=\frac{1}{2}vds^2$ ergs.

e.g. **Work done by heart.** Assuming the heart discharges per beat 75 c.c. from right ventricle against pressure 6 cm. of mercury and from the left ventricle 75 c.c. against 15 cm. pressure.

$$\begin{aligned}\text{These} &= 75\times 6\times(13{\cdot}6\times 981)+75\times 15\times(13{\cdot}6\times 981)\text{ ergs}\\ &= 21\text{ million ergs}\end{aligned}$$

where $13{\cdot}6\times 981=$ dynes pressure of 1 c.c. of mercury on its base $=1$ cm. mercury pressure.

Further, taking speed of blood in both pulmonary artery and aorta, where pressures are measured, as 50 cm./sec., and density 1·05

$$\begin{aligned}\tfrac{1}{2}m\times\text{speed}^2,\text{ for both sides} &= 2\times[\tfrac{1}{2}\times(75\times 1{\cdot}05)\times 50^2]\\ &= 196{,}000\text{ ergs}\end{aligned}$$

a total of 21·2 million ergs; [2·12 joules, nearly 2 ft.-lb., half a calorie.]

The fluid stores the work quietly done on it as potential energy. That of mass m of fluid raised to height h is mh grm.-cm. or mhg ergs, just like any solid. A column of fluid of height h has an average height only $\frac{1}{2}h$, and therefore contains (total mass $\times\frac{1}{2}h\times g$) ergs. Volume v under pressure p can supply energy pv to a water-motor of any sort. [v c.c., p dynes/cm.^{2}, pv ergs.]

EXAMPLES.—CHAPTER VIII

1. A load of 700 lb. rests directly on the 3-in. diameter safety-valve. At what boiler-pressure will the valve lift ?

2. The working lever of a hydraulic jack is 27 in. from handle to fulcrum. At $1\frac{1}{2}$ in. from the fulcrum it is attached to a plunger $\frac{3}{4}$ in. diameter, and this forces oil into a ram-cylinder 3 in. diameter. What force exerted on the handle will enable the jack to lift 20 tons ?

3. What is the pressure due to a ' head ' of 180 ft. of water ?

[A column of 180 cu. ft. of water, each weighing $62\frac{1}{2}$ lb., exerts $180 \times 62\frac{1}{2}$ lb. on the square foot at its base $= 1120$ lb. per sq. ft. $= 78$ lb. per sq. in.]

4. To what height would water from hydraulic mains at 700 lb. per sq. in. rise in a stand-pipe ?

5. Express in grm./cm.^{2} and in dynes/cm.^{2} the pressure due to a 76-cm. column of mercury, of density 13·6.

6. Calculate the height of a column of air, density ·0012, which exerts the same pressure on its base as does 1 cm. depth of mercury.

7. Define ' pressure ' and explain ' pressure at a point ' in a liquid. What quantities determine its magnitude ? [D.]

8. Describe some form of siphon which can start automatically.

9. Calculate difference of blood pressure between head and feet of a man 1·7 m. tall; s.g. blood 1·05. [L.]

10. Explain how total pressure on bottom of a vessel full of water may exceed weight of water. A vessel is $5\frac{1}{2}$ in. deep and diameter of bottom is 4 in. Find total pressure on it when full of water, 1000 oz. per cu. ft. [Ab.]

11. Give a diagram of a pump that would raise water 50 ft. [L.]

12. Find the force on the 12 sq. in. piston of a pump to draw water from a well 20 ft. deep and deliver 40 ft. high. [Ab.]

13. If the atmospheric pressure be that due to 76 cm. of mercury, density 13·6, at what depth under sea-water density 1 03 will pressure be 2 atmos. ? [Ab.]

14. Find work done per 24 hours by a heart discharging 10 cu. in. blood 72 times per minute against $\frac{1}{5}$ atmosphere (atmos. $= 15$ lb./sq. in.). [L.]

CHAPTER IX

THE MEASUREMENT OF PRESSURE

§ 70. **Manometers.** For measuring small differences of gas pressure the **U** tube pressure gauge or Manometer of Fig. 35 is in common use. Gas pressure difference P—P′ is compensated by an equivalent rise h of the liquid, so as to maintain the equilibrium condition of equal total pressures on each side of an area drawn at the bend. Then since liquid below the lower level balances itself,

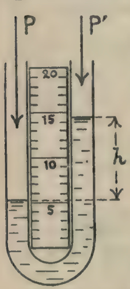


FIG. 35.

Gas pressure = diff. in level × density of liquid [in grm. per sq. cm.]

or $= hdg$ dynes per sq. cm.

By § 63 there is no obligation to have the limbs of equal diameters. Short **U**'s containing oil, inky water, etc., suffice for light pressures, such as furnace draught or domestic gas supply. Long tubes, running up towers or mine shafts, and filled with the far denser mercury, form standard manometers for heavy pressure.

The statement of pressure is often left as so many 'inches of water' or 'cm. of mercury,' with obvious meaning.

§ 71. If one tube contains no gas pressure, but has a vacuum above the liquid, the instrument becomes a **Barometer**, and measures the absolute pressure of the gas which balances that of the column of liquid between the two surface levels.

The pattern in Fig. 36 (**S**) is called a **siphon barometer,** though the open tube is seldom left so long as shown. In the domestic **wheel barometer**, Fig. 36 (**W**), glass weights hang round a pulley, one rising and falling with the mercury surface on which it floats, and a pointer conveniently magnifies the motion. It lags a little behind the true reading until sticking at the pulley pivots, etc., is relieved by tapping.

For scientific accuracy one prefers to read the mercury column

direct. In a siphon this rises and falls only half the barometric change, for the short limb moves equally and oppositely, and both must be read. But if the short limb is broadened its variation of level is of course much less (Fig. **S**, as dotted; Figs. **H, M,** and **F**). Then in the **Kew or marine barometer M** the scale divisions are deliberately shortened from true inches sufficiently to allow for

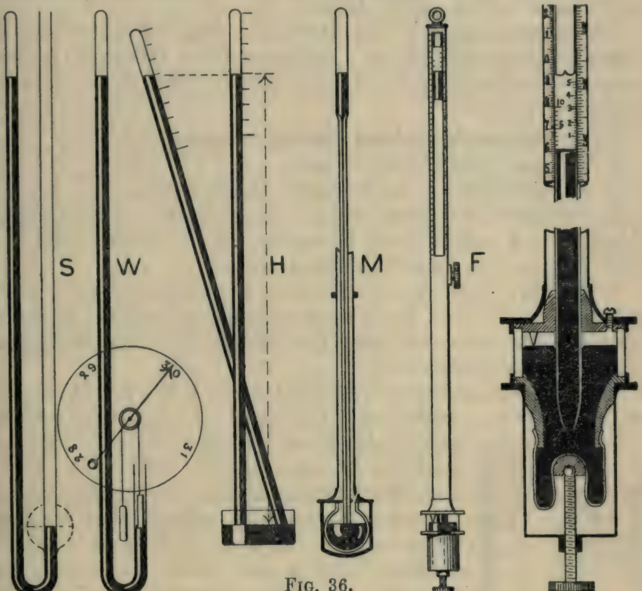


FIG. 36.

this small fall and rise in the 'cistern.' This instrument has a constricted tube to hinder oscillation of level when the ship rolls, and is hung on gimbals at the middle of the protecting brass sheath. For, as in Fig. 36 (**H**), inclining the tube alters the reading; it is the *vertical* height that remains constant. The vernier-head resembles that of the next pattern.

In **Standard Barometers** this shortening of the divisions is inadmissible. The mercury in the glass-walled cistern is always first adjusted to touch a fixed pointer which is the actual zero point of the vertical scale. In the **Fortin** pattern (Fig. 36 (**F**) shows one graduated for mountain use, and on a larger scale its cistern and vernier-head) this is effected by moving the leather bottom of

the reservoir. The scale is on a protecting brass tube and is read to 1/500 in. and to 1/10 mm. by a vernier shutter, borne on an inner sleeve and racked down until it just cuts off light over the middle of the meniscus. [For carrying, the tube is slanted till it fills completely, the bag is screwed up till mercury exudes at the little air-screw shown on the right, this is screwed home, and the instrument carried upside-down.]

The **normal height** of the barometer after all corrections are made is taken to be 76 cm. The uncorrected British 30 in. very nearly corresponds to it. It represents a pressure of 1,016,000 dynes per sq. cm.

§ 72. **Barometric readings must be corrected** :—

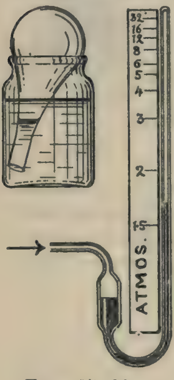
1. For any error of scale or zero.

2. For temperature, most important, see § 137.

3. For capillary depression, see § 252, variable and troublesome. Averages $+\frac{1}{4}$ mm. in 1-cm. bore tube, but standards should have 1-in. tube and no appreciable depression.

4. Mercury vapour pressure in 'Torricellian space' above column is negligible. But if air manages to stray there enough to form a bubble persistent when the tube is slanted at 45 degrees, the tube must be refilled.*

5. For variation of gravity with latitude, § 29. It is the fashion to express in cm. mercury at 0° C. as in lat. 45°, instead of straightaway in dynes/cm.^{2} Add at lat. 40° $-\cdot034$ cm., lat. 50° $+\cdot034$ cm., lat. 55° $+\cdot067$ cm.



FIGS. 37, 38.

6. For meteorological comparisons only, correct to sea-level by adding 1 mm. for every 11 m., or ·1 in. for every 90 ft. above sea, § 76.

Since water is 13·6 times less dense than mercury, the **Water barometer** is $13{\cdot}6 \times 2\frac{1}{2} =$ 34 ft. high. **Glycerine** (which has less vapour pressure, etc.), stands at 28 ft., lubricating oil about 40 ft., and these are the utmost heights to which the normal atmospheric pressure would force up these liquids into a pumped-out vacuum.

§ 73. We now turn to some pressure-gauges depending on elasticity instead of on dead-weight.

In the popular weather-glass of Fig. 37 water is forced up the

* Clean, dry, invert, fill with mercury and boil from below upwards to wash out adherent air bubbles; best done under reduced pressure.

neck of the flask against the elasticity of the enclosed air as the atmospheric pressure outside increases. But the contrivance need be kept in a corner at a steady temperature, for increasing warmth expands the air and drives the water down, 20° F. more than compensating any ordinary barometric change.

The gas in the closed tube of the little **compressed-air manometer**, Fig. 38, halves its volume every time the pressure on the outer end of the mercury thread is doubled, according to Boyle's Law, § 101.

§ 74. The **Aneroid** (=without liquid) **Barometer** (1848) is light and easily portable. Fig. 39 shows the mechanism of a good pocket aneroid (an interesting travelling companion). Attached to the base plate is a flat vacuum box R of thin metal, corrugated for flexibility. The atmospheric pressure would crush it in but for the pull of a folded spring C to which it is hooked. As it is, barometric rise compels this to yield a trifle. A long arm A attached to C magnifies the motion three or four times, and is linked to a shaft rocking on pivots PP. The distance of its point of attachment from the shaft's axis (length of lever arm) is variable by a screw which forces away the elastic free leg of the forked rocking shaft from its stiffer pivoted leg: this modifies the total magnification so that the pointer is driven round neither too fast nor too slowly. From the rocking shaft projects a longer upright arm; from this a chain passes round a pulley on the pointer axle and is kept stretched by hair-spring H. The end of the pointer is thus made to magnify the motion of the box-lid several hundred times.

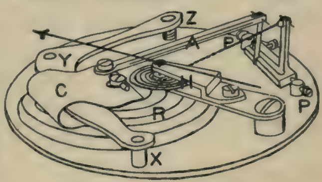


FIG. 39. *Nat. size.*

Aneroids must be compensated for temperature. Warmth weakens the spring, which gives way too much and lets down the end of A, producing unduly high readings. This is counteracted by making A a compound bar (§ 130) of brass and steel (on top) so that its end bends up as much as the weakening would let it drop.

The lower fold of C is fixed to an L-shaped bar supported on the base-plate by two steel posts at XY and a ' setting ' screw beneath Z, accessible from the back of the instrument. Adjusting Z rocks

the bar on its posts, tending to fold or unfold C a very little, and immediately moving the pointer. The aneroid is thus initially set to agree with a standard barometer, a zero adjustment which most aneroids require every year or so, since the spring slowly and persistently alters under the constant strain. Whymper, living in the Andes above 8000 ft., found this ‘ creep ’ very serious there, and no reliance is nowadays placed on aneroids that have been subjected to such low pressures for any length of time.

Self-recording instruments have frequently a stack of aneroid boxes as barograph, and a Bourdon tube completely full of alcohol as thermograph.

§ 75 : The **Bourdon gauge**, Fig. 40, is used by engineers for all fluid pressures. Curled round in nearly a circle is a thin steel tube of very flat elliptic section. Increase of pressure inside this tends to fill out the elliptical shape, and this forces the tube to uncurl to some extent, the free end moves, and the pointer geared to it magnifies its motion.

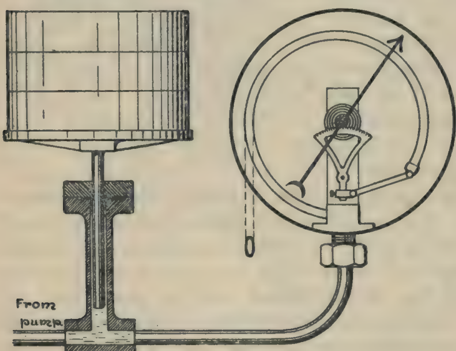


Fig. 41. Fig. 40.

For **heavy pressures** these are graduated by attaching them temporarily to an oil cylinder in which moves (easily, with rotation) a plunger of known sectional area loaded with known weights, as in Fig. 41.

Pressures in guns are estimated by the crushing of a small block of copper on which they have squeezed a steel plug. Better by mechanically or optically magnifying the elastic compression of a hollow steel plug, an inch or more of whose blind end projects into a cavity in the explosion chamber.

§ 76. **Determination of heights by the barometer.** As one climbs above the lower dense layers of the atmosphere the pressure of course diminishes by the weight (per sq. cm.) of these layers, and the barometer falls. It was the observation of this

fall, first made by Pascal in 1648, that established the true principle of the 'Torricellian tube.'

The calculation is this—what depth of air of known density, computed from its temperature pressure and humidity, must be removed from above the open limb in order that 1 mm. depth of mercury may be removed from the closed limb ?

The depths are inversely as the densities.

Taking ·0012 average for air and 13·6 for mercury gives $13{\cdot}6 \div {\cdot}0012 = 11{\cdot}3$ m. of air per mm. of mercury.

The diminution of pressure and also of temperature of the atmosphere at greater heights causes a change of density which complicates the full calculation. The result is :—

Height in feet=[log. barom. reading at bottom—log. ditto at top]$\times$56,200$\times$[1·004$\times$temp. centigrade].

This, up to 3000 feet, gives approximately

$$\text{Height in feet} = \frac{\text{diff. of readings bottom and top}}{\text{sum of readings bottom and top}} \times 49{,}000 \times 1{\cdot}004t^{\circ}$$

or about an inch fall of barometer for 900 feet rise.

At greater heights, in rarer air, the rate is slower.

In variable weather climbers' readings are useless, of course, unless afterwards compared with records simultaneously made at a fixed level.

EXAMPLE.—CHAPTER IX

1. If the atmosphere sustains the barometric column, how is it that a barometer tube is heavy to lift from place to place in a basin of mercury ?

CHAPTER X

SPECIFIC GRAVITY

§ 77. **Archimedes' principle.**—Consider the fluid contained in a closed volume marked out inside a quantity of fluid at rest, for instance the water contained in a submerged net. It is acted on by the pull of the earth and by the pressures of the adjacent fluid, and these just balance each other, for it remains at rest. That is, the pressures of the surrounding fluid just exactly bear up the weight of the fluid filling the volume.

Suppose the volume to be emptied of its fluid and filled with some other material. The surrounding pressures are quite unaltered, i.e. this foreign substance *is borne up with a force equal to the weight of fluid it has displaced, or apparently it loses that much of its usual weight.* This is the '**Principle of Archimedes,**' though his sudden and historic discovery seems to have been merely that, with equal masses, the denser material displaces less water (from a brim-full vessel).

If the foreign substance is more massive (denser) than the fluid it has displaced it will still require some other support, but if less massive it must be held down, or it will rise and float and displace only that fraction of its own volume of fluid which has a weight equal to its own.

§ 78. Evidently a body cannot rest midway in a fluid of constant density not precisely equal to its own. For instance, a torpedo cannot be *weighted* to remain 6 ft. under water; that depth must be kept by active mechanical control.

But it may happen that between one position and another (1) the immersed volume of body changes, (2) fluid density changes, (3) mass of body changes.

(1) happens when it rises and projects above the surface of a liquid; it bobs up and down and then floats stably at a constant load-line.

(2) is exemplified by pouring water on brine and only imperfectly stirring together. The liquid varies from water at top

to strong brine at bottom. In this an egg would float entirely immersed at the particular level where the concentration of the solution gave it the egg's density, and if disturbed would return to that level.

(3) occurs in ballooning. The altitude of a balloon has to be actively controlled by throwing away ballast. The tail of a rope trailing on the ground is of course a reduction of air-borne mass: some dirigibles can take on ballast by slightly compressing air into large rigid envelopes.

§ 79. Archimedes' Principle applies not only to gravity, but e.g. to centrifugal force, the employment of which for separating bacteria from liquids, or cream from milk, is well known.

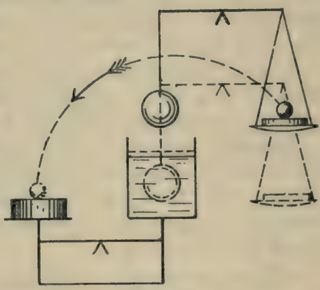


FIG. 42.

The Principle can be experimentally verified as in Fig. 42. The ball and the can of liquid are first separately counterpoised, then the ball is lowered into the liquid, and to restore equilibrium it will be found that the same weight that has to be removed from the first scale pan on the right must be put into the last pan on the left. The liquid is bearing just exactly the missing weight.

SPECIFIC GRAVITY

§ 80. *The* **specific gravity** *of a substance is the ratio of the weight of a volume of it to that of the same volume of water.*

Being a mere ratio it is the same whether in British or c.g.s. measure. In the latter it is equal to **Density,** for this is the mass of 1 c.c., and 1 c.c. of water is 1 grm.

Variations in the composition and in useful properties of substances are frequently accompanied by characteristic slight changes in their densities. Hence the accurate measurement of specific gravity or density is of great technical importance as it very often affords the quickest means of discrimination and valuation. It is the refinement of the familiar guessing at what a substance is by its 'weight,' of detecting spurious coins by their lightness. The mineralogist uses it as a guide to the nature of minerals—gem stones, metallic ores, etc. The apothecary, the analyst, the technical chemist, the brewer, the exciseman, all

possess tables drawn up to give the concentrations of the particular solutions they are dealing with in terms of their hydrometer readings, and find it vastly more convenient, and often more accurate, to make use of this instrument rather than to undertake any chemical analysis. Quite recently Waller has suggested controlling the chloroform dosage of a patient under that anæsthetic by passing the vapour-laden air through the case of a balance bearing a large counterpoised glass bulb, when the presence of the heavy vapour in the surrounding atmosphere buoys up the bulb a milligram for every 0·1 % present, so that the pointer of a quite ordinary balance plainly shows variations of this extent from the supposed normal 2 % of chloroform.

Speaking strictly, specific gravity is reckoned from water at 15° C. or 60° F., and neglects the fact that the weighings are all done in air. Hence small corrections to water at 4° C. and to true weights in vacuo are needed to make the specific-gravity measurement one of density.

All specific-gravity determinations must be made very near the standard temperature, for liquids are very expansible.

§ 81. An obvious mode of finding the specific gravity of a liquid is by means of the **Specific-gravity Bottle** or **Pyknometer.** This is a bottle which can be filled with always exactly the same volume of a liquid either, Fig. 43, (i) to a flat plate (a scholastic con-

FIG. 43.

trivance), or (ii) up to a mark on a narrow neck, or (iii) completely up to the stopper (perforated for overflow when dropped in), or (iv) from nozzle to file-mark in the Sprengel pattern.

The dry bottle is counterpoised on a balance, then the net weight of cold water filling it is found, W. It is rinsed and filled with the liquid, whose net weight proves to be L. *Then sp. gr.* = L÷W. The English apothecary saves calculation by using a bottle with W = 1000 grains at the ordinary temperature.

For **insoluble solids (powders)** M grm. are weighed into the previously counterpoised bottle, which is then filled up with water. Its contents now of course weigh less than M+W by the weight of water which cannot get in on account of the presence of the solid, i.e. which would occupy the same volume as the solid. *Then* M÷*this shortage=sp. gr. of solid.*

With **soluble solids** any limpid oil would be used, and then M÷shortage=ratio sp. gr. of solid : sp. gr. of liquid, this last being found as before described.

§ 82. **'Hare's apparatus' of balancing columns of fluid.** If two non-miscible fluids are poured into a U tube, Fig. 44, they will come to rest at different levels. Omitting the changes of pressure below their contact level (§ 63) the pressures on either end of the portion of (denser) liquid in the bend below this common level must be equal. The air pressure is the same in both open tubes and can be left out of account. Then by § 62 $h_1 \times d_1 = h_2 \times d_2$

$$\text{or } \frac{d_1}{d_2} = \frac{h_2}{h_1}$$

and therefore if liquid (2) is water, d_1 the *density of the first liquid=height of liquid÷height of water, both from the common level.*

For miscible liquids the form of apparatus shown on the right is preferable. The atmospheric pressure — the reduced air pressure in the bend (sucked out)=the pressure due to either column of liquid, hence as before *heights* H_1 H_2 *above the levels in their respective reservoirs are inversely as densities.*

Notice that the sizes of the tubes are quite immaterial, § 63.

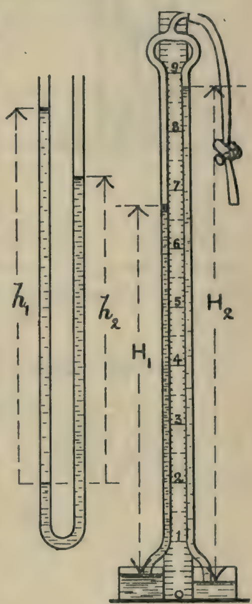


FIG. 44.

The Barometer is a 'Hare's apparatus' with one column miles high.

§ 83. The **Hydrostatic Balance** method of weighing a body in air and then in water applies Archimedes' principle directly. A balance is arranged as in Fig. 45 with a 'stirrup' of thick copper

wire (flexible and heavy to enfold and sink things that want to float, when W exceeds M) hanging by a stout silk fibre under cold water, and this is counterpoised (s). The water should be distilled and boiled air-free, but ordinary tap-water serves to 1 part in 5000.

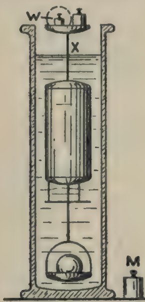


FIG. 46.

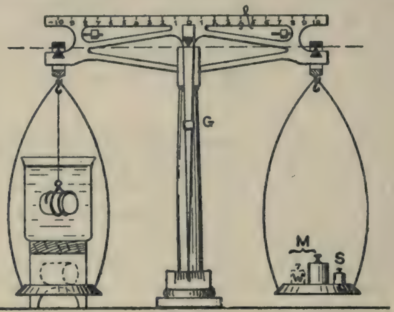


FIG. 45.

The body is laid on the pan and weights=its M placed on the other pan till equilibrium is restored. The body is removed from pan to stirrup, it is found that a portion W of the weights must be removed from the weights pan *

$$M \div W = \textit{Specific Gravity of body.}$$

For **soluble solids,** weigh in oil and *sp. gr.*$=\dfrac{M \times \text{sp. gr. of oil}}{\text{loss in oil}}$

[An ingenious, though not very accurate, way is possible with Ice. Wrap round it a known weight of copper wire and throw it into plenty of water. It gradually dissolves and loses buoyancy: at the moment it begins to sink it is quickly fished out and weighed. Then mass of substance+wire—the 1/9 wt. of wire (sp. gr. 9) lost in the water=wt. of water displaced.]

The apparatus can be used to find the **specific gravity of liquids,** for this is *the loss of weight in the liquid, of a 'sinker' or plummet, divided by its loss of weight in water,* these being the weights of the two liquids that the same bulk displaces.

Very convenient glass plummets displacing exactly 10 c.c. (or less, to 1), i.e. 10 grm. water are obtainable. The weight that has to be hung on the same side as the plummet (previously counter-

* *Or else* the last weight is made up by adding W in the pan the body has just vacated.

poised in air) to keep it under the liquid, divided by 10 (or less, to 1)=sp. gr. of liquid.

§ 84. **Nicholson's hydrometer** is an old modification of the hydrostatic balance. Instead of hoisting the body by beam and counterpoise it is borne by a floating buoy. Weights are placed in the top pan, Fig. 46, till the buoy sinks to mark X ; then **solid body** being placed there too, weights=its M have of course to be taken off to prevent foundering. Removing body to lower pan or cage, the water now directly bears the weight of the W grm. it displaces, and this W has to be restored on the top pan to keep the buoy down. It can also be used for liquid specific gravities, but it is a troublesome instrument in practice.

§ 85. The **common Hydrometer,** used for liquid specific gravities, consists of a glass buoy ballasted by a load of shot or mercury at the bottom and having a thin stem projecting above the liquid, Fig. 47. It floats, therefore always displacing a weight of liquid equal to its own *constant* weight, or

volume displaced × *density of liquid*
= *mass of hydrometer*

hence it displaces less, i.e. floats higher, in a denser liquid.

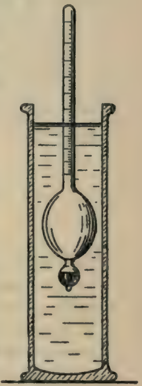


Fig. 47.

A scale of specific gravities is therefore graduated on the stem, with the largest readings at the lower end, and in such a way that the volumes of the instrument up to the scale divisions are inversely as the specific gravities marked on them : the divisions get rather wider apart towards the top. [The liquid's specific gravity is the reading at which the stem cuts the surface.] For a given size of bulb, their length (i.e. possible delicacy of reading) is greater on a thinner stem, being inversely as its cross-section.

Hydrometers are commercially obtainable of various degrees of sensitiveness, over various ranges of specific gravity (e.g. ·7 to ·9 ; 1 to 1·1, etc.) under different names—lactometer, salinometer, alcoholometer, etc.—and to various arbitrary scales, e.g. Twaddell's or Beaumé's.*

Sykes's hydrometer, used by the exciseman, works like the

* Specific gravity = 1 + degrees Twaddell ÷ 200
Specific gravity if above 1 = 144·3 ÷ (144·3 − degrees Beaumé)
„ „ if below 1 = 144·3 ÷ (134·3 + degrees Beaumé)

common hydrometer, but can be loaded with collar weights which make the same instrument available for denser liquids.

§ 86. **The specific gravity of substances of which only small chips or drops are available** is found by preparing a jar full of a mixture of liquids of the same density as the substance, as determined by placing a fragment or drop of it in the midst, when it must show no appreciable intention of either rising or sinking.

Thus the specific gravity of ice can be found as that of a mixture made by stirring alcohol into water until the ice just ceases to float.

A mixture of chloroform (sp. gr. 1·526) and benzole (sp. gr. ·889) is made up till a drop of human blood floats undecided ; then a hydrometer *or small plummet* (§ 83) *in the mixture gives the specific gravity of the blood.*

In finding the specific gravity of mineral fragments, or separating the constituent minerals in a powdered rock, very dense liquids are used, such as mercuric iodide in potassium iodide solution (max. 3·2), cadmium borotungstate solution (3·6) or methylene iodide (3·3) ; diluted with water or alcohol.

EXAMPLES.—CHAPTER X

1. A tube 3 ft. long and 1 in. bore contains $\frac{3}{4}$ lb. liquid. Calculate specific gravity, given 1 cu. ft. water weighs 62·3 lb. [L.]

2. Given a quantity of beads about the same size, how would you determine their average bulk and their specific gravity ? A quantity is contained in a narrow-necked litre flask, suggest a method of finding the number.

3. 30 grm. of metal sp. gr. 8·4 are melted with 20 of sp. gr. 7·2 and contraction of 2 % occurs in the formation of the alloy. Find its specific gravity.

4. State the principles of Pascal and Archimedes. What advantages are derived by the floating of the brain in the cerebro-spinal fluid ?

5. State Archimedes' principle and show how to prove it (a) experimentally, (b) theoretically. A solid ball sp. gr. 5·5 floats in mercury 13·6 and oil 1·46 is added till the ball is covered; what fraction of volume of ball is now in mercury ? [Ab.]

6. A can of water stands on a balance pan. Into it is lowered a glass ball sp. gr. 2·5 counterpoised on a second balance by 200 grm. What alterations in the weights are necessary to restore equilibrium ? [M.]

7. Calculate the lifting power of a balloon of 10^9 c.c. capacity in air of density 0·00129 when filled with hydrogen of density 0·00009. [L.]

8. Two bodies of sp. gr. 1·4 hang from a balance and are immersed in liquids of sp. gr. 1·1 and 0·8. The balance is in equilibrium, compare volumes of bodies. When liquids interchanged 45·5 grm. must be added on one side, calculate actual volumes. [L.]

9. A U tube contains mercury sp. gr. 13·5 in the bend ; on it in one limb stands 20 cm. salt water sp. gr. 1·1, in the other 10 cm. of ether sp. gr. ·73. What is difference in mercury levels and what height of ether added would make them the same ? [L.]

10. Explain how the 'gas-pressure' in the pipes is greatest at the top of the house.

11. Define specific gravity and density. A solid weighs x, y, and z in air, water, and a liquid respectively, find specific gravity and density of solid and liquid, density of water being w. [M.]

12. A solid weighed in air 14·86, in water 8·67, in a liquid 9·85. Find densities of solid and liquid, explaining why they are densities. [D.]

13. A 12-oz. piece of wood and $5\frac{1}{2}$-oz. piece of lead together weigh 2 oz. in water. Lead weighs 5 oz. in water. Find specific gravity of wood. [Ab.]

14. 1 oz. of wood sp. gr. ·5 is just sunk in water by a stone of sp. gr. 2·5. Find weight of stone. [Ab.]

15. A body floats in water $\frac{5}{6}$ immersed. A 3 cu. in. cavity is made in it and it now floats with $\frac{3}{4}$ its apparent volume immersed. What was its volume ? [Ab.]

16. A hydrometer to measure specific gravities from 1·2 to 1·4 has stem 5 in. long. Find length of stem which would have volume equal to bulb. [Ab.]

17. A hydrometer with a uniform stem sinks 10 cm. farther in a liquid of density 1 than in a liquid of density 2. In a third liquid it sinks 3 cm. farther than in second liquid. Calculate density of third liquid. [L.]

18. A fluid 3 in. deep sp. gr. 1·4 floats on a fluid sp. gr. 3. A cylinder 1 ft. long floats in them with one end projecting 2 in. into the air. Find its specific gravity. [M.]

CHAPTER XI

FLUIDS IN MOTION

§ 87. **Fluids in motion.** Fluids are set in motion by differences of pressure in different parts. The momentum gained per second by any portion = the difference of the forces acting on its opposite sides; this is the statement for them of the Second Law of Motion.

Evidently to get into motion the fluid has to convert some of its potential energy due to altitude or pressure (§ 69) into kinetic energy of motion. Conversely when the moving fluid is gradually slowed down, without any wasteful eddies, the energy returns to the potential form, i.e. the pressure rises again. Thus if water is flowing along a pipe with gentle bulges in it the pressure at the bulged part where the motion is slower is greater than at the narrow necks, rather contrary to most people's expectation.

The Venturi water-meter ascertains the pressure diminution in a narrow neck, and hence calculates by clockwork the total flow of water.

The pressure at the bottom of a water-tank or in a steam-boiler may be considerable, but the fluid pressure in jets from orifices in them is no greater than the atmospheric. For if it were, the unrestricted jet would instantly burst and splutter in all directions. But all the energy due to pressure (above atmospheric) has gone into energy of motion.

§ 88: Let us calculate the **relation between the fall in pressure and the speed of outflow** it causes (but neglecting any elastic expansion due to release of pressure).

The energy available is PV (§ 69) or in 1 c.c. = P. The mass of the 1 c.c. $=d$ the fluid density, therefore its energy of motion at speed $v=\frac{1}{2}dv^2$. *Neglecting friction* these are equal.

$$P=\tfrac{1}{2}dv^2 \text{ or } v=\sqrt{\frac{2P}{d}}$$

NOTE.—Energy in ergs, P in dynes/cm.^{2}, v cm./sec.

If P is due to gravity it $=h\times d\times g$ (§ 62) where h is the 'head' of liquid above the orifice, and hence

$$v=\sqrt{2hdg\div d} \text{ or } v=\sqrt{2gh}$$

This does not involve d; hence jets of water and mercury would issue at the same speed from tanks of equal depth, a speed which would throw them in vertical fountains up to the original height (cf. § 63) but for friction. The speed is the same whichever way the jet points.

§ 89: **Effusion.** A way of comparing the densities of gases has been based on the relation $v=\sqrt{2P\div d}$. A graduated glass cylinder is filled with a gas and floated, open end down, in mercury. The top of the cylinder is closed by a very thin metal plate in which is a very small sharp hole. The weight of the sinking cylinder causes a pressure in the gas and blows it (causes it to 'effuse') through the hole. The time of escape of the gas between, say, 20 cm. and 15 cm. marks, is noted. Then the experiment is exactly repeated with the other gas.

The velocity of escape is of course inversely proportional to the time a given quantity takes to escape; it is also inversely as $\sqrt{d}$, because P has been the same for both, therefore

$$\textit{Time of escape} \propto \sqrt{\textit{density of gas}}$$

[Compare the law of gaseous diffusion, an utterly different process.]

The hole must be very sharp-edged, not tubular in the least, or the viscosities of the gases interfere.

§ 90. A stream of fluid exerts pressure on any obstacle that checks its motion. The brook runs against the water-wheel and turns the mill. The great water pressures available in mountain districts are utilized to produce jets which impel the cupped 'Pelton wheel' with great speed and power. Steam jets drive the Laval turbine wheel up to 500 revolutions per second. Of course, as with solids (§ 13):—

Force on obstacle = momentum destroyed on it per second = *mass of fluid delivered per second* × *its loss of forward velocity.*

If the fluid be brought exactly to rest after it strikes the wheel (the ideal towards which the engineer shapes his floats and blades)

the force pressing on the surface is equal to that which originally set the fluid in motion, viz. the mass delivered per second × its velocity of outflow.

§ 91: Now the mass delivered by a jet per sec. per sq. cm. cross-section $= v$ c.c. $\times d$, and moving at v this carries momentum $mv = vd \times v = v^2 d = (2P/d) \times d = 2P$.

This seems paradoxical and contradictory of what has just been said. How can a stream exert double the pressure that started it, when by the Newtonian second law its whole momentum when stopped can only reproduce the original force?

Recollecting, however, that pressure = force ÷ area, it must mean that the cross-section of the stream where v is measured is only a fraction of that over which the driving pressure was exerted. In the Fig. 48 fluid is travelling to the aperture from all sides; having begun to acquire energy of motion its pressure has already diminished some distance away from the orifice, hence the pressure on the wall around the orifice is less than that at the same level on the back wall, and the total force engaged in expelling the stream = difference of forces on back and front walls, *exceeds* (hydrostatic pressure × area of orifice only).

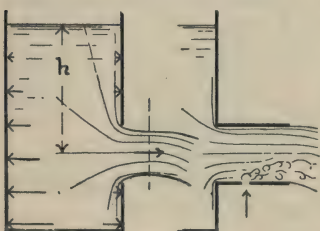


FIG. 48.

The Vena Contracta. The fluid rushing in from all sides cannot immediately lose its inward momentum. This pinches the stream into a 'contracted vein,' which at half the diameter of the orifice outside it has only about $\frac{2}{3}$ its area. It is in this narrow part that the calculated v is reached (less, in water, 3 % for friction).

A short pipe attached increases the outflow: a small hole bored in it near the vena-contracta position explains why, for air rushes noisily in, showing that the pressure in the pipe is there less than the atmospheric: hence the stream is being driven out by a greater difference of pressure than before.

§ 92. **The Sprengel dropping pump and jet pumps,** Fig. 49. If drops fall from a jet A into the mouth of a long narrow tube B the surrounding air of course finds its way in between them, and a

succession of alternate 'plugs' of water and air slips down the tube. This removal of air from C continues until its pressure is so reduced that the contents of the tube are held up from falling. But if this tube is somewhat longer than the barometric height ($2\frac{1}{2}$ ft. mercury, 34 ft. water) the weight of liquid in it can always overcome the outside atmospheric pressure at its foot and the falling will go on and completely exhaust C and any attached vessel. This the effective, but slow, Sprengel vacuum pump.

If instead of drops a violent stream rushes from A and drags air with it, this very rapid stream will produce, if gradually checked in a conically widening outlet tube, a pressure greater than the atmospheric, i.e. it will go on removing air from C and will prevent any being forced back from the atmosphere; thus dispensing with the long fall tube and working much faster.

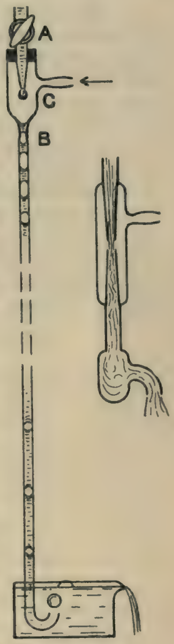


Fig. 49.

The glass 'Filter Pump,' Fig. 49 (right), which with an adequate head of water will exhaust down to merely the vapour pressure of water, or will pump water, or deliver air under pressure for blowpipe use, is a familiar instance. Steam jet 'ejectors' are used to remove air from the pipes of the 'vacuum brake' and steam 'injectors' drive feed-water into the locomotive boiler.

§ 93. **Reaction from jet.** Equal and opposite to the force the jet can exert when stopped is of course, by the third Law of Motion, a reaction on the vessel from which it started, § 91.

A firework display illustrates this to perfection from beginning to end, for the driving force of rockets, catherine-wheels, and all fly-about fireworks is their recoil from the outrushing powder gases. And a fireman has to support a reaction of several pounds weight on the nozzle of his hose; if the latter happens to wrest itself from his hands its lively backward writhings become widely interesting to the crowd.

The rotary lawn-sprinkler, whose radiating water pipes continuously retreat from the jets they deliver at a tangent from holes in their sides, is the modern form of Barker's Mill, the original 'reaction turbine.' So is the spinning ventilating cowl; actuated by the hot air passing out through it. The **Turbines**

of the present day are elaborations and combinations of this 'reaction' machine, of the 'impulse' wheel of § 90, and of the forward 'action' pressure wheel exemplified in windmills, old and new.

EXAMPLES.—CHAPTER XI

1. Roughly compare the speeds of outflow of air and water, from the same pressure, through a tap.

2. Compare the speeds of outflow of water and mercury from the same pressure.

3. Calculate maximum speed at which salt-water density 1·018 would be driven into a vacuum by the atmospheric pressure.

4. A reservoir is filled 6 m. deep. What is the greatest speed of outflow from a tap in the bottom, and if the stream had a cross-section of 3 sq. cm. how much would flow out in a minute? [L.]

5. A jet carrying 1 kg. of water per sec., leaving a tank vertically at 2 m. per sec., falls 5 m. on to a table, without splash. Calculate pressure on table. [L.]

6. Calculate the force of reaction on the fireman of a hose from which a 1·4 cm. diameter stream of water is issuing at 30 m./sec.

CHAPTER XII

ELASTICITY

§ 94. In elementary mechanics one thinks of solids as *rigid*, retaining their shape perfectly, and of liquids as *incompressible*, retaining their bulk perfectly, whatever forces act. Such substances do not exist. For if two perfectly rigid bodies ever met the absolutely instantaneous change of momentum on contact caused an infinite force which broke them, as they would not otherwise yield. Thus the sea rounds its hard pebbles. Thus the stream of sand splinters off the glass surface at which it is blown in 'sand-blasting,' but falls harmless from the soft gelatine covering the parts to be left plain.

All solids and fluids yield more or less to force, all elastically regain their bulk when the force is removed. And those that have a shape of their own (solids) either elastically regain also that shape or have been plastically moulded into another.

§ 95. **Hooke's Law.** Elasticity was studied by Hooke (ca. 1660) in what at first sight seems its simplest form. He hung weights on a wire and measured its elongation, and summed up his results in the law—*Ut tensio sic vis*—'as the stretching so is the force,' i.e. stretch and force causing it are proportional to each other. Two long wires of the same* metal hang from the same* hook, one is stretched by a constant load and bears a scale, alongside this moves a vernier attached to the second wire and reads its elongation as its load is progressively increased.

Plotting load against extension gives *a straight line*, retraced as the load is reduced. See the dotted line OE for wrought iron, Fig. 51.

A long thin heavily loaded wire of course stretches more than a short thick lightly loaded one. To get a number depending on the nature of the substance alone one must adopt a standard size and force, viz. 1 cm. long and 1 sq. cm. cross-section (a 1-cm.

* Thus eliminating thermal expansion and yielding of support.

cube) and 1 dyne. The *coefficient of linear elastic extensibility* is the fraction of a cm. that the cm. length of the cube stretches in response to a pull of 1 dyne applied over the sq. cm. base. It is very small, and the smaller the less yielding and 'stronger' the substance. This is inconvenient, and one inverts it and defines instead **Young's modulus of elasticity, Y,** *as the ratio of the force per sq. cm.* (*the tension, or pressure*) *to the elongation or compression of* 1 *cm. which it produces.*

Then stretching force per sq. cm. $=$ Y $\times$ elongation of 1 cm.,

$$\text{or } \frac{\text{stretching force}}{\text{area}} = \text{Y} \times \frac{\text{total elongation of whole length}}{\text{whole length}}$$

§ 96: This sort of elasticity is, however, not the simplest. For as anyone can see with india-rubber, the substance contracts sideways as it is stretched lengthways, or bulges when compressed, changing shape, but evading much change of bulk. A body undergoes a more simple elastic change when subjected to uniform (fluid) pressure from all sides, it contracts in bulk without change of shape;* the diminution in volume per c.c. per dyne/cm.^{2} pressure = its *coefficient of compressibility;* the reciprocal of this is its **bulk Modulus, E.**†

The bulk moduli of various solids have been measured in the following manner: A stout steel tube, like a gun-barrel, is plugged at the far end and is bored out a trifle larger at the near end (like a cartridge-chamber) leaving a shoulder inside. A rod of the solid, thinner than the bore, is pushed down it and is held in contact with 'muzzle' plug by a spring. A split collar is put on the rod and pushed down till it rests against the shoulder in the tube. Rod and adherent collar are removed, a scratch is made on each, and the distance apart of these scratches is observed under the microscope. They are replaced, water is admitted to the tube and hydraulic pressure put on. The rod, pressed from all sides, shrinks and shortens, slipping a little through the collar which cannot pass the shoulder in the tube. The pressure is relieved and the rod and collar removed and re-examined; the shift of the collar towards the end of the rod

* Except crystals, which are unequally elastic in different directions.

† *Stress* = force per unit area.

Strain = change of length per unit length, or of volume per unit volume, as the case may be.

Then *a Modulus* = *stress* ÷ *strain.*

And Hooke's Law can be generalized to, **Strain** $\propto$ **Stress.**

shows the lengthwise compression, hence the lengthwise compressibility, and the bulk compressibility is 3 times this, by the argument of § 132. The stretching of the steel tube has to be allowed for; the distance between marks on its outside is kept under observation during the experiment by a comparator (§ 129, Fig. 61).

Roughly speaking, bulk modulus = Young's modulus.

§ 97: The slight compressibility of a liquid is measured by enclosing it in a bulb, Fig. 50, with graduated stem of stout small-bore tubing, volumes of bulb and stem divisions being known, cf. § 134. A mercury thread in the stem shuts the liquid in; beyond this gauged hydraulic pressure is applied and drives it down as the liquid compresses. But the fluid pressure would expand the bulb; to prevent this the whole bulb is immersed in the pressure water and sustains the same pressure outside as in. This, however, does not keep its volume constant, for its outer surface is larger than its inner, however thin it may be. Thus there is a surplus of force which compresses every particle of it, and the whole bulb shrinks to the same extent as would a solid lump of glass, so that the compressibility of glass must be added to the apparent compressibility of the liquid. The apparatus is partly submerged in a constant-temperature water bath.

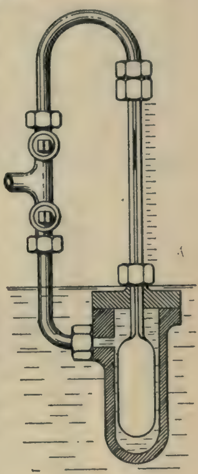


Fig. 50.

Liquids can be elastically expanded. For instance, water can be hermetically sealed 'in vacuo' in a stout glass tube which it all but fills. Warming makes it quite fill the tube, and this it continues to do when considerably cooled down. Presently it lets go suddenly and leaves a large bubble, having evidently been in an expanded condition under a 'negative pressure' or tension.

§ 98: Bulk elasticity is the only possibility in fluids, but is practically unimportant in solids as it cannot break them. Young's modulus controls the bending of beams, carriage and clock springs, etc., for the inner side is directly compressed and the outer stretched. Twisting brings in a third species of elasticity whose Modulus, that of **Rigidity**, is less than Young's.

It controls the strength of shafting, of helical springs directly pulled (which purely twists the wire), of resistance to shearing, etc. The history of a specimen twisted to destruction resembles that of one broken in tension, which follows :—

§ 99. Solids acted on in one direction by great forces presently reach an **Elastic Limit**. Thereafter their modulus has little or no meaning. They are overstrained, they do not immediately return to shape, they retain a deformation or 'set,' the extent and permanence of which depends largely on how long **time** the excessive stress was acting. The solid has begun to show the plasticity of a very viscous fluid. Watch and compare the yielding and the efforts to return to shape of a stick cut from the hedge after you have bent it a little, for however long ; and more severely, for different times.

Wrought iron and steel show this remarkably well. A **stress-strain diagram** for an ordinary 10-in. specimen on the testing

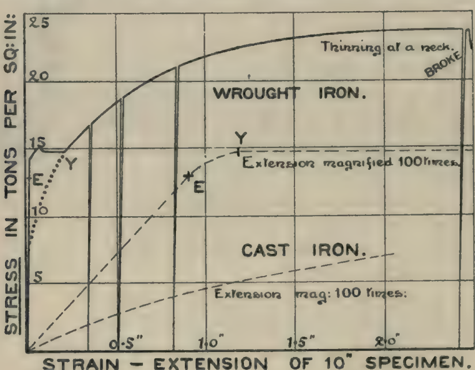


FIG. 51.

machine is given in Fig. 51. The specimen stretches, though hardly appreciably, with perfect elasticity up to a high limit E, the weigh-beam bouncing under the hand. Beyond E begins a permanent set, very slight at first, but at the yield-point Y the beam drops and the specimen stretches and stretches visibly though slowly. Presently, however, it recovers itself and picks up the load again. Increases of load now cause very large plastic extension, but slowly coming to a standstill for each load.

This forms a *new elastic limit* for the now altered and hardened specimen (as at 17, 18, 21 tons). Ultimately the specimen terminates the experiment by pulling out a neck and breaking there.

The elastic limit is sometimes poorly marked, e.g. the cast iron in Fig. 51 gives a line falling away from the direction it started in almost from the very beginning, i.e. it fails to obey Hooke's law as soon as any serious stress is put upon it; it has no definite modulus. And in most malleable metals the plastic stage starts gradually, without any remarkable yield-point [travelling say along the round dotted line in Fig. 51].

On quasi-fluid behaviour depends, of course, the possibility of drawing into wire (ductility) or extruding by steady pressure or by hammering (malleability). Under the microscope it is observed that the constituent crystals of the mass are cleaving into layers which glide over one another without loss of cohesion. Continued, this process develops a stream-line structure, recognized as the grain of wrought iron, and as a fictitious stratification in metamorphic gneiss, etc.—rocks probably crystallized from fusion with granite structure and then distorted by earth movements, while confined under pressure too great to permit their losing coherence and crumbling up.

Alternating or repeated stress (e.g. in bicycle forks) has been shown to be perfectly harmless if within the natural elastic limit, but rapidly destructive if a trifle beyond it.

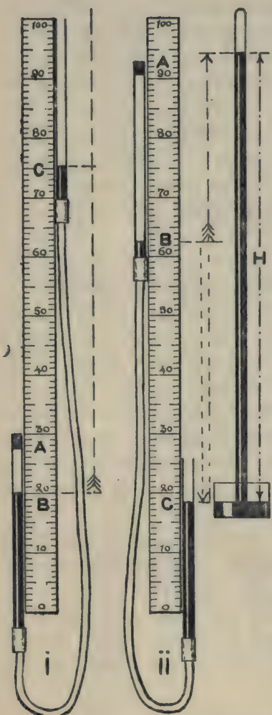
A true elastic limit in many materials, e.g. glass and rubber—colloid materials—is very low, if indeed it exists. They return quickly nearly to shape but not quite, the latter stages of the return may lag for minutes or hours.

§ 100. **Work absorbed in elastic stretching.**

If a specimen steadily stretches a distance e under a force which has steadily increased from zero to F, averaging therefore $\frac{1}{2}$F, the work done is the product $\frac{1}{2}e$F.

Thus elastic materials withstand a blow, absorbing its energy without fracture. Most of the energy is returned as the stress passes. Herein lies the use of springs of every sort.

India-rubber by reason of its enormous extensibility can store, per pound, 10 times as much elastic energy as spring-steel (instance its use in toy aeroplanes, etc.), but on account of its elastic lag does not restore it all, losing a few per cent in that internal friction which accounts for the lack of 'life' in a thick bicycle tyre, and for the heat developed in motor tyres.



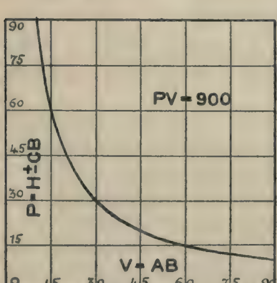


FIG. 52.

A material which has a high modulus and yields but little, and when overstressed cannot save itself plastically, is **brittle**.

Hard substances must have a high elastic limit and an enormous breaking stress, but their very inflexibility exposes them to the fate of the two rigid bodies. A mixture of hard and soft—glittering carborundum set in cement; or the natural mixture constituting a high-speed tool-steel—often displays their good qualities to best advantage. There are many special methods of estimating **Hardness**, but no good general method of all-round application; it varies too enormously.

§ 101. **The Elasticity of Gases** (volume compressibility) can be investigated throughout far greater change of bulk than can that of solids or liquids. Hooke's law still holds for small changes (e.g. § 288, compression in sound waves) but fails for greater, and is superseded by

Boyle's Law. *At constant temperature, the volume* V *of any particular mass of any gas varies inversely as its pressure* P.

$$P \propto \frac{1}{V} \text{ or alternatively } PV \propto 1$$

i.e. PV is constant for a constant mass of any particular gas at a constant temperature.

This relation between P and V is graphically expressed by the hyperbola of Fig. 52.

Robert Boyle discovered this (ca. 1660) by aid of a U tube containing air in its short sealed limb shut

in by an increasing height of quicksilver in its long open limb. He found that an extra 29 in., which amounted to doubling the barometic pressure, halved the volume of the imprisoned air.

The laboratory apparatus of Fig. 52 consists of two tubes a foot long connected by bicycle-pump tubing. Air or other gas is enclosed in the left-hand tube between mercury and the flat sealed-in stopper; its volume is proportional to its length AB.

The pressure it is sustaining is that due to the extra height of mercury BC, *plus* the barometic height H representing the atmospheric pressure on the top of that mercury in the open tube. With this apparatus, taking care not to warm the air by sudden compression or by handling, one can prove that

$$PV = (H \pm \text{height BC}) \times (\text{length AB}) = \text{constant}$$

[+ in (i) above atmospheric pressure and — in (ii) below atmospheric pressure H]. The two positions shown lie on the hyperbolic curve beneath.

For the relation, *modulus of compressibility of a gas = its pressure,* see § 311.

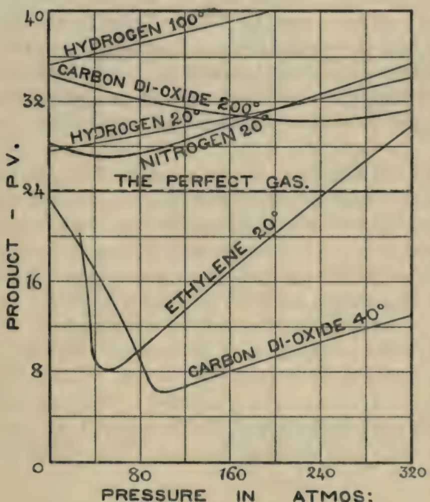


FIG. 53.

§ 102: Boyle's Law has been tested to **high pressures** by using very tall mercury columns. As the gas then compresses to an

inconveniently small bulk, the increase of pressure that just halves the volume is found, then keeping P constant more gas is pumped in up to the original volume, then P increased to halve this and so on, step by step, making all allowances for stretching of the containing tube, temperature and compressibility of mercury in the tall gauge, etc. Regnault found that the product PV decreased slightly with increasing P for all gases except hydrogen, where it increased. **Amagat**, going farther, found as in Fig. 53 that all gases ultimately followed the example of hydrogen. Anywhere near their liquefying points gases collapse with undue ease (CO_2), SO_2 does this noticeably even at 2 or 3 atmos.; hydrogen is of course farthest from liquefaction.

§ 103: According to the **Kinetic Theory**, which supposes all matter to be made up of separate minute molecules in rapid motion, a Gas consists of a swarm of molecules occupying a comparatively large space. In this they fly about at high speeds and free from one another's interference except for the brief event of a 'collision' when two of them come so close as to change each other's paths. Actual collision between hard particles is quite the crudest way of putting it, but it will suffice for us. [Elasticity, as we know it in matter, is a property of molecular crowds, the *perfect* rebound of molecule from molecule, whether of gas or of the wall of the vessel, is no mere elastic collision.]

Let a cubic centimetre of gas contain N molecules each of mass m, and let their average speed be v. Dealing with momentum, we may neglect the inter-molecular collisions, for at each collision there is a mere transference of momentum without loss (and we cannot follow an individual molecule). We may divide N into 3 equal groups, going N. and S., E. and W., up and down respectively; every molecule makes v journeys per second across the 1 cm., striking either wall $\frac{1}{2}v$ times; therefore each wall receives per second $\frac{1}{3}\mathrm{N}\times\frac{1}{2}v=\frac{1}{6}\mathrm{N}v$ blows. Each blow gives it momentum $m\times 2v$, since the molecule is stopped *and reversed*; hence the forward momentum destroyed on 1 sq. cm. per sec. = pressure on wall in dynes/cm.$^2=\frac{1}{6}\mathrm{N}v\times 2mv=\frac{1}{3}\mathrm{N}mv^2=\frac{1}{3}$ mass of molecules in 1 c.c.$\times v^2$, or

$$\mathrm{P}=\tfrac{1}{3}\ \text{density}\times(\text{molecular speed})^2.$$

Boyle's law follows from this, for the density is of course inversely as the volume into which the mass is crowded, and $\mathrm{P}\propto$ density if speed is constant.

§ 104: **Van der Waals** modified Boyle's law by taking into account cohesion in the gas. Such cohesion is strong in liquids, for a great amount of heat energy has to be supplied to tear apart their molecules (latent heat of vaporization, § 166), and as highly compressed gas and liquid can on occasion become indistinguishable (critical point, § 215) it is not absurd to assume its existence in gases. This slight mutual attraction holds the molecules back from striking a full blow on the walls, i.e. P observed is less than the true pressure, which may be written $(P+a/V^2)$ where a is a small constant. This correction, while very small at ordinary pressures, becomes rapidly larger as V is diminished by compression.

Further, the molecules are not mere mathematical points. Whether one thinks of them as hard colliding spheres or as centres of strong repulsion the effect is that each occupies a certain volume of its own, into which no other can penetrate. So that the space actually available for molecular wanderings is the measured volume V reduced by a small quantity b.

Van der Waals therefore writes

$$\left(P+\frac{a}{V^2}\right)(V-b)=\text{constant } [=RT, \ \S\ 154]$$

and with a proper choice of a and b (e.g. for CO_2 $a=\cdot 00874$, $b=\cdot 0023$) this equation does satisfactorily fit the experimental curves of Fig. 53.

Cohesive attraction and abrupt repulsion are difficult to reconcile at first sight. But the former is a property of the molecules themselves, while the latter is a consequence of their rapid motion. Compare the human desire for company, but no crowding.

NOTE.—Some values of *Young's modulus* are, in millions of millions of dynes per sq. cm.—Steel 2·0, copper, brass, and bronze ·77 to 1·0, glass ·4 to ·6.

Rigidity—steel ·8, copper, etc. ·3 to ·4, glass ·17 to ·24.

Some *liquid compressibilities* are, in millionths of millionths of the original volume for 1 dyne per sq. cm.—Water 50, glycerine 25, various oils 48, alcohol 90, ether 140, at ordinary temperatures.

EXAMPLES.—CHAPTER XII

1. Explain Strain and Stress; define Elasticity; give Hooke's Law, and verification. [Ab.]

2. Define a modulus of elasticity, explaining how measurements referred to are supposed to be made.

3. What is meant by Young's modulus, and by tenacity? A wire 1 m. long and ·5 sq. mm. section stretches ·5 mm. for 5 kg. and breaks with 30 kg.; calculate Y.M. and tenacity. [Ab.]

4. 1 grm. wt. stretches a spring 1 cm. Assuming Hooke's law calculate ergs stored in the spring stretched 10 cm. [L.]

5. Into a vertical cylinder area 12 sq. in., length 8 in., closed below, a 4-lb. piston is inserted. Where will it rest, atmospheric pressure being 15 lb./sq. in.? [Ab.]

6. A long narrow vertical tube is closed at the lower end and contains 2-ft. length of air shut in by 2-ft. length of oil. When inverted so that the open end is down the air expands to 2 ft. $2\frac{1}{2}$ in. Calculate the height of the oil barometer. [L.]

7. Torricellian space in a barometer at 30 in. being $2\frac{1}{2}$ cu. in. and cross-section of tube $\frac{1}{2}$ sq. in., an air-bubble measuring ·1 cu. in. at atmospheric pressure is admitted. How far will mercury fall? [M.]

8. A faulty barometer, with tube 40 in. long, contains air and reads 29 in. instead of 30. What will it read when true barometer reads 29? [M.]

9. Air at atmospheric pressure (32 ft. water barometer) is taken down to 30 ft. under water and liberated to form a spherical bubble. Show diameter of this has increased one-fourth when it reaches surface. [M.]

10. A cylindrical diving-bell is 6 ft. high and its top is 10 ft. under water. To what height has water risen in bell if mercury barometer stands at 30·5 in.? [M.]

11. A cylindrical diving-bell 18 ft. high, 24 sq. ft. cross-section, weighing 27,000 lb. is sunk till its top is 10 ft. under water. If water barometer stands at 33 ft., find how high water rises into bell, tension of chain, and volume of air at atmospheric pressure that must be pumped in to drive water out. [D.]

12. Calculate mean velocity of molecule of nitrogen at pressure of 1,000,000 dynes/cm^2, litre of gas weighing 1·2 grm. [L.]

Chapter XIII

THE MECHANICS OF APPARATUS OF PRECISION

In all apparatus of precision it is essential to make sure that a moving part moves to the extent and in the manner intended, and in no other way. The main principles underlying the construction of satisfactory mechanism are of an importance and interest that claims for them a short consideration here.

GEOMETRICAL THEORY

§ 105: **Degrees of Freedom**. Any motion of a free point or **particle** can be resolved into component motions in three directions at right angles; it is said to have **three degrees of freedom**. Confining it to a plane means depriving it of one degree of freedom, for it cannot now move perpendicularly to that plane. Confined to a line it has lost two degrees, moving only to and fro in one direction, and the loss of its third degree of freedom reduces it to a fixed point.

In addition to these possibilities of rectilinear motion in three directions **a rigid body** has those of rotation about three rectangular axes, N. and S., E. and W., and vertical. These three can combine in any way, § 57. Thus it has **six degrees of freedom.**

§ 106: **Constraints.** Let us see how, by progressively depriving it of these, its motions can be made very definite, with none of that wobble and slackness usually ascribed to bad workmanship, but really due to defective design.

The mode of destroying each degree of freedom is to make one point of the body touch a fixed surface.

I. *Constraining one point to move on a surface* deprives the body of the freedom of moving perpendicularly to it, e.g. Billiard-ball, spinning top.

II. *Two points confined to a plane* (leaving 4 *degrees*)—a dumb-bell rolling, spinning, or sliding either way on the floor.

One point confined to a line (leaving 4 *degrees*)—the ‘flying

pigeons' of the pyrotechnist, sliding on a wire; or a top with its toe running along a crack.

III. *Three points touch a plane* (3 *degrees*)—the usual tripod, can slide either way or rotate about a vertical axis.

One point fixed, permits 3 *rotations only*—a top spinning in a fixed socket. Actually this fixing of one point is effected by keeping 3 points on a rounded toe in contact with 3 planes meeting in an angle, a trihedral dent such as a blow from the corner of a cube would make.

[A hemisphere can geometrically touch a hemispherical cup in only one point, unless the radii are absolutely equal.]

IV. *Four points touch a cylinder* (2 *degrees*)—an axle lying in two Vees can turn, or slide axially.

[An axle in a cylindrical bearing can geometrically touch it at two points only and hence can sway and rattle as soon as it runs short of a sufficient 'packing' of lubricant.]

V. *Five points* (1 *degree; most valuable case*)—A cylinder *resting in Vees and butting against an obstacle* can rotate only.

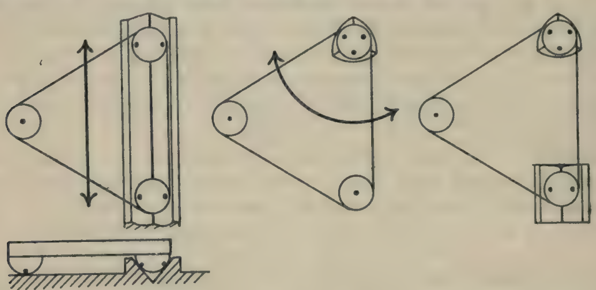


FIG. 54.

The body carries *two vees* (or V-grooved rollers) *resting on a round-topped rail* (*or else two rounded feet resting in a V groove*) *and a third foot resting on a flat rail* in one point, it can slide lengthwise only—all sliding and rolling carriages. The usual sliding carriage falls short of this in being supported, like a railway truck, at four points only, hence it can oscillate laterally and jam, like a drawer or a window-sash.

One point fixed (*see* III *above*) *and two feet resting on a plane*—rotates about one axis through the fixed centre.

VI. *Six points*. The best way of fixing the position of a body is by means of the *hole, slot, and plane*. One rounded foot rests in a conical, or rather, trihedral, hole; the second in a V-groove pointing straight away from the hole (along this it can slide if any expansion occurs, without slewing the body round), and the third rests on a flat plate.

In Fig. 54 a transparent plate with three rounded feet is shown in plan, supported as in V and VI. Points of contact are indicated by dots.

§ 107: Very often the weight of the body suffices to hold it in contact with these points of support, but it may be necessary to provide springs to press it against them. This will introduce several other points of contact, but these are only elastic points, for which the finger might be substituted.

Every rigid body to be securely held with the required number of degrees of freedom must have (six minus this number) rigid points of support and no more. It may in addition have any number of elastic points, but the pressures applied at them must leave the rigid points quite unmoved.

Ordinary non-geometrical holders fail by confusing these two varieties of supporting point. For instance, an object held by the fingers up to the fixed jaw of a vice should not be displaced in the least as the pressure of the movable jaw comes upon it, but how often is this attainable in practice? And as the flat fixed jaw can provide only 'three rigid points,' it has to have teeth to bite into the object and provide other 'three points' on their inclined faces.

MECHANICAL CONSIDERATIONS

§ 108: No material has the infinite strength and rigidity assumed above, and applications of the geometrical theory will be mechanical failures unless due attention is paid to this fact. There is a deal of apparatus on the market which gives the undiscriminating unbeliever opportunities of deriding the geometrical theory as unmechanical, but the fault is that the designer did not make adequate allowance for the non-rigidity of constructive material. He had not learnt that the production of a successful machine is a process of evolution.

That a mechanism wears loose means that its construction is ungeometrical; that it wears at all appreciably means that it is unmechanical.

It goes without saying that there must be no *shake* anywhere. A screw working in its nut is a favourite place for neglecting geometrical principles altogether, since they cannot be practically applied to it in full. At the least the nut should be 'split,' and the weak edges of the cut devoted to elastically pressing the screw against the stiffer part, or else there should be a spring as in Fig. 55 (or spring-washer for a set screw).

The forces that arise in the mechanism from whatever cause must be prevented from deforming it appreciably. *The driving force should be applied in line with the resultant of the weights, frictional resistances, etc., that it has to overcome.*

The common pattern of microscope 'fine adjustment' may be taken for criticism. When the microscope stands vertical

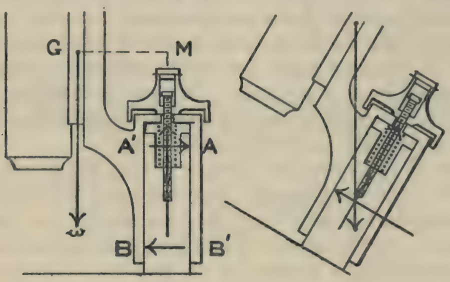


FIG. 55.

as in Fig. 55 the centre of gravity of the parts to be moved is 2 in. out in front of the micrometer screw and a couple, of moment $w \times \text{GM}$, is causing pressure between the sliding sleeve and the supporting post at A and B, inducing friction and tending to fitful working. A strong spiral spring, shown in section as a double row of dots, more than bears the weight of the moving parts and presses the upper faces of the threads in the milled nut against the lower faces of the screw threads. This makes for definiteness and no 'back-lash,' and fortunately gives the screw so much work to do that the variable friction at A and B makes less difference in the total.

When inclined, the three forces—spring pressure, weight, and post reaction—which act on the moving parts, meet in a point; the weight is carried by the spring and the upper face of the post, and the jamming force at A disappears. The mechanism fulfils

the condition specified, and undoubtedly owes its survival to its success in this familiar position.

When laid horizontal the overhang, and resulting frictions at A′ and B′, are equally bad. Small wonder that this contrivance is seldom capable of the minute steady adjustments indispensable in high-power photo-micrography. The most hopeful remedy is to carry a supporting belt under the centre of gravity of tube, etc. From this a string rises to a pulley high above and bears a counterpoise equal to weight of tube, etc.

There is on the market an obtrusively geometrical micrometer-microscope in which this fault is exaggerated, the driving screw is actually far behind the sliding ways, the overhang is doubled; mechanically, the contrivance is almost identical with the carpenter's 'valet,' and, were it not for the saving grace of the geometrical slides, would jam just as hard.

[Another objection to the usual fine adjustment is that it is attempted to make a V-grooved sleeve fit perfectly on the V-sectioned post, a '4-point' instead of a '5-point' contact. The result is a sidewise rocking which shows itself as the sudden lateral shift of the whole field of view the moment the direction of rotation of the screw is reversed.

To minimize elastic yielding all forces should be kept as small as possible. We have just seen how a faulty design brings into existence quite unnecessary stresses, all of course tending to deform the structure. It might be thought that a stoutening up of parts would compensate for additional stresses, but in addition to the increasing weight there is a consideration of 'lost motion' which a simple example will make clear. 1000 yards of wire moving 6 in. pulls down a railway distant signal, the increased stress stretches the wire an inch, say. Propose to do the same work by using rods six times stouter, moving one-sixth the distance; the stress per square inch, and hence the stretching, is the same as before, and the proposed inch of motion is wholly lost in stretch. Again, in some cycle brakes, the grip rotates a shaft through a small angle only, stressing it greatly and making it twist and slacken in its bearings till the brake soon becomes ineffective.

§ 109: How profoundly the finite strength of materials modifies geometrical form appears in comparing the slender shanks of a robin with the seemingly too-bulky limbs of the largest of land animals; a little rough calculation reveals about the same pressure per square millimetre in both.

Crushing or tensile strength is proportional to area of cross-section, i.e. to the *square* of linear dimensions. Weight is proportional to the *cube*.

Since a bar resists Compression or Tension far more stiffly than it does Bending, all forces should be taken by struts and ties as direct as possible. The stiffness of a bar to resist Bending is proportional to Young's modulus $\times$ breadth $\times$ (depth $\div$ length)^3 and is enormously greater if the bar is supported at both ends than if 'overhung.' Since stiffness depends on (depth)^3 the bar should be 'on edge.' Evidently the top and bottom layers are the most effective, since (their distance apart)^3 is so much the greatest. Hence the intermediate parts can be hollowed out to leave only the thin web, or triangular lattice-work, of the *Girder*.

The common force-diagram assumes all the forces acting in a plane; if they do not, Twisting must be met. The strength of a bar under twist is proportional to the fourth power of its effective diameter; hence the large thin tubes of a bicycle frame are the best form to resist pedalling stresses, while the thin flat strip of bronze suspending a galvanometer-coil resists its rotation much less than would a round bronze wire of the same tensile strength.

Local pressures must be kept within the crushing limit. Always of course something has to give way until (area of contact $\times$ crushing pressure of material) exceeds weight to be carried (e.g. a sharp-pointed tripod visibly settles down into the table top), but for permanent utility the stress should not reach the elastic limit of the material. *Here again small forces are far more easily dealt with than great.* The pressure on the knife-edges of a fine balance must be enormous, but the hard material sustains it and the balance is sensitive to 1 part in a million; what engineer would expect his testing machine to respond to 1 ounce in 30 tons?

§ 110: When movement is necessary under large pressures a layer of lubricant must separate the surfaces, and the area of contact, pressure, speed, and nature of lubricant must be so adjusted that the latter is not squeezed out, see § 240.

Chapter XIV

THE PRECISE MEASUREMENT OF LENGTH, TIME, AND MASS

The Measurement of Length

[Note.—In all accurate measurements of length or angle the thermal expansions of scale and object must be allowed for, and local inequalities of temperature must be avoided.]

§ 111. The methods of producing multiples and submultiples of the unit of length are of course developments of the process everybody uses with a pair of dividers.

The difficulty in using subdivided scales to read ordinary lengths accurately is that the subdivisions soon become too small to see. Without a magnifying glass it is better to guess at the decimals of a tenth-of-an-inch division than to attempt to read a hundredth-inch scale.

The **Diagonal Scale,** so familiar to the draughtsman, is an early device for coping with this difficulty. It is easy to make and use with fair accuracy but is troublesome when it comes to splitting lines. The 8-ft. radius mural quadrant provided with it at Greenwich in 1750 could be read no more accurately than a surveying theodolite of the present day.

§ 112. To a soldier of fortune, Pierre Vernier (ca. 1620), is due the contrivance most widely used in reading scales on all sorts of instruments. The main scale is graduated throughout into equal divisions as small as can be distinguished comfortably. Attached to the moving part, and sliding alongside the main scale, as in Fig. 56, is another, the **vernier,** each of whose divisions is one nth part less than those of the main scale. Evidently n divisions of this fall short n $n$ths$=1$ scale division; or n vernier divisions$=n-1$ scale divisions (in the simplest form 10 and 9, Fig. 56, top scale).

Now if the index mark on the vernier (either its edge or else marked with an arrow or 0) lies *in line with* a scale mark, then

the mark 1 on it falls short of a scale mark by $1/n$th scale division, mark 2 by $2/n$ths, etc. Pushing the vernier forward $2/n$ths will therefore bring the 2 into line with a scale mark, and so on, coincidence at the mth vernier mark meaning that it has been bodily pushed m/nths of a scale division beyond the last scale mark preceding its index.

Thus the Vernier always reads $n$ths of the smallest scale division, say tenths of the tenth of an inch, thirtieths of a half-degree, twenty-fifths of a twentieth of an inch (1/500ths), etc., e.g. the upper verniers in Fig. 56 are indicating 0·0 and 17·4;

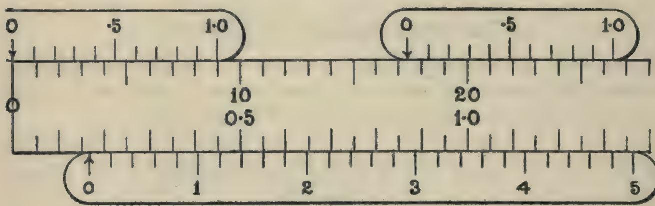


FIG. 56.

the lower vernier is reading 8/25ths of a lower scale division, and as 20 scale divisions make 1 unit this$=8\div25\div20=\cdot016$, which added to the 3 scale divisions (·15) before its index makes ·166.

Sometimes the vernier's 10 divisions$=$19 of the scale, this is merely to get more 'open' divisions and avoid dazzling the eye; it is a vernier to twentieths with alternate marks invisible, only even twentieths can be read, i.e. tenths.

Quite distinct is the vernier, to be found on older instruments, with its divisions $1/n$th *greater* than the scale divisions. Its index mark is at the far end and it is figured backwards; the reader can work out its theory.

§ 113. Better than a long vernier is the **Micrometer Screw.** A true screw will advance through a perfectly fitting nut $1/n$th its pitch for each $1/n$th of a revolution. In practice a well-made screw and nut are carefully ground together to smooth away irregularities; there must be *some* clearance between screw and nut and this gives rise to shake and 'back-lash'—travel of the screw without turning, or vice versa. To avoid these, lubricate well with grease, take readings with the screw going always in the same direction, and if possible have a spring arrangement to

press screw and nut together, always one way, with a steady pressure (cf. Fig. 55).

The head of the screw is enlarged and graduated into a large number of equal parts. A very common arrangement has a $\frac{1}{2}$-mm. pitch screw and 50 divisions on the head. A scale to read whole turns of the screw is provided, but it is often safer to count them.

In the **screw-gauge**, Fig. 57, the flat end of the screw works up to a flat anvil formed on an extension of the nut, the object whose thickness is required being put between, and *very gently*

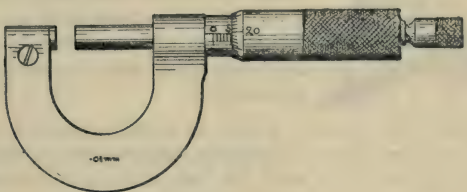


FIG. 57.

gripped. The large micrometer benches used by engineers are in principle screw-gauges with long adjustable gaps, which are first standardized by using 'end measure bars' of known length.

In the **screw spherometer** the screw is mounted so that it can measure small heights, and specially the height CZ$=h$, Fig. 58, of the arc of a curved surface between rigid projections AB* ('feet') on its nut. From this the radius of curvature R of the surface can be deduced thus :—

If AC$=$CB$=r$ (measured *carefully*, best by vernier callipers)
Remainder of diameter of curvature of surface$\times$CZ$=$AC$\times$CB

i.e. $(2R-h)h=r^2$ [Euclid, II 14 or III 35.]

$$\therefore\ R=\frac{r^2}{2h}+\frac{h}{2}$$

and in practice the $\frac{1}{2}h$ is usually negligibly small.

The Spherometer, Fig. 58, of physical laboratories usually has three feet ('points' or better, small steel balls) fixed at the corners of an equilateral triangle and each distant r from the screw point when standing on a plane. On a sphere this makes no difference, the foot B has virtually split into two, one has travelled 60° E. longitude and the other 60° W. along the small

* Imagine, for the time being, a foot at B, Fig. 58.

circle 'of latitude' in which the plane ACB cuts the sphere AZB. On a cylinder, however, they differ; the pattern with feet in line reads full curvature one way and zero the other, the tripod pattern averages up and reads half (giving 2R instead of R) in whatever position it stands on the cylinder [proof tedious].

Contact of the screw point is indicated by the instrument being able to spin round or to just perceptibly totter.

[NOTE.—h is proportional to $1/R$ the 'Curvature' of the surface, and equal increases of h mean equal increases in curvature.]

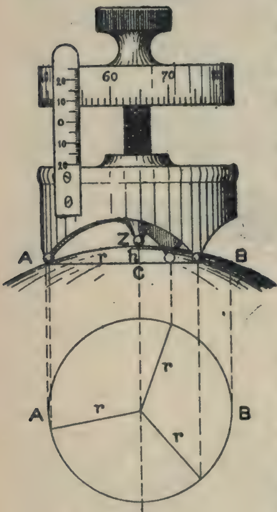


FIG. 58.

§ 114. In dividing engines, travelling-micrometer-microscopes, cathetometers, etc., the end of the screw presses on a carriage sliding on 'geometrical ways' and carrying the cutting tool, or a cross-wire microscope or telescope (§ 465).

Very small lengths are measured by first forming a real image of the object magnified a known number of diameters, and then travelling over this image a fine spider-line fixed across a frame pushed by the screw, image and spider-line being examined by a magnifying eyepiece. Eyepiece-micrometer-microscopes of this description are used to magnify and subdivide scale graduations far too small and delicate for verniers to be practicable.

The **Cathetometer** measures vertical distances (cathetos= upright). Up a long vertical stem slides geometrically a carriage, with vernier and cross-wire telescope. The latter has to be kept very exactly horizontal according to a sensitive spirit-level fixed to it, for a very slight tilt will cause serious error at a distance. Usually one can do just as well by setting up a scale close to the object and viewing both at once through the telescope, with its horizontal cross-wire, of a rougher ungraduated cathetometer.

§ 115. **Area.** Areas of irregular shape are measured :—

(1) By tracing on squared paper and counting squares.

(2) By tracing on ordinary paper of uniform thickness (machine made), cutting out, and weighing.

(3) By various rules proved in mathematics books, but seldom used.

(4) By Planimeters, instruments which in one way or another 'integrate' the area as their tracing point is carried once round it.

The area of a circle is $\pi r^2 = 3 \cdot 1416 \times \text{radius}^2$, the area of a sphere is $4\pi r^2$.

§ 116. **Volume.** The volume of a parallel-sided block, or cylinder, whether rectangular or oblique, is area of base $\times$ height perpendicular to it. Of a pyramid or cone, one-third this. Volume of a sphere $\frac{4}{3}\pi r^3$.

The volumes of irregular solids are easily measured by dropping them, like the thirsty crow in the fable, into water or any other liquid partly filling a jar graduated in cubic centimetres. See specific gravity, § 77.

§ 117. **The measurement of angle.** Angles are measured in degrees (360 to the circle), minutes ($60' = 1°$), and seconds ($60'' = 1'$). Practically, their measurement is that of distances round a circular scale, verniers, micrometers, etc., being employed. At Greenwich angles are quoted to a hundredth of a second, about the thickness of this paper at a distance of a mile.

The great fault of most divided circles is that their centres do not lie exactly in the axis of rotation. Simultaneous readings at both ends of a diameter are necessary to annul this error. In the best work readings by several equidistant microscopes smoothe out this and other irregularities.

The Measurement of Time

§ 118: **Time** is ticked out in successive equal intervals by the swing of a Pendulum controlled by gravity, or of a Balance Wheel controlled by a spring. In getting these intervals to average equality for any length of time the clockmaker has many difficulties to contend with.

In either case the time of one swing is proportional to the square root of the quotient of the moment of inertia by the controlling couple, § 55.

Hence the pendulum must have a constant moment of inertia about its point of support, therefore its Expansion with rise of Temperature has to be compensated, § 130, Fig. 62.

Rise of Atmospheric Pressure, increasing the density of the air, has three effects. The mass of air pushed into motion by the pendulum in its swing increases, i.e. its effective moment of inertia

increases. Secondly, the bob is buoyed up more by denser air, 1 in. rise of barometer reduces the weight (the controlling force) of a lead bob 1/300,000 part, § 77. Both effects tend to make the clock lose, but fortunately the extra labour of pushing about the heavier air shortens the arc of swing, and this reduces the 'circular error,' § 38, and speeds it up; with a seconds pendulum swinging $3\frac{3}{4}$ inches this automatic compensation is almost complete. Or the clock can be boxed air-tight and pumped up to 31 in. pressure after every winding.

The escapement must give the pendulum equal impulses, or else the arc of swing and the 'circular error' will vary. This means driving by a weight or very steady spring through a train of wheels and an escapement which never vary in their friction; a requirement difficult to meet, seeing how large a proportion of the energy is spent in them before any reaches the pendulum. Large clocks, with outdoor dials to drive in all weathers, actuate their pendulums through 'gravity remontoires'; the clock merely lifts every 30 sec. (or 1 sec.) a small weight through a constant distance, this then driving the pendulum through only one wheel (or direct) with a minimum of friction.

And further, the escapement must be such as to supply the impulse very near the middle of the swing and then let the pendulum alone. For if anywhere else, the pendulum is practically bumping against a spring and being sent back before finishing its swing, any variation in the driving force now affecting it badly. The common recoil escapement of mantelpiece clocks violates this condition, and that is why such clocks change five minutes a week when stood the least bit out of level.

In some very successful Electric Clocks the pendulum pushes a light scape-wheel which at the end of 30 sec. releases a weighted lever. A little wheel at the end of this lever falls on to a horizontal shelf on the pendulum and at mid-swing runs over the end of the shelf, thus giving it a sideways push. The falling lever then strikes an electric contact permitting a battery current to circulate in an electro-magnet, which throws the lever up again, and also in the electro-magnets of any number of dials, jerking them $\frac{1}{2}$ minute forward.

In Chronometers and Watches the expansion difficulty recurs, but a score times more serious is the weakening with warmth of the balance spring. Springs of a new nickel-steel, however, actually strengthen, and can be made to just eliminate the expansion trouble.

Chronometers lie on their backs in gimbals, but in watches the balance-staff (axle) is horizontal, and wear on the pivots lets staff and wheel sag pendulum-wise below the axis of rotation (and the wheel itself was probably never perfectly true), and gravity now assists the hair-spring [try any cheap clock on its back, and upside down]. In 'tourbillon' watches this error is got over by making the whole escapement mechanism slowly turn over and over and equalize the wear all round.

§ 119. For continuous time records involving fractions of a second recourse is had to a **chronograph,** a rotating paper-covered drum. As it would require elaborate mechanism to secure quite uniform rotation this is not looked for, but the paper is electrically marked off in seconds by a clock pendulum as in § 315, Fig. 127. It is best to utilize only the alternate marks, 2 sec. apart, for the contact-maker may not be just in the middle of the pendulum's swing.

A second electrical marker is connected direct to the experimental apparatus or else to the observer's press-button. Direct connection is preferred where possible, for there is always a fraction of a second interval, called the observer's **personal equation,** between his seeing or hearing a signal and his pressing the button. This interval can be measured by contriving that a signal, imitating that which he will have to observe, shall record itself on the chronograph which immediately after receives his record of its occurrence. Fortunately a practised observer's personal equation for the same sort of signal remains fairly constant for a few hours, so that the time lost at the beginning of a record is made up at the end; but between morning and evening, between day and day, between sight and sound signals, between one observer and another, personal equations vary and must be measured and allowed for.

The electrical marker itself has a personal equation, but constant and seldom exceeding ·02 sec. Records can be scratched on smoked paper, or made in ink, or pricked in. For high speeds a standard tuning-fork is timekeeper, Fig. 127.

Small calculable intervals of time are easily obtained by letting a heavy pendulum knock down two triggers placed a definite distance apart in its path, near mid-swing.

Note.—Technically the *Error* of a clock is the amount it is slow at noon, its *Rate* is its loss per day.

The Measurement of Mass

§ 120. The comparison of masses with each other and with the standard is effected by comparing their weights with the **Common Balance,** in the full faith that gravity acts on all substances equally.

The stiff balance beam, Fig. 45, bears three 'knife edges'—sharp-edged prisms of steel for heavy weights, agate or rock crystal for light. The middle inverted one rests on a flat plate of the same material on the supporting pillar, the others carry flat plates from which hang the pans, etc. To preserve the delicate edges from crushing, mechanism (not shown in the figure) is provided for lifting edges and plates out of contact except when actually testing the equilibrium.

The balance informs us when the turning-moments of the forces applied at the outer knife edges are equal and opposite. The intention is that this shall mean equal weights, and this, since they are vertical forces, involves equal horizontal distances from the centre.

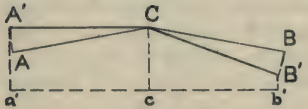


Fig. 59.

This involves

(a) knife edges parallel to one another;

(b) their distances, the balance 'arms,' equal;

(c) edges all touching a straight line, the theoretical beam, dotted in Fig. 45. For if not, let them be ACB, when tilted they move to A′CB′ (Fig. 59) and now horizontal $a'c$ is not equal to cb'. Such a balance alters its readings according to the zero to which one adjusts it to work, or unless levelled with extraordinary care.

(d) beam should be so stiff that working loads do not appreciably bend this straight line.

(e) empty pans+arms must exert equal moments. Horizontally adjustable weights on the beam regulate this.

(f) there must be a bias towards mid-position (feebly stable system). If the centre of gravity of the beam is in the central edge it rests indifferently at any tilt, which is useless. A central 'gravity-bob' G is lowered till the c.g. is below the edge to any desired extent: when the beam tilts the c.g. swings out towards the up side and has a moment tending to turn it back.

Evidently the lower the c.g. the less the tilt for which this moment attains a given value, i.e. for a given overload in one pan.

Thus one controls the **sensitiveness** of the balance, which is *the number of scale divisions the pointer is displaced for a definite small overload* (*often* 1 *milligram*).

A **Rider** of aluminium wire weighing 10 mg. can be placed anywhere on one arm, which is divided into 10 equal parts (sometimes 12 mg. and 12 parts). Placed directly over the end knife edge this has a moment 10 mg. × full distance 1 at which the weights in the pan act. Placed say at the third division from centre of beam its moment = 10 mg. × distance ·3 which is equivalent to 3 mg. × distance 1. Thus it now turns the balance just as much as a weight of 3 mg. in the pan. The one rider saves fiddling with weights from 10 mg. down to ·1 mg. The thing is a common steelyard in miniature.

§ 121. No actual balance completely fulfils the conditions laid down above, and for really accurate weighing one of two devices has to be employed :—

I. **Weighing by substitution.** The body is carefully counterpoised by shot, etc., in the other pan. The body is removed and weights are put in its stead until the original equilibrium is re-established ; evidently these amount to the weight of the body.

II. '**Double weighing.**' If the imperfectly equal lengths of the balance arms be r and l the body in the left pan will be balanced by l/r times its weight placed in the right. For in a lever the forces are inversely as their distances from the fulcrum. Now changing the body to the right pan it is counterpoised by r/l times its weight in the left. Multiplying the two weights together and taking the square root, this, $\sqrt{\frac{l}{r}\mathbf{w} \times \frac{r}{l}\mathbf{w}} = \mathbf{w}$. Or, what comes to the same thing in practice in any balance not too glaringly lopsided to be used, add them together and divide by 2.*

All these procedures demand is a balance, of suitable delicacy, consistent in its readings when asked to do the same thing twice running ; it is sufficient that the instrument have clean sharp knife edges firmly affixed, and be sheltered from draughts and sudden changes of temperature.

* And evidently dividing one weight by the other and taking the square root gives the ratio of the arms,

$$\sqrt{lw/r \div rw/l} = l/r = 1 + \tfrac{1}{2} \text{ difference of weight} \div \text{weight}.$$

NOTE.—In actual weighing, to be sure that there is no sticking anywhere, the beam is kept swinging gently, and the rest-place obtained as the mean from scale readings of successive swings left-right-left. To save time the balance is made to swing more quickly by shortening its beam and so reducing its moment of inertia, and by lowering the gravity-bob and so increasing its controlling force, see § 55. These proceedings have reduced its sensitiveness, but that can be restored by using a very fine scale read by a magnifying glass or mirror.

It is useless to attempt to weigh hot bodies, on account of the rising draughts they create.

For ordinary work the balance case should not be kept specially dry by hygroscopic chemicals, for it has to be recognized that glass apparatus is coated with a film of moisture *impossible to get rid of* but likely to evaporate to an uncertain extent in a dry atmosphere. In fact it is recommended to wipe glass with a very slightly damp cloth shortly before weighing.

§ 122. The **Density** of a material is the mass of unit volume of it.

In the c.g.s. system this is in grammes per cubic centimetre and ' specific gravity ' is almost identical with it, § 80.

In British measure one can take one's choice of a multitude of unit measures of capacity.

§ 123. **Correction for weighing in air.** Finally it is necessary to correct for both body and weights being buoyed up by the air around them to an extent equal to the weight of air they displace. This prevents the weights exerting their full face value, while the body itself appears lighter than it should.

1 c.c. of dry air at 0° C. and 76 cm. mercury pressure weighs ·001293 grm., and this requires correction for temperature, pressure, and humidity ; commonly 1 c.c. of air may be taken as weighing ·0012 grm.

Therefore the force with which the body presses on its scale pan

$$=\text{its true weight } \mathbf{w} - \text{its volume} \times \cdot 0012$$

$$=\mathbf{w} - \frac{\mathbf{w}}{\text{its density}} \times \cdot 0012$$

And the force with which the weights press on their pan

$$=\text{their true weight (face value) } \mathbf{G} - \frac{\mathbf{G}}{\text{their density}} \times \cdot 0012$$

And these forces are equal

$$\therefore \mathbf{w}\left(1-\frac{\cdot 0012}{\text{body's density}}\right)=\mathbf{G}\left(1-\frac{\cdot 0012}{\text{weight's density}}\right)$$

$$\text{or}\quad \mathbf{w}=\mathbf{G}\left(1-\frac{\cdot 0012}{\text{weight's density}}\right)\div\left(1-\frac{\cdot 0012}{\text{body's density}}\right)$$

$$\text{or}\quad \mathbf{w}=\mathbf{G}\left(1-\frac{\cdot 0012}{\text{weight's density}}+\frac{\cdot 0012}{\text{body's density}}\right)$$

since the corrections are small (see note, p. 112),

and using brass weights, density 8·4,

$$\text{True weight}=\text{face value of weights}\times\left(1-\frac{1}{7000}+\frac{\cdot 0012}{\text{body's density}}\right)$$

§ 124. With very light and bulky bodies it may become necessary to alter the ·0012 to allow for barometric variations, etc., but this trouble is sometimes avoidable by using an equally bulky counterpoise. For instance, in determining the **Density of Gases** a sealed-up dummy glass bulb is hung with the weights to counterpoise the equal bulb in which the gas is contained. Then the mass of gas filling the latter=excess over its weight when pumped to a vacuum, with *only* the following corrections :—

(a) A practically negligible 1/18,000 to the platinum gramme-fractions probably used.

(b) The bulb shrinks when evacuated, owing to the then unbalanced atmospheric pressure. This small shrinkage can be found when the volume is being measured by weighing full of water; when connected up for a moment to a vacuum vessel a little of the water will be sucked out, and the bulb is weighed again, when grammes lost=shrinkage. Then shrinkage × ·0012 must be added to the apparent weight of gas, and true weight ÷full volume of bulb=density.

Actual weighing in vacuo is a long and costly business.

EXAMPLES.—CHAPTER XIV

1. Describe the use of the electric chronograph for measuring short intervals of time. [D.]

2. Describe the requisites of a good balance. Prove that true weight = geometrical mean [or for all practical purposes the arithmetic mean] of the apparent weights in the two pans, and show that ratio of arms = square root of ratio of apparent weights. [M.]

3. Describe an accurate balance and explain the principle of the rider and divided beam. [L.]

4. A balance with 10 gr. in each pan rests several divisions out of centre. How would you find whether this is due to defects of balance or inaccuracy of weights? [L.]

5. Against brass weights in air a litre flask, of glass sp. gr. 2·4, weighs 150 grm. Find its true weight in vacuo.

$$\text{From § 123, } w = 150(1 - 1/7000 + \cdot 0012/2\cdot 4)$$
$$= 150(1 + \cdot 00036) = \underline{150\cdot 054.}$$

6. The flask is then filled with liquid and appears to weigh 950 grm. Find true weight of liquid.

7. A 100-grm. weight of rock-crystal is being tested against a 100-grm. of platinum. What allowance should be made for air buoyancy, if the quartz has been found to weigh about 62 grm. in water and the platinum 95·6 grm.?

NOTE.—Approximately, when a and b are small

$$\mathrm{N}(1 \pm a)(1 \pm b) = \mathrm{N}(1 \pm a \pm b)$$
$$\mathrm{N}(1 \pm a) \div (1 \pm b) = \mathrm{N}(1 \pm a \mp b)$$
$$\mathrm{N}(1 \pm a^m) = \mathrm{N}(1 \pm ma)$$
$$\mathrm{N}^m\sqrt{1 \pm a} = \mathrm{N}(1 \pm a/m)$$

These simple results of the binomial theorem are often very handy.

HEAT

Chapter XV

THE EXPANSION CAUSED BY HEAT

§ 125. That most familiar of scientific instruments, the Thermometer, measures the temperature or the hotness of its immediate surroundings. The primitive utterly cold substance being unattainable, its scales of degrees have been made to start from points at which it stands when surrounded by some arbitrary cold substance, just as our chronology starts from a zero comparatively recent in time. The scale we shall mostly use starts from zero in freezing water and reads 100 degrees in (ideal) boiling water; it is hence the 'Centigrade' scale; it is preferred in physics as minimizing numerical complications.

As the common thermometer depends on expansion (of a liquid past a scale) we had better study the expansion of substances in general, caused by heat, before dealing with thermometry. For the present, temperatures will be supposed measured with a bought centigrade thermometer of certified accuracy. For true measurements it will be wholly immersed in quickly circulating fluid.

§ 126. **The Expansion caused by heating things.** Illustrations of this abound on every hand, though the expansion is too small to be directly visible except on long lengths, for the great swelling of a red-hot poker or of a lamp filament is an optical illusion due to local dazzling of the eye. We warm the neck of a bottle to ease out a stuck stopper. Telegraph wires sag noticeably more in summer. On a hot day the 'distant signal,' pulled down through 1000 yards of wire, only languidly indicates a clear line. The rods to distant 'points,' where half-way motion cannot be tolerated, have to be contrived so that half their length pulls and half pushes. The gaps at the ends of rails visibly close up, and very exceptionally more than do so; quite recently I have

known the traffic delayed while fifty yards of line was being persuaded to lie down flat again.

Conversely, the reduction of temperature is accompanied by contraction. The tire grips the cart-wheel tightly when quenched from the blacksmith's bucket, cranks are sometimes shrunk on to shafts, and formerly jackets of great guns on to the inner barrels, over which they just slipped when hot.

Liquid expansion is instanced by the thermometer itself or by the overflowing of a saucepan, quite full of cold water, long before it boils. Smoke rises, for its Gases have expanded till they are less dense than the cold air around, and solid sparks are borne upward by the little invisible 'balloons' of hot air which they themselves produce.

§ 127. **Forces involved in thermal expansion.** All substances yield to force and the stretching of a brass wire, say, due to a pull may be compared in the laboratory with that due to heating. Small as the latter is, it will be found to equal that produced by very large forces. Red-hot rivets hammered up tight draw the plates together with a pressure of many tons as they cool. If the engineer cannot allow for free expansion in his structures—6 in. is allowed in the Forth bridge—he must design them to meet heavy stresses. Tram rails are bolted up tight and prevented by weight of surrounding road metal from the lifting that expansion would otherwise cause; they are in a state of compression on a hot day and of tension on a frosty one. The Assouan dam endures stresses from temperature not less severe than those from the weight of water. Glass ware has to be cooled slowly (annealed) or else it may contain strains that are released in explosive breakage following some trifling blow. Anyone who has overheated a thermometer need not be reminded of the pressures that arise in liquids when their thermal expansion is impeded.

§ 128. **The expansion of solids.**—The expansion of solid rods can be quickly measured with the apparatus of Fig. 60. The rod is geometrically gripped at one end and the other flat end presses on the spring point of an ordinary optician's spherometer clamped on the framework. In this, by the multiplying gear described in the aneroid (§ 74) the motion of the end of the rod is magnified about 200 times, one dial division corresponding to ·001 cm. The rod is 50 cm. long.

Ice-cold water is run through a jacket corked on the rod and in a minute or two the dial reads steadily. The water is run out

and steam blown through, the dial read when steady and the temperature obtained from the barometric height, § 204, Fig. 83. [In this way one can be independent of thermometers, but commonly one uses tap water, and steam or any hot liquid or vapour, running them out over a thermometer.]

If the wooden framework increased in length from heat or moisture during the experiment this alteration would subtract itself from the rod's expansion. This is a very weak point in most forms of apparatus sold for this experiment, they seem expressly designed to facilitate and measure warping. The teak frame shown is well ventilated and away from the changing temperatures, and secured by its form against change in length by warping. Further, hot and cold readings can be alternated at three-minute intervals, as no time is spent in fiddling about with the measuring apparatus.

Dividing then the difference in dial readings by 1000 gives the elongation of the rod in centimetres. Now if it is merely left on record that a certain rod expanded so much for such a rise of temperature, it will involve a compound-proportion sum when we want to reckon out the extension of some other piece of the same material for a different rise. As the whole rod is equally heated (for the little ends outside the jacket reach practically full temperature in a few seconds by conduction) each centimetre is contributing an equal share to the whole extension, i.e. in our case each contributes one-fiftieth part. Again, if we divide the observed expansion by the rise of temperature that caused it we get the average expansion caused by 1° rise. And by experiments at several temperatures in succession it will be found that the expansions caused by changes of temperature are *very nearly* proportional to those changes. That is, the observed extension expressed in centimetres (or whatever units the length of the rod is measured in) divided by the length of the rod and divided by the rise of temperature which caused it, gives us the **expansion of one unit length when heated one degree, the ' coefficient of expansion in length ' or the ' linear expansibility,'** a, **of the material of the rod.**

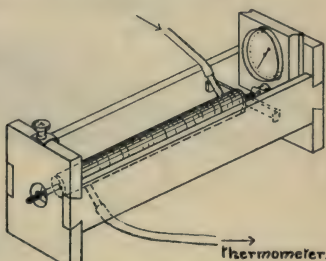


FIG. 60. *Scale* $\frac{1}{16}$.

Linear Expansibility per °C., in parts per million.

Fused silica	·7	Copper	17	Pine (along grain)	5·5
'Invar' steel	·9	Brass	18·5	" (across)	34
Platinum	9	Aluminium	23	Glass	8·5
Steel	11	Lead	29	Hard rubber	70
Iron	12	Zinc	29	Paraffin wax	120

Per °F. the expansibilities are ten-eighteenths of the above. At *very high* temperatures a increases considerably.

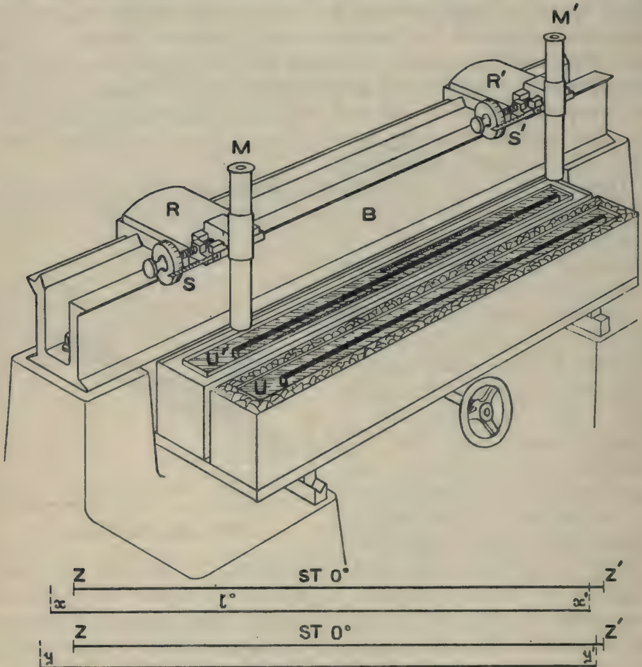


FIG. 61. *Scale* $\frac{1}{16}$.

To calculate the expansion of a length of material then, we must multiply its a by the length and by the rise of temperature, e.g. a length L_o in ice at 0° C. expands $L_o \times a \times 100°$, i.e. increases in total length to $L_o + L_o \times a \times 100°$ when put into boiling water.

L_o *at* 0° *becomes* $L_o + L_o aT$ *at* T°

and L *at* t° *becomes* $L + La(T - t)$ *at* T°

§ 129: **Exact measurement of linear expansion. The Comparator.**

Very exact measurements of linear expansion are nowadays made with the **comparator** sketched in Fig. 61, a machine used in the comparison of measuring bars, which of course involves knowledge of their expansions.

Two tool rests R R′ 'universally' adjustable on a massive lathe bed B carry vertical cross-wire microscopes M M′ (§ 114) which can be moved by micrometer screws S S′ graduated to ·00005 cm., or less. B bridges a short railway on which runs a truck carrying two troughs, containing narrower oil troughs, U U′. U contains a bar with the standard lengths marked on it and is packed round with melting ice, U′ contains the graduated bar under test and is surrounded by circulating water automatically kept to any desired temperature within ·05°.

In use, R R′ are clamped at convenient places on B, M M′ are focussed on the standard bar and moved by S S′ till their cross-wires lie on the centre of the graduations scratched on it. S S′ now show readings z z'.

The truck is moved by turning the handwheel till the test bar lies under the microscopes. It is adjusted to their focus. S S′ are turned till the cross-wires lie on the test-bar graduations and now read x x'. Thermometers in the oil alongside the bar show the temperature $t°$.

The truck is moved back under the bridge and z z' repeated.

If B has expanded they will not exactly repeat, in which case the means of the before and after are taken. Then the test bar at $t°$ exceeds *the standard at* 0° by $(z-x)-(z'-x')$, the difference of two very small distances on the micrometers which both read from left to right.

The test bar is now raised to T° (occupying several hours) and the measurements repeated, giving z_1 z_1' for the standard in ice and y y' for the bar. Then the test bar at T° exceeds *the standard at* 0° by $(z_1-y)-(z_1'-y')$ and the difference of this and the former excess gives the expansion, which is dealt with as before.

Thus, by referring every time to the invariable standard bar the necessity of keeping the whole machine perfectly unchanged is avoided.

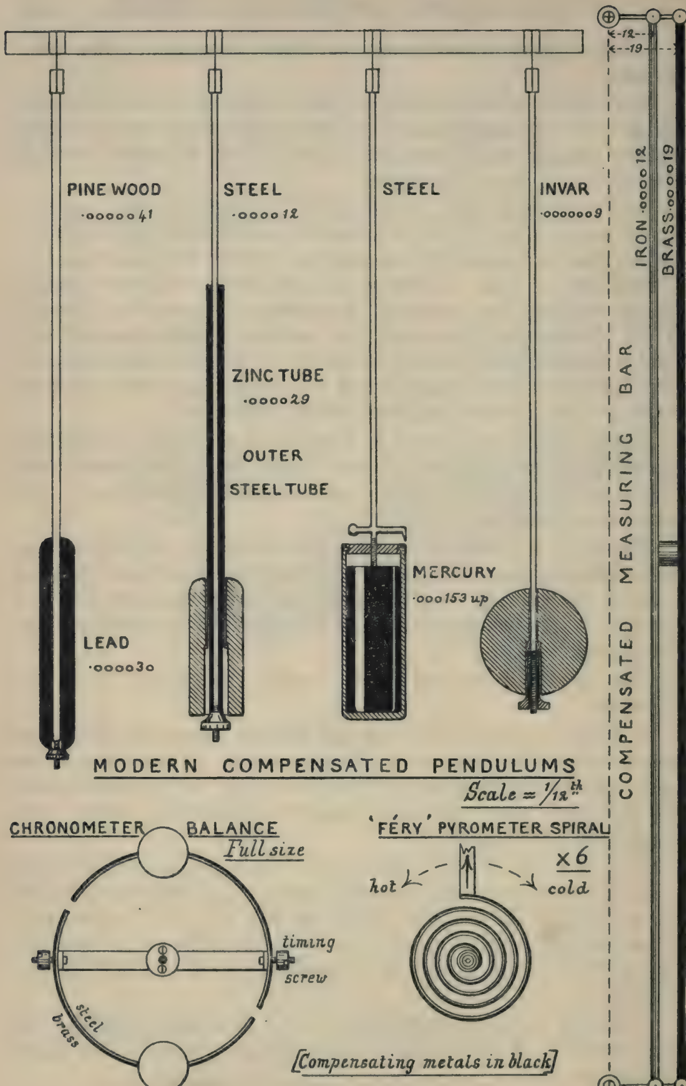

PINE WOOD
·0000041
STEEL
·0000012
STEEL
INVAR
·0000009
ZINC TUBE
·0000029
OUTER
STEEL TUBE
MERCURY
·000153 up
LEAD
·0000030
IRON ·0000012
BRASS ·0000019
-12-
-19-
COMPENSATED MEASURING BAR
MODERN COMPENSATED PENDULUMS
Scale = 1/12th"
CHRONOMETER BALANCE
Full size
timing
screw
steel
brass
'FÉRY' PYROMETER SPIRAL
x 6
hot
cold
[Compensating metals in black]

FIG. 62.

§ 130. **Compensation contrivances.** Fig. 62 shows various devices in which, by taking advantage of the different expansibilities of different materials, an unvarying length is obtained.

On the right is a MEASURING BAR like those used in measuring the base-line for the trigonometrical survey of this country. The reader will see easily how the brass expanding faster pushes out its end of the pin-jointed lever more than the iron does, with the result that the cross-wire microscopes (becoming the vertices of pairs of similar triangles) do not move at all.

In PENDULUMS the problem is to prevent the centre of mass (practically) being lowered when the rod expands, which would make the clock lose. The expansion upwards of (the lower half of) a long lead or mercury bob raises the centre of mass as much as the wooden or steel rod lowers the bottom of the bob, which bears on the nut at the end of the rod. In the modern representative of the old 'gridiron' pendulum, a zinc tube rests on the nut and expands up as much as the inner steel rod and outer steel tube (hanging from its top) expand down, and the bob is unmoved. With 'Invar' steel a very short zinc tube suffices.

In a COMPOUND BAR two thin strips of different expansibilities are securely soldered together throughout their lengths. When heated the bar of course has to bend more and more with the highly expansible strip on the outer longer curve.

By this means the masses on the BALANCE WHEEL of a chronometer or 'compensated' watch are brought in nearer the centre, in warmth. This reduces the moment of inertia and compensates for the enfeebled elasticity of the balance spring, a twenty times more serious hindrance to time-keeping than the mere expansion of an iron wheel or pendulum. [Those who expect a few milligrams of oil to last a watch for ever need not be surprised to find it *losing* when cold.] See also § 118.

The *Féry Pyrometer* is turned towards a small hole in a furnace-wall and the heat of the furnace is focussed by a concave mirror on to a diminutive compound spiral, which curls and moves a long pointer attached to its free end over a scale of furnace temperatures.

Example 1. A half-metre aluminium rod expands 1·15 mm. between 0° and 100°. Find a.

50 cm. become 50·115 cm.

L ,, ,, $L + L \, . \, a \, . \, 100°$

$\therefore 5000\, a = \cdot 115. \quad \therefore \underline{a = \cdot 000023.}$

Ex. 2. A steel foot-rule is correct at 15°. What correction will be necessary in boiling water ?

12 in. at 15° becomes $12+12\times \cdot 000011\times(100-15)=12{\cdot}0112$ in.

$\therefore$ the rule is $\frac{1}{90}$ in. too long.

Ex. 3. What length of lead bob compensates a pendulum made of 44 in. of pine and a 2-in. suspending spring ?

Centre of lead must remain nearly fixed, i.e. its lower half expands as much as the wood and steel, i.e.

$$2\times \cdot 000011+44\times \cdot 0000055=\cdot 000264 \text{ in. per degree.}$$

$$\therefore \text{ half length}\times \cdot 000029=\cdot 000264. \quad \therefore \text{ length}=18 \text{ in.}$$

Ex. 4. A tram rail 40 ft. long is heated. Its normal expansibility is ·000007, but expansion is prevented by the end-pressure of adjoining rails. Find the increase in this pressure between 40° F. and 90° F., given that 16 tons shortens the rail $\frac{1}{16}$ in. [L.]

Free rail would expand $40\times \cdot 000007\times 50°\times 12=\cdot 168$ in.

If 16 tons shortens it $\frac{1}{16}$ in., what force would shorten it ·168 in. ?

§ 131. **Expansion of an area,** Fig. 63 (A). If a square of side 1 expand to a square of side $1+at$ its area increases from 1^2 to $(1+at)^2=1+2at+a^2t^2$. Now at is small (less than ·01) therefore a^2t^2 (is less than ·0001 and) is insignificant compared with $2at$.

Therefore the area of the square increases twice as fast as the length of its side; any area can be built up of little squares; **the areal expansibility is twice the linear expansibility.**

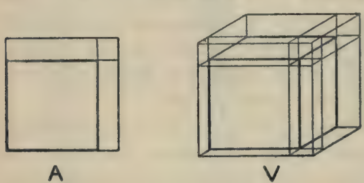


FIG. 63.

§ 132. **Expansion of a volume,** Fig. 63 (V). If a cube of edge 1 expand to a cube of edge $1+at$ its bulk increases from 1^3 to $(1+at)^3=1+3at+3a^2t^2+a^3t^3$. As before a^2t^2 and a fortiori a^3t^3 are insignificant compared with at.

Therefore the volume of the cube increases 3 times as fast as the length of its edge; **the volume or cubical expansibility is three times the linear.**

NOTE that the internal volume of a hollow vessel has the same volume expansibility as the material of the walls. For it might be filled with a solid mass of their material which would then expand with them and always exactly fill the cavity.

§ 133. **Expansion of liquids.**

The expansion of fluids is of course *volume* expansion.

Taking that quantity of liquid which occupied 1 c.c. at 0° C., its expansion in c.c. when heated 1° is its expansibility, E.

Or The expansibility or coefficient of thermal expansion of a liquid is its increase in volume when heated 1° expressed as a fraction of its volume at 0° C.

Notice carefully that it now has to be specified at what temperature the original volume is measured. This is because liquids expand so much more than solids, e.g. 1 c.c. alcohol at 0° becomes 1·015 c.c. at 15° and if only 1 c.c. at 15° were taken E would work out $1\frac{1}{2}$ % too small.

As in § 128, V_0 *at* 0° *becomes* V_0+V_0ET *at* T°,

But now V at t° does *not* become $V+VE(T-t)$ at T°,

It has to be dealt with in two steps :—

(1) V at $t° = V_0+V_0Et = V_0(1+Et)$, from this calculate what its volume V_0 at zero would be ; then (2) as above.

The expansibilities of most liquids increase rather fast at higher temperatures and E usually given is only an average value over some ordinary range of temperature (which ought to be specified).

§ 134. **'Apparent' Expansion.** The vessel containing a liquid complicates measurements by expanding and leaving more room for the contents. [If a flask filled with water to somewhere in its long narrow neck be suddenly plunged into hot water the liquid in the neck goes down for an instant, the glass has got heated first.] *The apparent expansibility e of a fluid in glass is therefore less than its true or absolute expansibility* E.

Three ways of finding the former, which concerns us most in practice, will be given, and subsequently means of calculating and experimentally measuring the latter.

Measurement of net or 'apparent' expansibility in glass vessels.

I. **The Volume Dilatometer.** This is practically a big thermometer containing the liquid. (A) in Fig. 64 is neat, but difficult to fill and empty, (B) is a half-hour's exercise for the amateur glass-blower and is more easy to manage. A glass bead in the bulb helps to stir up its contents and spread a uniform temperature.

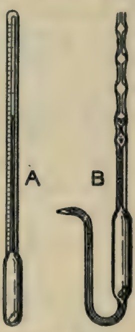


FIG. 64.

Before use the dilatometer must be calibrated, i.e. the volume of its various parts found. It is cleaned, rinsed, and dried by a current of warm air. The bent end is dipped under mercury which is sucked in till full. It is now left for an hour in the balance case to take up a steady

temperature, then the amounts of mercury run out as it is emptied mark by mark are weighed. The weights are proportional to the volumes between marks, and the last one to the volume of bulb, etc., up to zero mark.

The ice-cold liquid is now drawn in near to the zero and the fine-drawn end sealed by touching with a flame. Dilatometer and thermometer are deeply immersed in a well-stirred water bath, very slowly heated, and the temperatures taken at which the liquid passes each mark. This is repeated during slow cooling and mean temperatures obtained.

Ex. 5. A dilatometer weighs 12·50 grm. empty. Filled with mercury at the ordinary temperature to the zero mark it weighs 243 grm., to the 1 mark 244·2, to the 2 mark 245·6. When containing a liquid, this reaches 0 mark at 4°, 1 at 14°, and 2 at 25°. Calculate its expansibilities between these ranges of temperature.

From net weights of mercury

$$\text{Vol. bulb to } 0 : \text{vol. } 0 \text{ to } 1 : \text{vol. } 1 \text{ to } 2 = 230{\cdot}5 : 1{\cdot}2 : 1{\cdot}4$$

For first rise of 10° vol. 230·5 expands $230{\cdot}5 \times e \times 10° = 1{\cdot}2$

$$\therefore\ e \text{ from } 4° \text{ to } 14° = {\cdot}000522$$

For second rise of 11° vol. 230·5 expands $230{\cdot}5 \times e' \times 11° = 1{\cdot}4$

$$\therefore\ e' \text{ from } 14° \text{ to } 25° = {\cdot}000553$$

The difference in bulk at 0° and 4° has been ignored, as it only alters the 230·5 a very little, and does not come directly into account.

II. **The weight or overflowing dilatometer.** This is neither more nor less than a specific-gravity bottle, § 81. Its weight empty (or containing a few bits of glass to act as stirrers) is b. It is stood in ice, filled with ice-cold liquid, wiped with a cold cloth, and weighed, $b+w_\circ$. It is warmed in a water bath and kept several minutes at a steady $t°$ till no more exudes, wiped and weighed (after cooling), $b+w_t$. This may be repeated at several rising temperatures.

w_t is the weight of liquid at $t°$ filling the bottle of volume v, which $w_\circ$ filled at 0°.

We are of course agreeing to neglect the change of v with temperature.

$\therefore$ volume of 1 grm. at $t°$ is v/w_t c.c.; at 0° is $v/w_\circ$ c.c.

$$\therefore \text{ expansion of 1 grm. per degree is } \frac{1}{t}\left(\frac{v}{w_t}-\frac{v}{w_\circ}\right) \text{c.c.}$$

This is the expansion of volume $v/w_\circ$, $\therefore$ to get expansion of what was 1 c.c. at 0° divide it by $v/w_\circ$ and we have

$$\frac{1}{t}\left(\frac{v}{w_t}-\frac{v}{w_\circ}\right) \div \frac{v}{w_\circ} = \frac{1}{t} \cdot \frac{w_\circ - w_t}{w_t} = e$$

[Or, look at it this way. Cooled to 0° again w_t only partly fills the bottle, a part which would now weigh $w_o - w_t$ being left empty. w_t raised to $t°$ again expands and fills this part, i.e. expands $(w_o - w_t) \div t$ per degree, or 1 unit at 0° expands $(w_o - w_t)/t\,w_t$.]

Notice it is not w_o in the denominator, for part of the expansion of the whole mass w_o occurred in the thrown-away overflow after it got outside.

Ex. 6. A sp.-gr. bottle contained 40 grm. of a liquid at 0°; after keeping at 35° till no more exuded it contained only 39·5 grm.

$$39{\cdot}5 \text{ expands } (40-39{\cdot}5) = {\cdot}5 \text{ for } 35°.$$
$$\therefore\ 1 \text{ expands } {\cdot}5 \div (39{\cdot}5 \times 35) = \underline{{\cdot}000362} \text{ for } 1°.$$

III. When a liquid volume 1 expands to $1+et$ its specific gravity decreases in the ratio $1/(1+et)$, hence measuring its specific gravity at different temperatures with a common glass **hydrometer** [or a glass ball hung from a hydrostatic balance] enables e to be calculated.

Ex. 7. A hydrometer in terebene at 0° read ·870 and at 61°, ·820.

$$1 : 1+61e = {\cdot}820 : {\cdot}870.$$
$$\therefore\ {\cdot}820 \times 61e = {\cdot}05, \underline{e = {\cdot}00100}.$$

Note.—If a fluid is enclosed in a long tube its *increase in length* denotes not a linear but a *volume expansion.* For the tube being unyielding, all three-ways expansions are added into one. In speaking of the linear expansion of solids their sideways swelling is ignored.

§ 135. **The correction for the expansion of the vessel.**

If V_o c.c. of a fluid actually increase to $(V_o + V_oET)$ c.c. at T° but the containing vessel expand with cubical coefficient g so that its c.c. all become $(1+gT)$ c.c., the expanded liquid will only occupy $V_o \times (1+ET) \div (1+gT)$ of these false c.c. Since g is small this is nearly enough $V_o(1+ET-gT)$ or $V_o + V_o(E-g)T$.

Therefore the apparent expansibility $e = E - g$, the true expansibility less that of the vessel.

Some reductions caused by the vessel are, per cent :—

		In glass	In aluminium	In fused silica
	E	g. ·000025	g. ·000075	g. ·000002
Mercury ..	·000182	14 %	42 %	1 %
Alcohol ...	·001050	2·5	7·5	·2
Air	·003667	·7	2	·05

Thus the use of dilatometers of fused silica, now quite cheap, all but abolishes the vessel correction.

§ 136. **True or 'absolute' expansibility experimentally.**

A Hare's apparatus of balancing columns is used, the legs being filled with the same liquid, cold and hot. As explained in §§ 63 and 82, this is quite independent of the sizes of the tubes, therefore the swelling of the hot glass does not affect it at all (provided, of course, that the scales are not on the tubes themselves; this is essential). In the apparatus of Fig. 65, designed as a laboratory illustration of the method, two lengths of glass joined by a short narrow rubber tube form a U tube, kept at 100° on one side by a steam jacket and cooled on the other by ice water. The glycerine which at the start is at the same level on both sides finally stands at 69·6 cm. on the cold and 73·1 on the hot.

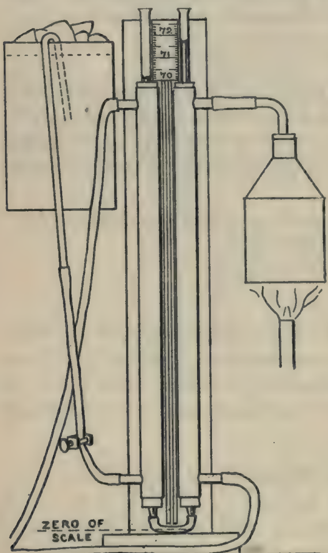


FIG. 65. *Scale* $\frac{1}{12}$.

Since each represents the same hydrostatic pressure, i.e. the same weight per square centimetre cross-section of tube, a volume equal to the 69·6 at 0° has expanded 3·5 for 100°, or ·035 per degree. Therefore 1 at 0 expands per degree $\cdot 035 \div 69 \cdot 6 = \cdot 00053 = E$.

The **absolute expansibility of mercury** was determined by Regnault with an elaborate form of this apparatus, but an excellent plan is simply to 'boil the barometer,' utilizing the atmosphere as the cold balancing column.—

A syphon barometer of the shape shown in Fig. 36 (S) is enclosed in a jacket (out of which only the end of its open tube protrudes) and is read, at various temperatures of circulating fluid, by a cathetometer and scale kept at the constant room temperature. Any small variation of the atmospheric pressure during the experiment must be observed on the laboratory barometer and allowed for, and there must be added to the height at each temperature the small depression due to the increasing pressure

of mercury vapour in the Torricellian space (·03 cm. at 100°, 1·83 at 200°, etc., see § 209).

Beside the barometer is the long bulb of a gas thermometer. Here are two substances, mercury and air, expanding, as well as the mercury-in-glass thermometer. In the next chapter we shall find reason for setting aside the latter and taking a gas as the standard thermometric substance.

NOTE.—There is no need to measure the absolute expansibility of any other liquid by this method, for E of mercury once known a glass dilatometer can be filled with it and g of the glass $=E-e$, the observed falling-off in expansibility. Then g is added on to other liquids examined in the same dilatometer.

Ex. 8. Mercury fills a glass cylinder to a depth of 20 cm. at 0°. At what true height will it stand at 60°?

Let area of cross-section of jar at 0° be A sq. cm.; at 60° this becomes $A+A\times(2\times{\cdot}0000085)\times60=1{\cdot}00102A$ cm.^{2}.

The bulk of mercury is 20A c.c. at 0° and at 60° this becomes $20A+20A\times{\cdot}000182\times60=20{\cdot}2184A$ cm.^{3}.

$$\therefore \text{Height}=\text{volume}\div\text{cross-section}=20{\cdot}2184A\div1{\cdot}00102A.$$
$$=\underline{20{\cdot}198 \text{ cm.}}$$

Ex. 9. If in the above the scale were etched on the glass what would be the reading at 60°?

The volume up to an etched mark now expands with the cubical coeff. $3\times{\cdot}0000085=g$. All we need do is to subtract this and get apparent e of mercury $={\cdot}000156$, which assumes the glass invariable.

$$\therefore \text{Reading}=20+20\times{\cdot}000156\times60=\underline{20{\cdot}187 \text{ on the glass scale.}}$$

Ex. 10. The **Barometric Column** stands at 76 cm. at 0°, what will be its true height at 25°?

The problem is to keep the hydrostatic pressure, i.e. the weight of a square centimetre column, the same. The expansion of the glass has nothing to do with it.

1 c.c. of mercury at 0° becomes $1+1\times{\cdot}000182\times25=1{\cdot}00455$ c.c. at 25°.

$\therefore$ 1 c.c. at 25° weighs only $1/1{\cdot}00455$ of the c.c. at 0°, and $\therefore$ 1·00455 times as many c.c. must be piled up on the 1 sq. cm. base.

$$\therefore \text{True height}=76\times1{\cdot}00455=\underline{76{\cdot}346 \text{ cm.}}$$

Ex. 11. In Ex. 10 if the scale were etched on the glass what would be the reading?

The true height has now to be measured on a false scale on which each centimetre has become $(1+1\times{\cdot}0000085\times25)$ cm. $=1{\cdot}00021$ cm. long.

$$\therefore \text{apparent height}=76{\cdot}346\div1{\cdot}00021=\underline{76{\cdot}330.}$$

§ 137. **Temperature correction of barometer.**

In these last two examples the barometer reading 76·330 is

$$\frac{76(1+1\times{\cdot}000182\times25)}{(1+1\times{\cdot}0000085\times25)} = 76[1+({\cdot}000182-{\cdot}0000085)\times25]$$

i.e. in correcting the barometer we have used the absolute expansibility of mercury minus the *linear* expansibility of the scale. (The sideways swelling of the tube does not come into account.)

Ex. 12. A barometer reads on its brass scale 76·50 cm. at 16°, what is the reading at 0° ?

This will be the true height, since the brass metre scale is correct at 0°

H_0 at 0° becomes $H_0 + H_0(\cdot000182 - \cdot000018) \times 16° = 76{\cdot}50$ at 16°.

$\therefore$ inverting this calculation $H_0 = 76{\cdot}50 \div (1 + \cdot000164 \times 16°)$

$= 76{\cdot}50(1 - \cdot000164 \times 16°)$ nearly $= \underline{76{\cdot}30 \text{ cm.}}$

This shows the **Practical Rule for reducing the height of the barometer to its true value at 0° C. From the observed height subtract observed height × (·000182 – linear expansibility of scale) × temperature.**

Ex. 13. The British barometer scale needs a tedious correction. Find the true height at 32° F. of a barometer reading, at 70°, 30·124 in. on a brass scale correct at 62° F. (Expn. F. $= \frac{5}{9}$ of their C. values.)

True length of scale at $70° = 30{\cdot}124(1 + \cdot000010 \times 8°) = 30{\cdot}127$ in.

$\therefore$ True height at $32° = 30{\cdot}127 - 30{\cdot}127 \times (\cdot000010 - 0) \times (70 - 32)$

$= 30{\cdot}127 - \cdot114 = \underline{30{\cdot}013 \text{ in.}}$

§ 138. Water.

Water expands increasingly faster at high temperatures and contracts increasingly slower at low, as do most liquids, but it gradually ceases to change at all, and thereafter begins to expand on the way *down* to its freezing point. Thus there is a temperature at which its volume is least, and therefore *its density a maximum.* This is 4° C. or 39·1° F. Altered either way it *very slowly* expands : it is because the change for 1° is hardly measurable that this temperature is taken in defining the gramme. Conversely, the slow change makes it difficult to find the maximum-density temperature accurately.

Joule used an apparatus of wide balancing columns 6 ft. high, and instead of attempting to observe difference in level he opened a cross-channel at the top and watched which way a floating bulb drifted (i.e. towards the denser column down which the water sank). Arguing that at equal distances either side of the maximum the water would be equally lightened, he found that with one column at 2° and the other at 6°, the bulb did not move, and the mean of all such pairs of temperatures was 4°.

In glass dilatometers, or with the very sensitive hydrometers employed to study the question, the apparent maximum density is reached at 6°, the water not catching up to the expansibility of the glass till 2° above the resting-place.

This idiosyncrasy of water has an effect in nature which can be illustrated by a tall jar of water and floating ice, with a thermometer dropped to the bottom and another held near the top. Both run down, but the bottom one slows up and stops near 4° while the top continues to 0°, the water getting lighter.

When the ice is fished out the top rises to 4° before the bottom begins, for although the surrounding air warms all parts of the jar, yet as long as there is water at 4° anywhere it sinks to the bottom. Provided with a waist-band for ice this jar is called Hope's apparatus, but this is an undesirable elaboration.

In consequence, fresh-water fish can luxuriate in 4° C. a foot below the ice-shield, while fish in the salt marsh must endure −1·9° C., for sea water then begins to freeze, before having reached its maximum density (−3·2° C.).

§ 139. **Expansion of gases.** In finding the thermal expansibility of a gas care has to be taken not to permit elastic expansion on account of diminution of pressure. The definitions in § 133, with the words '**at constant pressure**' inserted, apply to gases.

In a simple apparatus the gas partly fills a horizontal graduated capillary tube, Fig. 66, being shut in between its sealed end and a thread of coloured sulphuric acid. The volume of the gas is proportional to the length it occupies in the uniform tube. The tube is raised from ice to T°, the trifling rise of the acid in the wide turned-up end does not appreciably add to the pressure, and provided the whole, i.e. the *barometric*, pressure has not altered, $V_t = V_0 + V_0 eT$, and e is so large that g of the glass can be ignored in this apparatus.

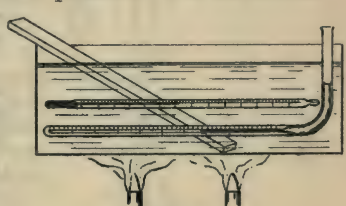


FIG. 66.

Experimenting in this way Gay Lussac and Charles found that all gases expand equally, and what is commonly known as the **Law of Charles** states that **All gases expand $\frac{1}{273}$ of their volume at 0° C. for each degree rise of temperature, the pressure being constant.** The gases must not be too near their liquefying temperatures and of course no chemical changes (e.g. N_2O_4 into $2NO_2$) are allowable.

§ 140: Subsequently Regnault, and Chappuis, have used apparatus of which an efficient laboratory copy is shown in Fig. 67.

A gas bulb under well-stirred water communicates with one limb of a U tube graduated in cubic centimetres. The other limb is open and oil is kept to the same level in both by aid of the tap and 'wash-bottle' below. The graduated tube is surrounded by a wider tube of water.

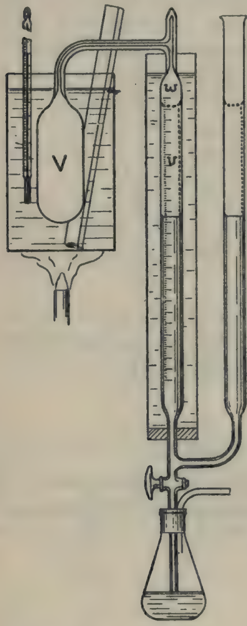


FIG. 67. *Scale* $\frac{1}{12}$.

The bulb has had its volume V measured by weighing full of water, it has then been dried by repeated pumping out, heating, and readmission of the gas through drying tubes. If there is moisture present it will liquefy at low temperatures and evaporate at high, and as an ordinary drop of water would completely *fill* a 150 c.c. bulb with steam the need for careful drying is obvious.

The connecting tube gets heated to an uncertain average temperature, therefore it is made of small bore so that the expansion of the little gas it holds can be ignored. Its volume + the few cubic centimetres of gas initially in graduated tube at $t° = w$.

It simplifies matters greatly to start all at the room temperature $t°$. Now heat bulb to T and running out oil till again at same level* on both sides v c.c. of gas are found driven out into the graduated tube kept at $t°$.

Gas that filled $(V+w)$ at $t°$ now fills V at T° and $(w+v)$ at $t°$.

Subtract $(w+v)$ at $t°$ from both,

$\therefore$ gas that filled $(V-v)$ at $t°$ now fills V at T°

$\therefore$ its expansion per degree $= v \div (T-t)$. This would also be its contraction per degree if cooled.

$\therefore$ its *volume at* 0° would be $(V-v) - t° \times v \div (T-t)$.

$$\therefore e = \frac{\text{expn. per }°}{\text{vol. at }0°} = \frac{v/(T-t)}{V-v-vt/(T-t)} = \frac{v}{(V-v)(T-t)-vt}$$

* If the barometer had changed we should compensate it here by raising the outer oil level $(13{\cdot}6 \div \text{sp. gr. of oil})$ times the barometric fall.

To allow for expansion of glass of bulb write $V+Vg(T-t)$ at T° instead of V at T°, the expansion per 1° becomes $v\div(T-t)+Vg$, the volume at 0° becomes $(V-v)-t\times$ expansion

$$\text{and } \therefore \; E=\frac{v+Vg(T-t)}{(V-v)(T-t)-vt-Vg(T-t)}$$

In the above it should be noted that the expansions per degree are not equal *if drawn off and cooled before measurement.* This is a drawback to the use of this instrument as the **constant-pressure gas thermometer**, equal rises of temperature do not show equally in the tube. At 1400° e.g. a Victor Meyer bulb, which the chemist will see is practically the same apparatus, has poured five-sixths of its contained air into the tube and has lost 35/36ths of its efficiency as an indicator of furnace temperature.

EXAMPLES.—CHAPTER XV

14. Explain how the coefficient of expansion of a solid may be found experimentally. A metal sphere is found to have a volume of 1000 c.c. at 0° C. and 1003 c.c. at 100° C. Calculate linear expansibility. [L.]

15. Find expansion per 1000 yd. between 20° F. and 100° F. of a wire of expansibility ·000012 per °C.

16. A solid of linear expansibility ·000015 contains a 500 c.c. cavity in which is a 200 c.c. body of linear expansibility ·00003. How much does volume of air-space left change per degree? [M.]

17. Find increase in volume of a glass litre flask between 4° and 40° C.

18. Why does the rate of a pendulum clock depend on temperature, and how can a clock be constructed so as to be unaffected? [M.]

19. A brass pendulum beats seconds exactly at 10°, if its linear expansibility is ·000018 show that at 25° it loses $11\frac{1}{2}$ sec. a day. [Ab.]

20. What length of zinc tube is necessary to compensate the expansion of (its own length + 42 in.) of steel?

21. What depth of mercury must there be in the cast-iron bob of a pendulum so that centre of mass of mercury may rise one-third faster than 42 in. of iron expand and lower bottom of bob? [This is to allow for cast-iron bob itself weighing one-third as much as mercury.]

22. How much would a 3-in. rivet shrink in cooling from 500° C. to zero?

23. The heavy iron rim of a fly-wheel is heated 100° hotter than the spokes by the friction of a testing brake. What will be the extra

stress in a straight spoke which one ton would lengthen by 1 part in 4000 ?

24. A sp.-gr. bottle weighing 8·75 grm. empty weighs 33·8 grm. full of liquid at 0° and 33·0 full at 40°. Find expansibility of liquid.

25. Explain how to find the coefficient of expansion of a liquid by weighing a solid in it. [L.]

26. Define coefficient of absolute expansion of a liquid and describe how to find it. [D.]

27. A barometer which stood at 75 cm. at 0° stands at 76·33 cm. (true) in steam at 100°. Adding on ·03 cm. for vapour pressure of mercury, calculate absolute expansibility.

28. Describe the temperature corrections of the barometer. [Ab.]

29. Explain how the irregular expansion of water influences the distribution of temperature in a freezing pond. [L.]

30. Volume of bulb of air thermometer 41 c.c., stem $\frac{1}{4}$ sq. cm. area. Between 0° and 10° index moves 6 cm. up stem. Find expansibility of air.

31. 50 c.c. of air at 15° C. are expelled from a constant-pressure air thermometer by changing from 0° to 100° C. Calculate temperature of thermometer when 10 c.c. are expelled, neglecting expansion of bulb. [L.]

32. Verify the statement at the extreme end of § 140.

33. A small volume of a fluid, of expansibility a, is at t' ; rest of fluid at t. If density at $0° = d_0$ find resultant force per c.c. on the warmer portion. What forces tend to diminish resulting motion ? [M.]

CHAPTER XVI

THERMOMETRY

§ 141. The branch of our subject which deals with the measurement of heat from the point of view of hotness or temperature is known as thermometry. Many definitions have been proposed for **Temperature,** but it is utterly needless to define so fundamental an idea, for the first physical necessity of active life in any organism is a certain degree of warmth and accordingly a sense of temperature is found in all animals.

Our own temperature sense is located in small 'warm spots' and 'cold spots' on the body surface, sensitive respectively to temperatures above and below that of the skin (whatever it may happen to be) and together averaging about twelve in number per square centimetre of skin. Excited simultaneously with the more numerous 'pressure spots' these tell us that we are touching a hot or cold object; without the pressure stimulus we feel hot or cold ourselves. Their first response is quick, but quickly falls to a much smaller value and ceases to attract attention if the stimulus is protracted: the coldness of the water ceases to afflict the bather after the first few seconds. Accordingly the temperature of the skin can be altered considerably without our knowing much about it, and a medium temperature which affects the warm spots on a cold hand may affect the cold spots on a warm hand, as in the familiar process of adjusting the temperature of the bath water. And what reader has not made the quite sudden discovery that the fire is out and he is very cold? A badly conducting layer, such as the thick epidermis of the finger-tips, or the cloth of the kettle-holder, slows the rate at which stimulus is applied and enables hot objects to be handled *for a time* with impunity.

That the sense is cutaneous only appears from the feeling of heat when perspiring freely, while the clinical thermometer shows a body-temperature hardly higher than usual, and in the case of fever actually lower than in the preceding shivering fit.

§ 142. Altogether our protective temperature-sense is not to be relied upon for the unequivocal measurements demanded in physics. Actions in inanimate matter have to be employed. Those mostly made use of are—

(1) Solidification and its converse, Melting, Boiling, § 142.
(2) Colour and other changes due to chemical action.
(3) Magnetic change, § 517.
(4) Change in size, expansion or contraction.
(5) Change in resistance offered to an electric current, § 620.
(6) Change in power of producing an electric current, § 645.
(7) Change of colour and brightness of emitted light, § 500.

(1) **Solidification. Melting. Boiling.**

Use of these temperature indications is familiar enough. There is the consultation of the puddles in winter, whether they be water or ice: there is the butter at tea-time fatuously informing us that the weather is warm: there is the sprinkling of water on the hot flat-iron, etc. etc.

Technically there are sold 58 'Seger cones,' little conical pastilles of various compositions which melt at temperatures going up by 30° steps from 590° to 950° and thence by 20° steps to 1850° C. These are still extensively used in controlling kilns and furnaces; three successive numbers are put in together, the first must collapse, the second may nod, the third must remain upright.

(2, 3) **Colour, chemical, and magnetic changes.**

The **colour changes** due to thickening of the oxidation film are the most usual guide in tempering steel. Frequently, however, the sudden **ignition** of a heavy oil is a temperature mark in the process, and occasionally small steel objects are heated till at 780° C. they lose their **magnetism** and drop off the suspending magnet into the quenching water.

(4) As a measure of temperature occupies the rest of this chapter (5, 6, and 7) must be left till much later on in the book.

§ 143. (4) **Expansion. The liquid-in-glass thermometer.**

Let us now apply the work of the last chapter to the liquid-in-glass thermometer which for long gave the standard measure of temperature.

Filling a thermometer.—The thermometer consists of a glass 'stem' of fine and very uniform bore, with a suitably sized and shaped 'bulb' at one end, and the problem is to fill this narrow-

necked bottle. A cup to contain some of the liquid is formed at the top of the stem ; the bulb is warmed, air bubbles out, and on cooling some of the liquid draws down to replace it. This is repeated and then, as we know perfectly well that air will be sticking to the glass or dissolved in the liquid, the bulb is strongly heated till its contents have nearly boiled away, the vapour 'washing out' this air. As it condenses the warmed liquid descends and fills the whole. It is now heated a little above the highest temperature it is destined to measure, and the top of the stem below the cup is sealed off in the blowpipe.

On cooling there is only liquid and its vapour inside, but this total absence of permanent gas is not essential. Most common thermometers retain an accidental trace of air, and high-temperature thermometers have their stems deliberately filled with nitrogen before sealing. Indeed, a mercury thermometer with a broken top works quite well till dirt gets in or mercury spills out, but an alcohol one would dry up.

Annealing and ageing. The instrument is now baked for a day at its highest temperature. This annealing gets rid of strains in the glass, for glass gradually yields to stress even when cold—a long tube resting at the ends sags year by year—and this used to show itself in thermometers as an unsteady crawl upward of the readings for many years. The high heat of annealing cuts years down to hours.

§ 144. **Scales of temperature.** Fahrenheit of Amsterdam in 1720 apparently took the greatest cold he ever reached as zero and the temperature of a healthy man as 100, but it was realized that these temperatures were too uncertain, and the **Fixed Points** are now the temperature of ice melting, and the temperature of steam from water boiling at the normal atmospheric pressure, which maintains 76 cm. of mercury in the barometer. On the Fahrenheit scale these are 32° and 212°. On the scale invented in France by Réaumur and now in domestic use in Germany they are 0° and 80°, on Newton's thermometer they were 34° apart, De l'Isle of St. Petersburg in 1733 fixed them at 150° and 0° respectively ! On the 'Centigrade' scale, due jointly to Celsius and his illustrious fellow-countryman, Linnæus, and now mostly used in scientific work, they are 0° and 100°.

§ 145. **Graduating or testing thermometers, Fig. 68.**

For the **freezing point** the thermometer is in finely broken ice standing nearly full of the pure water of its own melting. Solid

ice without water may be below its melting point and water containing dissolved salts lowers the melting point, § 273.

For the **boiling point** the whole thermometer must be in a current of steam, free from spray, and gives the boiling point of the 'distilled' water which forms in a film on its bulb. The 'hypsometer' in the figure is provided with an outer sheltering jacket and has a little water gauge to indicate that the steam

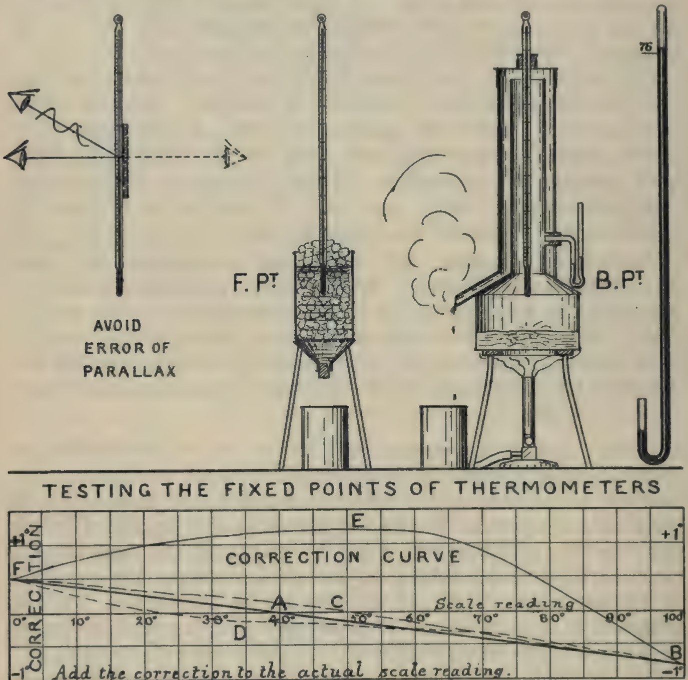


FIG. 68.

pressure is not appreciably above the atmospheric (from too much fire and a choked spout).

Beside it is the BAROMETER. Allowance has to be made that (among ordinary heights) the boiling point goes up or down with the barometer at the rate of 1° C. for every 2·7 cm. (roughly an inch) above or below the normal 100° C. at 76 cm. of mercury,

or 1° F. for ·6 inch. *A thermometer's boiling point cannot be tested without consulting the barometer.*

The interval between the marked points is now divided into the requisite number of equal degrees. These are intended as equal volume-increases in the tube, but the intention fails if the bore is not equal throughout. This the maker tested by running a short thread of mercury through; if it kept the same length throughout the bore was uniform, if it shortened the bore was bigger thereabouts, and a most tedious process of *calibration* would be necessary to devise a scale which should compensate for this irregularity. But modern tubes are very uniform, and in his best thermometers the maker avoids calibration by testing against a standard (itself compared with the 'hydrogen scale') at several temperatures and if necessary compressing or stretching the scale in parts so as to fit all these readings.

Testing Thermometers.

In this way the National Physical Laboratory will test your thermometers at a shilling a point. But common thermometers, which would not pay for this, and frequently show errors at both ends owing to inadequate annealing, can be sufficiently corrected as shown in Fig. 68, lower part.

At each end of their scale, on squared paper, is set up or down the *plus or minus correction which has to be* added *to the false readings to get true temperatures*, e.g. the thermometer shown reads $-\cdot5$ in ice and the correction $+\cdot5$ is therefore set up; it reads high in steam and $-\cdot7$ is set up (i.e. ·7 down).

Rule the straight line FAB, its height above or below the horizontal scale at any reading gives the correction to be added ($\pm$) to that reading.

I have heard it urged that you are no better off, for perhaps the maker's scale has incidental errors bigger than your corrections. To this one must answer that it is very probable that the true correcting line may resemble FCB or FDB rather than the straight FAB, but that it is as far out as FEB is *most improbable.* Let B be a beehive and F a nature-studying person. No single bee strictly follows the bee-line BAF, but they swarm along tracks like BCF and BDF, while not one in a hundred will go by E half round the garden. So in a swarm of thermometers only very few will be far out in the middle after the ends are checked. And having only two points, all one can do is to draw a straight line between them and be content that the odds are quite 20 to 1 that this correction is better than none.

§ 146. **Change of zero after heating.** That by no means ideal solid, glass, does not immediately shrink to its proper size after being heated for some time. The result is that the bulb remains larger and the thermometer reads perhaps half a degree too low near the lower end of its scale (e.g. in ice) for several hours. This disappears in a day with well-annealed glass, but is an annoyance. Thermometers are now obtainable which have sealed inside their bulbs a rod, of suitable size, of a variety of glass in which this lag is very pronounced and compensates the undue bulk of the bulb, so that the annular space occupied by the mercury is made free of this error.

For common use, clinical purposes, etc., where thermometers are never taken anywhere near boiling, this error need never be feared.

§ 147. **Choice of thermometric substance.** The earliest thermometer appears to have been a long-necked flask of *air* inverted and dipping in water, which stood some way up the neck and fell as the temperature rose. Several other air thermometers followed, all of them liable to great error due to barometric changes (§§ 101, 139, etc.).

The Florentine Academicians used spirits of wine and sealed the tube.

Alcohol is still in common use; it expands a lot and, tinted with dye, gives a large bold column; it runs quickly and never freezes (f. pt.—150° C.).

Per contra, it is a bad conductor of heat, and a big bulb of it heats slowly; it boils at 78° C. (175° F.), and worst of all, it begins to distil long before this, so that it is not unusual to find a few degrees of it snugly hidden at the top of the stem (colourless perhaps) and the thermometer reading too low by that much.

As it cannot be carried to the upper fixed point, and as its expansibility increases at higher temperatures, its thermometers have to be graduated by comparison with a mercury standard, and their bold plain reading then makes them preferable for domestic use.

Linseed oil, *sulphuric acid*, etc., have been used to get a longer range of temperature, but are too viscous and hang about on the tube.

Water of course is hopeless on several accounts.

Mercury, which Fahrenheit brought into use, freezes only in Arctic winter (—40°) and does not boil till 360° C., having there-

fore a long range in which its expansion is reasonably assumed to be steady. It runs easily and leaves nothing on the glass. It heats quickly, being a good conductor and having small heat capacity (§ 155). Its expansion is small, permitting only a slender thread, but this is perfectly opaque. It does not distil much below 300°. In thermometers for use above this the tube must be 'packed' with nitrogen, which, compressed by the expanding mercury, practically prevents its vaporization. Nitrogen-packed thermometers of fused silica are sold for use up to 750° C., when the nitrogen reaches 60 atmospheres.

§ 148. **Forms of thermometers.**

For domestic use a wooden scale, firmly attached without possibility of slip, gives a bold reading.

All-glass instruments are washable and non-corrodible. An outer protecting tube enclosing the paper scale gives legibility and cheapness; for scorching heat paper is superseded by a slip of opal glass. But a scale etched on the thick stem itself is the only sort sure not to come adrift. Avoid parallax in reading it (Fig. 68), and keep a penny tube of oil-black for refilling the marks.

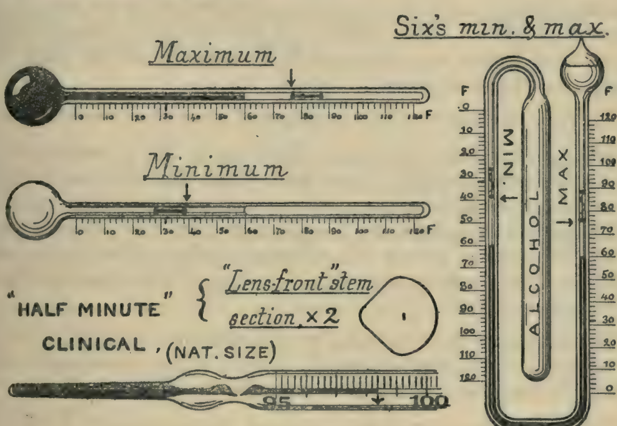


FIG. 69.

Registering thermometers are often useful.

In a pattern ascribed to Rutherford (Fig. 69, top) the mercury pushes a little black glass pin along the horizontal tube, leaving

it with its near end at the highest point reached; while in a companion alcohol thermometer the spirit drags a submerged pin down and leaves its head at the lowest point reached. These are reset daily by tilting them.

The instrument invented by Mr. Six in 1782 is an alcohol thermometer with a long thread of mercury shutting in the spirit. Beyond is more alcohol and at the end a bulb containing an air and vapour space. The mercury hardly alters in length, but acts as a flexible piston, forced out by the expanding alcohol or driven back after it by the air pressure in the subsidiary bulb, pushing either way little iron pins and leaving them at the highest and lowest points reached. Tiny wisps of elastic wire keep them stuck there till dragged back daily by the observer's magnet. The doubled-up vertical form given to this instrument in practice is purely a question of compactness, and the reader may find its action easier to understand if he redraws the figure with the tube straightened out.

In a Clinical Thermometer there is a very minute constriction between bulb and stem. The mercury is squeezed past this by the expansion pressure in the bulb, but its weight is not enough to squeeze it back. The reading of the patient's temperature is made at leisure and afterwards the mercury is got back far enough by violent swinging.

The clinical is an excellent example of a *sensitive* thermometer.

Its degree spaces are long. Now each °F. is only a ten-thousandth the volume of the bulb. But the bulb itself must be small and slender to take up the patient's temperature quickly. The bore of the stem must therefore be very fine; in the particular instrument drawn it is elliptical, $1/600 \times 1/900$ inch, and the glass of the stem is shaped so as to magnify its breadth when seen from the front.

* In recording thermographs a very flat curved Bourdon tube (§ 75) tends to straighten as the alcohol filling it expands and moves the short arm of the lever carrying the pen.

* Solid thermometers mostly depend on the compound strip of § 130, where an instance, the Féry pyrometer, is given. Short compound bars are employed in automatic fire-alarms, they bend when heated and close an electric bell contact.

* **Convenience, rather than accuracy, characterizes these contrivances.**

§ 149. **Conversion of thermometric scales.**

There frequently arises a necessity to convert a temperature from one scale to another. A centigrade reading is simply the percentage of the distance from freezing point to boiling point that the temperature has travelled, i.e. C° Centigrade is C per cent or C/100 of this fundamental interval.

On another scale let the ice point be i and the boiling point b, making the interval $(b-i)$. The corresponding temperature reading $x°$ on this scale has first to cover the handicap distance i and then runs the remaining $x-i$ up the fundamental interval, or the fraction $(x-i)/(b-i)$ of this interval.

$$\therefore \text{Fractional distance} = \frac{C}{100} = \frac{x-i}{b-i}$$

For example, on the Fahrenheit scale $i=32$, $b=212$.

$$\therefore \frac{C}{100} = \frac{F-32}{212-32} = \frac{F-32}{180}$$

or Degrees $C = \frac{5}{9}$ of (Degrees F less 32)

And to convert the other way from C to F, multiply by $\frac{9}{5}$ and add 32 ; both can be written as

$$\frac{F-32}{9} = \frac{C}{5}$$

Notice carefully, however, that a *difference* of 9° F. is the same as a *difference* of 5° C., for each F. reading is equally handicapped by the 32, which therefore disappears from their *difference.*

Consequently expansibilities and thermal units Fahrenheit are $\frac{5}{9}$ of their values Centigrade.

The Fahrenheit scale has a slight advantage for meteorological purposes in that minus temperatures are rarely attained on it during British winters. Thus one source of confusion is avoided. But whether the popular expression 'twenty degrees of frost' means 20° F. or 12° F. or —20° F. I leave to the reader to decide, if he can.

Example 1. Convert 98·4° F. into Centigrade.

$$°C. = \tfrac{5}{9}(98{\cdot}4° - 32) = \underline{36{\cdot}9°\ C.}$$

Ex. 2. Convert −185° C. into F.

$$\frac{F-32}{9} = \frac{-185}{5} \quad \therefore F-32 = -333 \text{ or } \underline{-301°\ F.}$$

§ 150: **Stem error of a thermometer.** A thermometer when being tested is entirely immersed, to secure a uniform temperature all over, but in common use its long stem stands out in a much cooler place. The mercury in the stem shrinks and the reading is too low. If the mean temperature of the stem can be ascertained, a correction can be calculated as in the following :—

Ex. 3. A thermometer sunk to its 20° mark in a bath reads 90°. Rest of stem averages 25°. Find true temperature of bath; e of Hg. in glass ·00015. How does the error depend on (a) length of degree divisions, (b) expansibility of thermometric liquid, (c) increasing difference of temperature of stem and bulb as that of latter rises? [L.]

The problem is to find the length, at about 90°, of a thread of mercury standing above the 20° mark which, at 25°, occupies (90−20) degree spaces. (The procedure for solid expansion, § 128, is quite near enough for calculating the small correction.)

$$L_{90} = (90-20) + (90-20) \times e \times (90-25)^\circ.$$
$$= 70^\circ + {\cdot}68^\circ \text{ correction.}$$
$$\therefore \text{ Corrected temperature} = 20^\circ + 70{\cdot}68 = \underline{90{\cdot}68^\circ}.$$

Evidently the correction does not depend on (a) at all, for we have not had to inquire their length (i.e. a long sensitive thermometer does not suffer excessively); (b) it is proportional to the expansibility (therefore large for alcohol); (c) it involves (T−low mark)×(T−low temp. of stem) i.e. is about proportional to T^2, *becoming very serious in high-temperature thermometers.*

§ 151. **Standard thermometers.** The degree centigrade was long defined in England as ·01 of the fundamental interval on a certain mercury thermometer at Kew, which means that the apparent expansion of mercury in a certain glass vessel was made uniform by Act of Parliament.

But a piece of that same glass does not expand quite uniformly with temperature as measured on that thermometer, which must therefore give a variable expansion to mercury itself. The standard scale becomes the difference of two imperfectly regular dilatations, a mixture of about 5 parts mercury and 1 of some sort of glass. And a different sort of glass will cause a discrepancy of as much as ·1° at 40° C.

Now **Gases** expand very much, so that change in the containing vessel has a much less disturbing effect. And they go on expanding all very nearly alike at temperatures below, and far above, the reach of mercury. Chiefly for these reasons a gas is now taken as the standard thermometric substance.

The apparatus of § 140 becomes rather a means of testing the mercury-in-glass thermometer. But as there pointed out its species of exaggerated 'stem-error' makes it unsatisfactory at high temperatures and another device is preferred.

The constant-volume gas thermometer.

Let a volume of gas which has expanded to $V_\circ+V_\circ\times\frac{1}{273}\times t^\circ$ be now compressed into its original volume.

By Boyle's law PV is constant at any fixed temperature,

$\therefore P_\circ\times(V_\circ+V_\circ\times\frac{1}{273}\times t)$ becomes $(P_\circ+P_\circ\times\frac{1}{273}\times t)\times V_\circ$ as is evident in multiplying out. That is, *if a gas is kept from expanding, its pressure rises with the same coefficient, $\frac{1}{273}$, as that of increase of volume at constant pressure.*

The whole of the gas is kept in the bulb, the rise of P with temperature goes on uniformly throughout and is easily read :—

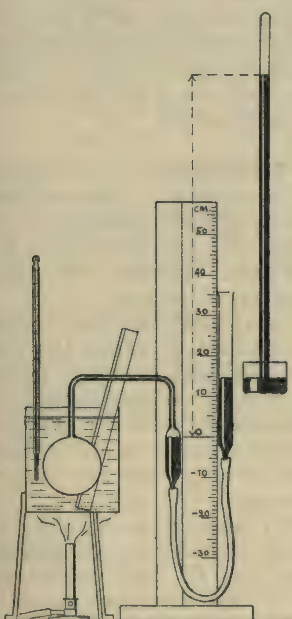


FIG. 70.

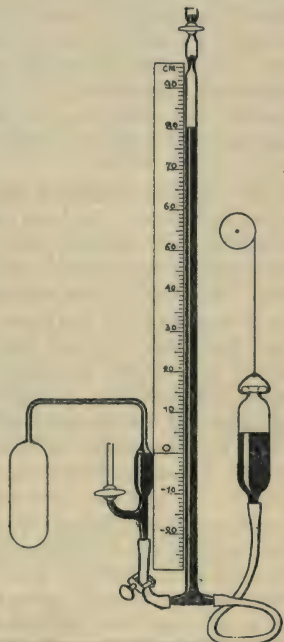


FIG. 71.

An efficient laboratory form of **constant-volume air thermometer** is shown in Fig. 70. There is the bulb and narrow connecting tube as before, but the latter opens into a wider vertical tube which communicates by a flexible pipe with a parallel open tube,

these containing mercury. To maintain the constant volume the mercury on the closed side is kept to a fixed scale division [or better, Fig. 71, to touch a glass claw sealed inside the upper end of the tube, where the walls slope in at 45° and the mercury surface is flat]. This is effected by raising or lowering the open tube. Comparing with Fig. 52, the reader will see that this resembles a Boyle's-law apparatus with the enclosed gas heated to prevent any shrinkage.

As with that apparatus, the pressure on the gas is $H+h$ cm. of mercury. This alters steadily $\frac{1}{273}$ ($=\cdot 00367$) of its value at 0° C. between the most extreme temperatures, e.g. suppose H was 75 and h 6·9 at 0°, making $H+h=81\cdot 9$ cm., then $H+h$ rises or falls ·3 cm. per degree, i.e. as long as the barometer stands still h alters this much. To obviate the double reading of H and h the barometer is sometimes incorporated in the apparatus as in Fig. 71, a form suggested as easy to fill and to use either with or without accessory barometer, and withal free from risk of leakage.

§ 152: In **The Standard Gas Thermometer** of the National Physical Laboratory the bulb is a litre cylinder of iridio-platinum with a long steel capillary tube running to the manometer, and passing into it through the flat iron plug top of the closed limb (about 1 in. diam.). The mercury stands in contact with a short spike projecting from the lower surface of the plug.

The open limb is a tall 1-in. tube; both rise from a closed reservoir into which mercury is pumped to adjust the level. Heights are read by a cathetometer, and corrected for the temperature of the mercury and of the brass scale (much as in § 137). The barometer is read separately with the same precautions. The slight dilatation of the bulb by heat and by pressure is allowed for and the little overflow of gas into the gauge as the pressure rises is taken into account.

The standard gas is Hydrogen very carefully purified and dried. Above red heat, however, this would leak through the platinum so that nitrogen is used instead to 1300° C. Both deviate from Boyle's law, and the difference of their deviations has to be applied to correct nitrogen temperatures to the normal Hydrogen Scale. (They do not differ more than ·01° anywhere between 0° and 100°.)

The degree centigrade is now defined as causing one one-hundredth of the increase of pressure observed in the constant-volume hydrogen thermometer between the melting point of ice and the temperature

of the steam over water boiling under the normal atmospheric pressure (76 *cm. of mercury in the barometer, corrected to* 0°).

On this scale a standard mercury-in-glass (*verre dur*) thermometer is ·1° high at 50°.

§ 153. **Absolute temperature.** The observation that a gas alters its volume or its pressure so uniformly by $\frac{1}{273}$ of its value at 0° C. per degree change has led to the conception of a temperature at which the perfect gas would have shrunk 273 273rds of its freezing-point value—it would have no volume at all at −273° C. This temperature is called the **absolute zero** of temperature. It has not been reached, for we have to rely on gases for cooling, and as a matter of fact all gases escape this annihilation by liquefying before reaching this low temperature. (What laws liquid and solid may follow then are unknown.) The most resistant of them, helium, liquefies at −268·6°. [Doubtful, because the petrol thermometer often used with liquid air has long frozen, and the effects obtainable from electric thermometers have dwindled almost proportionally to the gas volume.]

Indeed, all electrical effects in metals bid fair to become extinct at the absolute zero. And the best Radiation experiments fit in with a law which assumes the cessation of radiation at this same point.

But while there seems to be something very significant about −273° C. it would be rash and unjustifiable to prophesy that all thermal effects are going to cease there. All we need do here is to rechristen −273° C. 'Absolute Zero' and to start from it the 'Absolute Scale' in which each temperature is the centigrade plus 273°.*

$$°A = °C + 273.$$

§ 154. The *Law of Charles* may therefore be restated :—*The volume of a mass of gas at fixed pressure is proportional to its absolute temperature* T.

$$V \propto T \text{ when } P \text{ is constant.}$$

Now Boyle's Law states that PV is constant at fixed temperature, so having hitherto kept P constant we stop at any T we like, and, altering P, V changes so as to keep PV constant.

* Really 273 + a fraction unknown. Hence accurate thermometry keeps to °C. while the temp. °A, like the radian, is fundamental in theory.

We might, for instance, force V down to its initial size, for which we should have to maintain a P proportional to T, V being constant, as in § 151.

The two laws then combine into one statement, **the characteristic equation of a perfect gas,** $PV \propto T$, or

$$PV = RT.$$

The product of the pressure and the volume of a mass of gas is equal to R times its absolute temperature where R is a number which depends on masses, units, etc., but remains fixed when once fitted to the particular case in hand.

Ex. 4. Find R for 1 c.c. of air at 0° C. and 76 cm. mercury.

$PV = RT$ becomes $76 \times 1 = R \times 273$. $\therefore \underline{R = \cdot 279}$.

Ex. 5. Find R for 1 grm. molecule (mol. wt. in grm.) of any gas (occupying 22,120 c.c. at 0° C. and 1 atmo.).

1 atmo. = 1,016,000 dynes per sq. cm.

$1{,}016{,}000 \times 22{,}120 = R \times 273$, $R = 82{,}400{,}000$.

Now PV = energy (§ 69) in ergs, hence R = ergs per degree = about 2 cals. (§ 187) per degree, i.e. *the 'specific heat of a gramme molecule' of any gas is about* 2.

Ex. 6. Find R for 1 grm. of dry air at 0° C. and 1 atmo. given 1 litre weighs 1·293 grm. At what temp. will 1 grm. occupy 1 litre at 1 atmo.?

1 litre weighs 1·293 grm. $\therefore$ 1 grm. fills 1/1·293 litre.

$\therefore 1 \times 1/1{\cdot}293 = R \times 273$. $\therefore R = \cdot 00284$.

Again $1 \times 1 = \cdot 00284 \times T$. $\therefore T = 353$ A = 80° C.

EXAMPLES.—CHAPTER XVI

[Expansibility of mercury in glass ·00015.]

7. Define 0° C., 100° C., 23° C. [L.]

8. Describe the construction and mode of action of a mercury-in-glass thermometer, and explain how you would test the accuracy of one. [L.]

9. A mercury thermometer at 0° C. contains 2 c.c. of mercury and distance between fixed points is 30 cm. Calculate diameter of tube. [L.]

10. A thermometer immersed in a bath as far as zero mark reads 250° C.; mean temperature of stem 20°. Find temperature of bath. [M.]

11. What is meant by a scale of temperature and on what does the definition of any particular scale depend? Explain carefully the construction and mode of action of some form of constant-pressure air thermometer. [L.]

12. Convert 50° C. and −40° C. into Fahrenheit temperatures and 56° F. and zero F. into Centigrade.

13. Describe a method by which the temperature of any very hot place such as a furnace could be determined.

14. How would you measure the increase of pressure in a given volume of air between 0° and 100° C.? How would the presence of water affect it? Graph. [L.]

15. Upon what factors does the volume of a known mass of gas depend? To what pressure at 0° C. can a glass tube be filled to stand 400° C. if its bursting pressure is 20 atmos.? [M.]

16. State the laws of change of pressure, volume, and temperature of gases, and show that they may be expressed in the form of a single equation containing one constant. What is the value of the constant for 1 grm. of hydrogen at 0° C. and 760 mm. pressure; density ·0000895?

17. 15 litres of air are cooled from 45° to 15° C. and pressure is reduced from 795 mm. to 760. Calculate new volume. [Ab.]

18. A sample of a gas was found to have a volume of 100 c.c. at 18° C. and 72 cm. of mercury pressure, and a volume of 200 c.c. at 90° C. and 45 cm. pressure. Calculate at what temperature it would have a volume of 400 c.c. at 100 cm. pressure. [L.]

19. An open-mouthed litre jar at 100° is plunged mouth down into water and cooled to 15° and ultimately adjusted so that water inside and out is at same level. What volume is above water? [Ab.]

20. In $PV=RT$ express R in foot-pound-second units given that 1000 cu. ft. air at 0° and 14·75 lb. per sq. in. weighs 80·7 lb. and $g=32{\cdot}2$ ft. per sec.^{2}. [L.]

Chapter XVII

CALORIMETRY

In this chapter it is assumed that the substance under consideration undergoes no permanent internal alteration such as combustion, solidification, etc., and is not worked upon by any external mechanical forces. With this provision :—

§ 155. A body gets hotter or colder. It is natural to suppose that some entity passed into it and raised its temperature; or left it, as it cooled. We call that entity **Heat.** In physics, cold is not regarded as a separate entity, such as one commonly thinks of Frost—childhood's 'Jack'—it is merely deficiency of heat.

Temperature is measured in Thermometry by its degree, like height of water-level; Heat in Calorimetry by quantity, like water by the gallon.

Heat is always contained in matter and gives it a temperature. Empty space cannot contain heat and cannot have temperature (except transiently, see Radiation).

Heat travels from one portion of matter to another without change in total quantity, and a body cooling from one temperature to a lower gives out the same amount of heat as would raise it from the lower to the higher.

A large mass of a substance can contain more heat than a small, indeed, *Quantities of Heat are proportional to the masses of a standard substance which they can warm from one to another fixed temperature.*

The *Capacity for heat of a whole body* is the number of units of heat that must be poured into it to raise its temperature 1°.

The standard substance is Water.

The unit quantity of heat, called the calorie, warms 1 gramme of water 1°, viz. from 15° to 16° C.

The kilogram Calorie=1000 calories.

The engineer's 'British thermal unit' warms 1 lb. of water 1° Fahrenheit.

The capacity for heat per gramme, called **the specific heat (sp. ht.) of a substance, is the fraction of a calorie that warms 1 gramme of substance 1°.**

The specific heat of water (1 at 15° by above definition) exceeds that of every other substance except hydrogen. Specific heats increase at higher temperatures, but not excessively, and for most things we can speak of a 'mean specific heat between 0° and 100°' and reckon it constant.

The Quantity of Heat required to warm a body is therefore the product of its capacity for heat and its rise of temperature, i.e. the product of its mass, specific heat, and rise of temperature, $Ms(t_2°-t_1°)$.

Quantity of heat is primarily measured by catching it in water, when it=mass of water$\times 1 \times$rise of temperature produced.

§ 156. **Measurement of quantities of heat. Method of constant heat supply.**

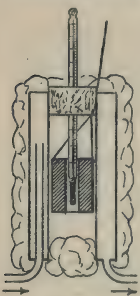
Some of the earliest calorimetric experiments were made by Black at Glasgow (ca. 1760) by using a source (e.g. a clear charcoal fire) which supplied heat at a presumably constant rate. The reckoning of the power of a stove by the shortness of time in which it brings a certain kettle to the boil is familiar enough; it is just one step farther in scientific exactness to express it in *calories per minute=grammes of water in the kettle×rise of temperature per minute.*

Black showed that quicksilver heated much faster than the same weight of water, so that its specific heat is small (as mixing hot quicksilver and cold water also proved), so small that even an equal bulk of the heavy metal did not take as much heat per degree as water. Indeed, it took not much longer to *melt* a pot full of lead than to bring the same pot full of water to the boil, at a far lower temperature.

In the next chapter one of his experiments on latent heats is given as performed in the laboratory with a gas-burner, and also the modern development of this method in which the heat is both supplied and exactly measured by electrical means (using a lamp or heating coil immersed in the calorimeter), a development which has altered a rough-and-ready method into one of the highest convenience and accuracy.

§ 157. **The method of mixtures** is frequently employed in finding specific heats, etc. A weighed mass M of substance is heated in some sort of steam or vapour jacket swaddled in wool

(or for high temperatures an electric oven or furnace) till it reaches a steady temperature T and is then dropped quickly into a 'calorimeter' held for the moment close under the heater (much closer than in Fig. 72). This calorimeter is a little pot of thin polished copper or aluminium, about two-thirds filled with W grm. of water, and furnished with a stirrer and a delicate thermometer. It is sheltered from draughts and from stray warmths (hand, flames, etc.) and from conduction, by standing on pointed corks inside a larger jacket. See Fig. 72.



Just before dropping the hot body in, the water is observed to be at $t_1°$ (near the room temperature); and soon after, it rises to a maximum $t_2°$ (kept stirred). Therefore the hot body has lost $Ms(T-t_2)$, the product of mass, specific heat, and fall of temperature, and the water has gained $W(t_2-t_1)$ calories, and to a first approximation these are equal. Hence s.

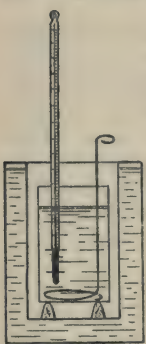


FIG. 72.

Example 1. Find sp. ht. of metal of which 300 grm. at 100° raise 500 grm. water from 14° to 20° C.

Calories lost by metal $300 \times s \times (100-20)$ fall $= 24{,}000s$.
" gained by water $500 \times 1 \times (20-14)$ rise $= 3000$.
These are equal, $\therefore$ $\underline{s = \cdot 125.}$

§ 158. **Method of mixtures, allowances for vessel and for cooling.**

But the water has not captured and held all the heat. I. Some was lost into the cooler air as the hot body passed from heater to calorimeter; II. some went to heat thermometer, stirrer, and the metal calorimeter; and III. some has already been lost from its walls, for as soon as it rose in the least degree above its surroundings it began to send them heat.

I. Of these, the first must be minimized by a short quick transfer.

II. The second is allowed for by adding in the capacity for heat or **Water Equivalent,** as it is called, of the pot, etc. The good-conducting metal speedily rises to the same temperature throughout * as the water, therefore multiply its weight c by its

* This is the objection to glass and crockery calorimeters; they are badly conducting, and get only partly warmed, and one does not know how much to allow.

specific heat ($\cdot 1$ Cu, $\cdot 25$ Al) and add the product on to W. And add $\cdot 5$ grm. for each c.c. of thermometer submerged.

III. **Cooling correction.** For the third, go on watching after reading t_2 for half to three-quarters the time it took to rise from t_1, and any small (fraction of a degree of) cooling observed, add on to increase and correct t_2. (See Cooling, § 173, from which it will also appear that it is an advantage to work on a rather large scale and to be content with a small rise of temperature, delicately measured.) Hence s from

$$Ms(T-t_2)=(W+\text{w. eq. of cal., etc.})\times(t_2 \text{ corrected}-t_1).$$

Ex. 2. In Ex. 1 the copper calorimeter (sp. ht. $\cdot 1$) and stirrer weighed 160 grm.; submerged part of thermometer $=2$ c.c. bulk; and it cooled to $19\cdot 8°$ in two-thirds time of experiment afterwards ($\therefore$ add $20-19\cdot 8=\cdot 2°$ on to the highest, $20°$).

Calories lost by metal $300\times s\times(100-20)=24{,}000s$

$$\left.\begin{array}{ll} \text{,, gained by water} & 500\times 1 \\ \text{,, ,, cal. and st.} & 160\times\ \cdot 1 \\ \text{,, ,, thermom.} & 2\times\ \cdot 5 \end{array}\right\}\times(20\cdot 2-14)=3200$$

These are equal, $\therefore$ $\underline{s=\cdot 133.}$

§ 159: **Method of mixtures; experimental variations.**

Liquids are heated in a beaker and poured into the water, but if chemically active are enclosed in sealed tubes treated like solids.

Or for **soluble substances,** sodium, sulphuric acid, etc., paraffin oil can be used instead of water, and its specific heat afterwards found (Ex. 4).

It may be preferable to heat the water and pour it into **inflammable substances** whose specific heat is required (Ex. 3), or into the cold vessel whose 'water equivalent' is to be found.

Unstable substances can be cooled instead of heated (Ex. 5).

The specific heats of **Gases** (free to expand) were measured by Regnault and others by passing slowly first through a coil of many yards of small gas-pipe in a heating bath and thence immediately through a similar coil in the calorimeter. The weight of gas was that lost by the steel cylinder whence it came.

Ex. 3. 50 grm. water at 90° stirred into 200 grm. paraffin oil at 15° bring the mixture to $40\cdot 7°$. Omitting corrections, find s of oil.

Water loses $50\times 1\times(90-40\cdot 7)=2470$ cals.
Oil gains $200\times s\times(40\cdot 7-15)\ =5140s$ cals.
These are equal, $\therefore$ $\underline{s=\cdot 48.}$

Ex. 4. 40 grm. sodium removed from oil at 80° are dropped into 200 grm. of this paraffin oil and raise it from 15° to 23°. Find specific heat of sodium.

$$\text{Sodium loses } 40 \times s \times (90-23) = 2680s \text{ cals.}$$
$$\text{Oil gains } 200 \times \cdot 48 \times (23-15) = 768 \text{ cals.}$$
$$s = 768 \div 2680 = \underline{\cdot 286.}$$

Ex. 5. 25 grm. of an insoluble explosive at 0° are dropped into 60 c.c. water at 15° (Al cal. and st. 16 grm., 1 c.c. of thermom.). If $s = \cdot 5$ find resultant temperature t_2.

$$\text{Explosive gains } 25 \times \cdot 5 \times (t_2 - 0) = 12 \cdot 5 t_2 \text{ cals.}$$
$$\left.\begin{array}{l}\text{Water loses } 60 \times 1 \\ \text{Etcs. lose } (16 \times \cdot 25 + 1 \times \cdot 5)\end{array}\right\} \times (15 - t_2) = 966 - 64 \cdot 5 t_2 \text{ cals.}$$
$$\therefore 77 t_2 = 966. \quad \therefore \underline{t_2 = 12 \cdot 5^\circ}.$$

§ 160 : Two modifications of this method have the advantage of reducing the troublesome cooling.

I. In one, which may be called a **Compensation method,** ice-cold water from a burette packed in ice is run into the calorimeter quickly after the hot body, until it restores the initial temperature. Then in calculation this water gains all the hot body loses, and no other water-values come into account.

II. In the **Constant-Flow method**, suited to experiments where heat is being steadily supplied, the calorimeter is kept down in temperature by circulating a stream of water through pipes inside it or a jacket outside. The mean *small* steady difference of temperature of ingoing and outcoming water is measured by a pair of sensitive electrical thermometers (§ 645) and multiplied by mass of water passed = calories removed.

§ 161 : **Specific heat of liquids by cooling.**

A small closed calorimeter contains the liquid of sp. ht. s; and in a second experiment, the same bulk of water. It hangs in a cold enclosure and in each case is timed as it cools from 60° to 50°. The rate of losing heat depends solely on the outside (§ 174), and if it takes say half as long with liquid s inside, then evidently to give up calories at the same rate the liquid has to cool twice as fast as the water, i.e. it contains per cubic centimetre only half as much heat as water. Its s per grm. is therefore $\cdot 5 \div$ mass of 1 c.c. To generalize, for half read $1/a$ and for twice read a times.

§ 162. **Dulong and Petit** discovered that the

Specific Heat of an element × its Atomic Weight = 6·4.

This of course is only approximately true, since specific heats

vary somewhat with temperature. The product is called the **Atomic Heat,** it is the capacity for heat of the 'gramme-atom' of any substance. Further, the Molecular Heat of a compound is found to be the sum of the atomic heats of its constituent atoms. The law is discussed at length in the chemistry books.

EXAMPLES.—CHAPTER XVII

6. 8 gal. water at 90° were poured into an iron bath (water equivalent 15 lb.) containing 10 gal. water at 14° C. Find final temperature.

7. Into an empty calorimeter at 15·8° 100 grm. of water at 54·2° were poured and the final temperature was 52·2°. Calculate water equivalent of calorimeter.

8. 50 grm. water at 60° were mixed with 50 grm. at 10°. Final temperature 32°, find water equivalent of calorimeter.

9. Define the specific heat of a body and explain how that of a liquid may be determined. [L]m.

10. 960 grm. of water at 15° C. is contained in a calorimeter weighing 200 grm. of sp. ht. 0·2. 500 grm. of water at 65° C. is mixed in. Calculate temperature. [L.]

11. 27·45 grm. of marble at 95° were dropped into 100 grm. of water in a calorimeter (of water equivalent 5·8 grm.) at 16·9°. Final temperature 20·8°, find specific heat of marble.

12. 41·4 grm. of lead at 100° were dropped into 71·2 grm. of water in a copper calorimeter of 67·1 grm. at 12·6° C. Final temperature 14·2°, find specific heat of lead.

13. 10,000 grm. of oil sp. ht. ·4 are to be raised from 20° to 200° by a burner giving 200 cals. per sec. How long will it take ?

14. 50·3 grm. of water at 15° were contained in a calorimeter of water equivalent 3·65 grm. The thermometer was removed, heated, and replaced when it read 70°. Final temperature 16·1°, find water equivalent of thermometer.

15. How much coal per 24 hours will raise to 15° C. the air of a building 100 m. $\times$ 40 m. $\times$ 25 m., the whole air being replaced every hour by air entering at 0° C. Sp. ht. air ·2375, sp. gr. ·00129 ; 6000 cals. evolved per grm. coal burnt. [L]m.

16. In using Siemens' Pyrometer, a copper cylinder average sp. ht. ·1 weighing 137 grm. was heated in the furnace, snatched out and dropped into 570 c.c. water at 14° in a vessel of water equivalent 30 grm. Temperature rose to 34·25°, find that of furnace.

17. A quantity of oil at 40° is poured on a 20-grm. lump of sodium at 10° and the resulting temperature is 35°. The metal is immediately fished out and 20 c.c. of water at 10° run in from a pipette. Temperature now 25°, find specific heat of sodium.

18. Explain how the specific heats of two liquids may be compared by the method of rate of cooling. What are the objections to this method ? [L.]

CHAPTER XVIII

LATENT-HEAT CALORIMETRY

IN this chapter the effect of the removal of part of the condition at the head of the foregoing chapter comes under consideration; it is the **Calorimetry of changes of physical state.**

§ 163. The chapter on Change of State must be anticipated thus far :—

As a solid is supplied with heat its temperature presently ceases to rise and *remains* at a steady melting point till all is melted, the heat meanwhile 'going into hiding,' so to speak. And again when the liquid's temperature reaches a boiling point it stops rising and the liquid *gradually* disappears.

The calories that have hidden per gramme of substance constitute the **Latent Heats** of these changes of physical state—fusion and vaporization.

The measurement of these latent heats is of importance. And once measured, they open up new and convenient calorimetric methods.

§ 164. **The Latent Heat of Melting** of ice is easily found by dropping dry lumps of it into the calorimeter. To avoid a big correction for heat derived from the air, Rumford's plan may be used—warm the water nearly twice as far above the air at first as you will cool it below the air at last. Then the loss of heat while hotter will be about balanced by the slower gain during the much longer time it is colder than the air.

Example 1. Into a calorimeter, w. eq. 12, containing 500 grm. water at 25°, 86 grm. dry ice at 0° were stirred and temperature fell to 10°.

Lost by water and vessel $= 512 \times 15° = 7680$ cals.

Gained by ice in melting $86 \times L$, ,, after melted, in rising to 10°, $86 \times 10°$ $\left.\begin{matrix} 86 \times L \\ 86 \times 10° \end{matrix}\right\} 86 \times (L+10)$ cals.

$\therefore L+10 = 7680 \div 86. \quad \therefore \underline{L = 79{\cdot}5 \text{ cals. per grm.}}$

Or the procedure may be reversed with advantage. A hollow is scooped in a large block of ice. Into the dried concavity,

water at the ordinary temperature is run, covered with an ice lid and left to cool to 0°. Then it is pipetted out, the last traces removed with a dry ice-cold sponge and the whole weighed, when the extra weight is that of the ice melted by the heat brought in by the water.

Ex. 2. 20 grm. of water at 16° are run in and 24 grm. at 0° removed.

Lost by water $20 \times 16 = 320$ cals.

Gained by ice $4 \times L =$,, $\therefore$ $\underline{L = 80.}$

§ 165. Owing to the prevalent use of ice as the typical solid, students are apt to overlook the fact that not all solids are on the point of spontaneously melting, but most require warming up, as solids, before any question of latent heat comes in. Then the following example shows that the melting point (unless the specific heats are equal) and the specific heats both as solid and as liquid must have been found already :—

Ex. 3. How much heat warms 1 kg. of glacial acetic acid from 5° to 25° ? It melts at 16·6° with latent heat 46·4 ; sp. ht. solid ·62, liquid ·50.

Solid absorbs before melting	$1000 \times \cdot 62 \times (16 \cdot 6 - 5)° =$	7190 cals.
Melting requires	$1000 \times 46 \cdot 4$ $=$	46,400 ,,
Liquid absorbs after melting	$1000 \times \cdot 50 \times (25 - 16 \cdot 6)° =$	4200 ,,
	Total	$\underline{57,790 \text{ cal.}}$

§ 166. The **Latent Heat of Vaporization** of a substance can be measured by (Black's) **Method of constant-heat supply** :—

Ex. 4. (Very rough.) A little calorimeter contained dry ice, broken small. It was placed over a steady bunsen and sheltered from draught. In 2 min. the ice had just disappeared, at $4\frac{1}{2}$ min. water boiled, at 19 min. all boiled away. Find latent heats of melting and boiling.

Water rose 100° in $4\frac{1}{2} - 2 = 2\frac{1}{2}$ min. $\therefore$ bunsen supplies each gramme of it with 40 cals. per min.

$\therefore$ it takes $40 \times 2 = 80$ cals. to melt 1 grm. ice.

and $40 \times (19 - 4\frac{1}{2}) = 580$* cals. to boil away 1 grm. water at 100°.

Ex. 5. An electric lamp using ·50 ampère at 94 volts was immersed in a can of ether which it kept steadily boiling at 34°. The can stood on a balance pan and the time at which it tilted above the counterpoise was noted. 5 grm. was removed from the counterpoise and the next time of passing of the pointer over zero noted, and so on. The intervals were 50 sec., 50, 50, 50, 52, 50, 48, 48. (Subsequently the ether was kept steadily near 23° by running lamp for 10 sec. every 2 min., but as it was found to be evaporating 1 grm. every 2 min. all this ' cooling ' was due to evaporation, and therefore no other cooling loss need be allowed for.)

On the average then 1 grm. was boiled off every 10 sec. by an energy supply $(94 \times \cdot 50 \times 10) \div 4 \cdot 2$ cals. [§ 639],

or $\underline{L = 112 \text{ cals. per grm.}}$

* High result due to neglect of ' cooling ' loss.

§ 167. More usually the latent heat of vaporization is measured when it is all being given out again during liquefaction. For instance, a jet of steam from a boiling flask is plunged into the calorimeter water and allowed to raise the temperature 10° or 20°, after which the increase in weight is the steam condensed; corrections as in § 158 are required, and also a correction for steam condensing in the pipe and dropping into the calorimeter merely as hot water.

Ex. 6. A copper calorimeter weight 159·8 grm. weighed 571·0 grm. when containing water at the room temperature 16° C. Steam at 100° blown in raised it to 27·3° in 2 min., and it afterwards cooled at the rate of ·15° per min. Final weight 579·0 grm.

579·0−571·0=8 grm. of steam gave up $8 \times [L+(100°-27·3°)]$ cals.
Calorimeter received $[159·8 \times ·1+(571-159·8)] \times [(27·3+·15)-16]°$
$=4900$ cals.

Equating these

$$8L=4900-582. \quad \therefore L=\underline{540 \text{ cals. per grm.}}$$

Condensation in the pipe is often troublesome and some such device as Fig. 73 (J. A. Harker) should be adopted. The swaddled and *steam-jacketed* pipe slopes upwards, so that any drops condensed in it run back to the boiler. There is a heat-insulating piece of rubber and then the pipe in the calorimeter turns downwards to a box at the bottom. This forms a ' condenser ' which can be taken out and weighed separately, and has the advantage of keeping the substance under experiment separate from the heat-collecting water. Practically no vapour escapes from the upcast spiral. The apparatus should be large. By putting this calorimetric condenser in place of the reflux condenser of Fig. 84, Regnault and others have measured the latent heat of vaporization of water at different temperatures, with result now given as [between 0° and 200°].

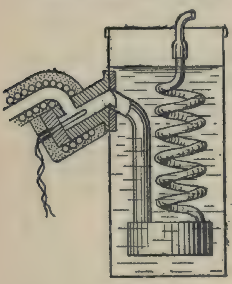


FIG. 73.

Latent Heat of Steam=596·73—0·6010 t° C. cals. per grm.

Thus the latent heat at 100°=536·63 and is less at higher temperatures, threatening ultimately to vanish altogether (§ 215). But the ' total heat ' to convert water at 0° into saturated steam at t° of course increases with temperature, being about 597+·4 t° cals. per grm.

§ 168. **Ice Calorimeters.**

Black's, Fig. 74. The specific heat of a body may be found very conveniently by dropping a known mass of it at a known temperature (say the room temperature) into the dry cavity in the block of ice of § 164, and after leaving covered with the ice lid for several minutes, removing and weighing the water produced.

Weight of body $\times$ sp. ht. $\times$ temp. °C. = grm. ice melted $\times$ 80.

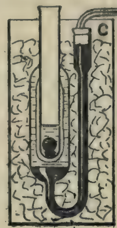


Fig. 75.

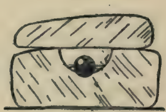


Fig. 74.

Bunsen's, Fig. 75. In this the well-known contraction of ice on melting is made to indicate the weight melted. An inner tube is surrounded by a jacket containing air-free water, mercury fills the bend and extends along a graduated capillary tube.

A freezing mixture is circulated through the inner tube till a cap of clear ice forms round it, then the instrument is packed round with melting ice and left a day or two to settle down to 0°. Then

Ex. 7. 5 c.c. of water at 15° are run into the inner tube, the mercury thread retreats and comes to rest 150 mm. nearer the bulb. 1 grm. of platinum at 100° is now dropped in and the thread retreats 6·5 mm. farther. Find sp. ht. platinum.

Water emits $5 \times 15 = 75$ cals. as it cools to 0°.

Index moves 150 mm., $\therefore$ 1 mm. corresponds to $75 \div 150 = \cdot 50$ cal.

$$\therefore 1 \text{ grm.} \times s \times 100° = 6{\cdot}5 \times {\cdot}50 \text{ cal.}; \quad s = {\cdot}0325.$$

The mercury thread may be driven out along the scale for further experiments by squeezing in the cork C.

§ 169 : In **Joly's Steam Calorimeter** the cold body whose specific heat is to be measured lies on a light balance pan hung inside a box to which steam at 100° is admitted by a large pipe. The steam condenses on the body, warming it to 100°.

Ex. 8. 180 grm. of metal originally at 21° increase to a weight of 183 grm. Of this ·15 grm. is known to be due to condensation on the pan itself. Find specific heat of metal.

2·85 grm. steam condensing to water at 100° emit $2{\cdot}85 \times 537 = 1530$ cals.

$$\therefore 180 \times s \times (100 - 21)° = 1530. \quad \therefore s = {\cdot}108.$$

Joly measured the *Specific Heats of Gases at Constant Volume.* From both sides of the balance hung 3-in. copper spheres, with

catch-pans, in the steam enclosure. Into one a few grammes of gas were compressed, and the increased weight (about ·1 grm.) condensed on this side was due to the heat absorbed by this gas. He found for air ·172, oxygen ·155, hydrogen 2·40.

§ 170. Another calorimetric operation is the measurement of the heat absorbed or produced during **Solution, Combustion,** or **Chemical Action** of any sort.

Quantities of the powdered salts to be dissolved are dropped into water in a large calorimeter, stirred around, and the changes of temperature noted. Allowance is made in calculation if the specific heat of the solution is sensibly different from 1. This would have been measured as in § 159.

Or the reagents in two separate tubes immersed in the calorimeter (so as to start at its temperature) are gradually mixed in one.

Or the substance to be burned (a food-stuff e.g.) is enclosed in a submerged steel 'bomb' with compressed oxygen and fired electrically.

The engineer tests Coal by powdering it and mixing with saltpetre in a cartridge, lights a touch-paper, fixes over it a miniature 'diving-bell,' and plunges it into water, up through which the smoke gases presently stream and give up their heat: a useful method of moderate accuracy.

Fuel-Gas is passed through a delicate meter and burnt in an elaborate sort of constant-flow 'geyser.'

Of course all weights and temperatures have to be observed, with 'water equivalents' and cooling corrections.

Some **Heats of Combustion** are, in calories per gramme: hydrogen 34,000, paraffin oil 9800, anthracite 8400, common coal 7000 to 8000, coke 7000, fat 9500, butter 9200, lean meat, etc. 5800, alcohol 6900, sulphur 2300, iron 1575, zinc 1300, dynamite 1300, black gunpowder 715.

§ 171. **Animal Calorimetry.** The calories emitted by an animal can be measured by shutting it in a box with a double wall filled with water and packed round with shavings to hinder access of heat from without. The water is kept stirred and a record made of its temperatures. Or the animal may be put in an 'ice-safe' and the rate of liquefaction of a contained lump of ice noted. The heat removed in the regulated ventilating current must be added in. The apparatus is standardized by burning inside it a known weight of alcohol.

Elaborate experiments with the human subject have shown that the body converts the net potential energy of food (as measured by heats of combustion) into thermal and mechanical energy as quantitatively as does any inorganic engine. The output of energy as hard mechanical work may be about one-eighth of the energy given off as heat. See also § 176.

EXAMPLES.—CHAPTER XVIII

9. 64 grm. ice at 0° reduced from $32\frac{1}{2}°$ to 20° the temperature of a 120-grm. copper calorimeter containing 500 grm. water. Find latent heat.

10. A copper calorimeter of 41·5 grm. weighed 118·0 grm. when containing water at 18·0° C. Ice was put in till the temperature was 10·4° and the weight 124·82 grm. Find latent heat of ice.

11. Ice at 0° was mixed with 2 kg. water at 25°. How much melted ?

12. 1 kg. of ice at 0° was melted in 2 kg. water at 55°. Find temperature.

13. 27 lb. of iron at 100° sp. ht. $\frac{1}{9}$ are dropped into 2 lb. of ice and 9 lb. of water. Find final temperature. [Ab.]

14. If specific gravity of ice is ·918, at what rate per square metre is heat escaping from a lake when a layer of ice 2 mm. thick is formed in an hour on its surface ? [L.]

15. How much sea-water at 6° is required to melt at $-2\frac{1}{2}°$ C. 100,000 tons of ice ?

16. 4 lb. ice (sp. ht. ·5) at −20° C. were mixed with 3 lb. paraffin (sp. ht. ·67) at 17° C. Find temperature.

17. Sketch Bunsen's ice calorimeter. Find travel of mercury in the tube when 10 cals. are given to the ice, diameter of tube being 0·4 mm. Ice density ·916. [L.]

18. A calorimeter, water equivalent 6 grm. contained 101·2 grm. water at 14·5°. 3·38 grm. of steam at 100° were condensed and raised temperature to 32·3°. The heating took 3 min., $1\frac{1}{2}$ min. later temperature had fallen ·3°. Find latent heat of steam.

19. Find the approximate weight of steam that would warm from 0° to 20° C. a room $15 \times 12 \times 10$ ft., air weighing 0·08 lb. per cu. ft., and total latent heat of steam at 20° being 590.

20. Compare the quantities of water, originally at 15° C., necessary to condense 100 tons of steam at 39° C. when its total latent heat is 580, and at 26° C. when its total latent heat is 588. [L.]

21. Define thermal capacity and latent heat. A metre cube of ice at 0°, density ·91, is converted into steam at 100°. How much coal is necessary if 1 grm. gives 7500 cals.? [Ab.]

22. 118 grm. of copper at 14° is suspended in steam at 100°. How much steam condenses? [M.]

23. By how much will the weight of a kilogram iron weight increase when weighed in steam at 100° (initial temperature 15°, sp. ht. ·12)? [L.]

24. Boiling point of a liquid is 156°, mean specific heat 0·46, latent heat 68. Find quantity of vapour at boiling point passed into a copper vessel weighing 30 grm., which contains 250 grm. of the liquid at 15°, to raise the temperature to 27°. [L.]

$$Q[68+\cdot 46(156-27)]=[250\times\cdot 46+30\times\cdot 1]\times[27-15].$$

25. How much heat will convert a kilogram of ice (sp. ht. ·5) at −20° C. into steam at 100° C.?

26. A silver vessel weighing 40 grm. contains 45 grm. ice, steam is supplied till ice all melts. Find total weight of vessel and contents at end (sp. ht. silver ·056). [Ab]m.

27. What weight of steam at 100° is required to raise 1000 kg. of oil sp. ht. ·4 from 0° to 80°?

28. Ditto, fat, if it melts at 45° with lat. ht. 25, and has sp. ht. ·5 sol. and liq.?

29. Superheated steam (sp. ht. ·3) at 150° C. is blown into ice at −20° C. (sp. ht. ·5). How much will raise 1 ton to 100°?

30. Describe a method of determining the specific heat of a gas at constant volume. How would you calculate the specific heat of a gas at constant pressure from that at constant volume?

CHAPTER XIX

COOLING

THERE are various processes by which heat travels from place to place; the joint effect of these in promoting the cooling of a hot body will now be considered. Afterwards they will be taken individually.

§ 172. Sir Isaac Newton was led by his experiments to a statement now known as **Newton's Law of Cooling** that *The rate of cooling of a hot body is proportional to the excess of its temperature above that of its surroundings.*

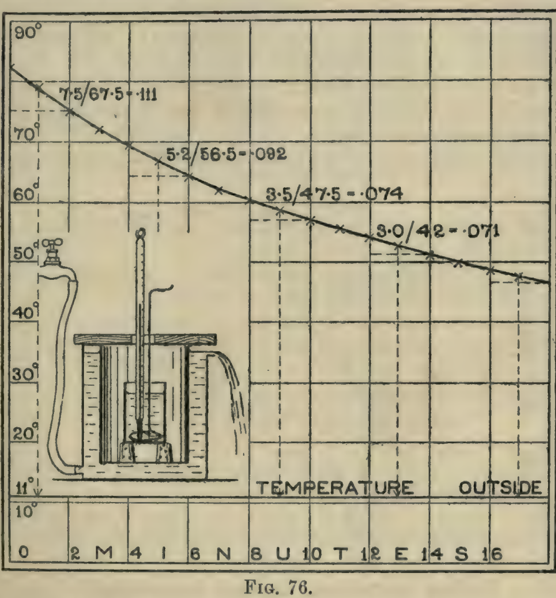


FIG. 76.

To test this a calorimeter of hot water can be stood inside a larger vessel through whose double walls cold water is circulated,

so as to get surroundings at a definite temperature. The hot water is stirred and its temperature is read every minute, and a curve like Fig. 76 plotted, a curve which always has the same general shape though its actual gradients depend on the particular apparatus employed (e.g. becoming flatter for a larger vessel).

According to the smooth curve plotted, the temperature fell during the first 2 min. from 82·5° to 75°, i.e. the rate of cooling was 7·5° per 2 min. The average temperature meanwhile was 78·5°, which was 67·5° in excess of the surrounding 11°.

$$\therefore \frac{\text{rate of cooling}}{\text{temperature excess}} = \frac{\text{height of 2-min. step}}{\text{total height to be gone down}} = \frac{7{\cdot}5^\circ}{67{\cdot}5^\circ} = {\cdot}111.$$

This ratio has been worked out on the diagram for several 2-min. intervals. According to the law it should be constant. It is only roughly so, being greater for greater temperature elevations, i.e. hotter bodies cool faster and faster than the law suggests.

But limiting ourselves to small differences of temperature, and recollecting the variety of processes involved in ordinary cooling, Newton's Law is good enough for common uses.

As a matter of history, Newton cooled his hot vessels in a strong draught, and a recent reinvestigation has shown that the law then holds much more closely.

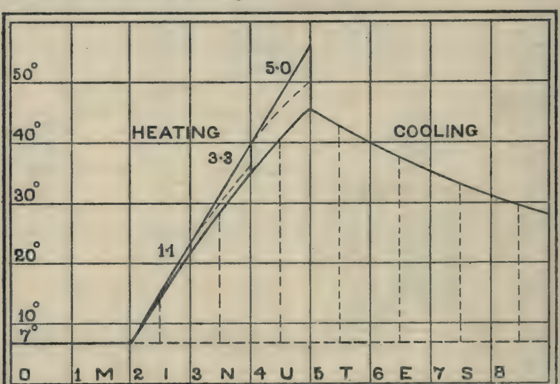


FIG. 77.

§ 173: **Cooling corrections in heat experiments.**

The instruction given in § 158 was to add to the top temperature the fall that afterwards took place in half the time of

heating. For heating steadily from the room temperature the average excess has been half the final excess above the surroundings, and by the Law cooling has averaged half as fast as it is now going on at the finish. The fall from the top temperature for half the time is therefore equal to the fall from the average temperature for the whole time.

More accurately, temperatures are taken every minute of heating and the average temperature for each minute found. To each is added the fall appropriate to that temperature taken from the cooling curve, or the sum of all these is the total correction to be added to the top temperature. In this way only narrow use has been made of the Law. Commonly, however, one uses it to fill in the long tail of the cooling curve, which would take a long time to observe and is required only for the smallest corrections.

Thus the mean temperatures in the 3 min. of heating in Fig. 77 are 14°, 28°, 41°. Cooling curve shows at mean temp. 41° a drop of 5° per min.; it does not continue far, but shows for mean temp. 33°, =26° above surroundings at 7°, a drop of 4° per min., or $4/26° = \cdot 154°$ *per min. per 1° temperature excess.* At 14° this gives loss $(14-7) \times \cdot 154 = 1\cdot 1°$, at 28° a loss $(28-7) \times \cdot 154 = 3\cdot 3°$; these are marked up from heating curve and give the straight rise to $1\cdot 1 + 3\cdot 3 + 5\cdot 0 + 45\cdot 5° = 54\cdot 9°$.

§ 174: **Emissivity.** The loss of calories per second is the loss of temperature per second × heat capacity (per degree) of hot body = fall per second × mass × specific heat. Dividing this by the measured-up area of surface of the body gives the loss per second per square centimetre. According to the Law, dividing this by the temperature excess above the surrounding medium, etc., we find what is called the *Emissivity of the particular sort of surface under conditions like those of the experiment; i.e. its loss of calories per square centimetre, per second, per degree excess.*

$$\text{Emissivity} = \frac{\text{fall of temp.} \times \text{mass} \times \text{specific heat}}{\text{cooling surface area} \times \text{time} \times \text{temp. above exterior}}$$

But emissivity under ordinary conditions depends so much on how the surrounding air or water (into which it is much greater) can circulate (see Convection) that calculations based on it have only limited application.

§ 175. The processes referred to as promoting cooling are those by which heat travels from place to place :—

Evaporation, Convection, Conduction, and Radiation.

Evaporation from wet surfaces has been already dealt with. On the small scale it helps cool your tea, on a large scale it assists in the great 'cooling towers' now a common and far from architectural adjunct of electric power stations (see § 213).

A little evaporation takes away a lot of latent heat : conversely, condensation of dew on a cold body warms it effectually.

At the boiling point it is all-important.

Convection currents in quiet air account for seven-eighths of the cooling of a closed hot-water vessel, etc., and in a good draught for a much greater proportion.

The effect of **Conduction** varies very greatly according to the things *in contact with* the hot body. Except with metal it is slow. An instance is the comforting application of cold metal or stone to a bruised or inflamed surface, another the 'chilling' of iron cast in an iron mould. Another, where it is quite abnormally effective, is this : a short copper fuse-wire takes two or three times the calculated current to melt it on account of conduction into the thick metal clamps at its ends. And everyone knows the part it plays in the Davy lamp.

Radiation goes on through a vacuum and is merely hindered by the presence of matter. Radiant 'heat' is therefore quite unlike the heat that we have measured in a calorimeter, and the whole subject is left to Chapter LI.

That radiation plays but a small part in cooling at temperatures below 100° is seen from the fact that, contrary to the usual statements, a blackened metal vessel cools only about one-eighth faster than a polished one, which *radiates* much less ; and is emphasized by the success of the popular Vacuum Flasks, which cool only by the radiation from a silvered surface traversing a vacuum jacket.

But Radiation becomes enormously effective above a red heat. While the common hot-water 'radiator' merely warms the air rising in convection currents past it and sends but little 'in rays' to cold hands held in front of it, an open fire warms by radiation only, unless 'the chimney smokes.'

As suggested here, radiation is distinguishable by going on equally in all directions : Light is one form of it.

§ 176. **The Heat Loss of the human body.** The best possible observations of these four processes are obtainable under experimental conditions of unusual simplicity. You require no apparatus whatever; on the contrary, you divest yourself of everything.

You are aware of an immediate disinclination to step on metal or stone, on smooth oilcloth or into a splash of water. You are avoiding good **conductors** and also that closeness of contact that enables even an indifferent conductor to snatch away a little heat.

You do not stand in the cold wind. Cold air merely rising past you in streams caused by your own warmth is tolerable, but a more active ventilation **conveys** away more heat than you care to lose.

Then when the water happens to be rather on the cold side you may be glad of the perceptible protection which even a thin costume affords, as the badly conducting film entangled in the meshes of its fabric fends off the rush of water.

And the speed with which you towel down in the friendly lee of anything that breaks the wind a little is an admission of the great additional effectiveness of **evaporation** as a cooling agent.

Finally, if you are lucky and the sun shines out, a long scorch in its **radiant** warmth compensates all your hardships. Shivering —which appears to be an involuntary (reflex) exercise of the muscles, counteracting the tendency to the restriction of their blood supply and chemical activity, and so to keep warm—ceases; the constricted capillaries of the skin dilate as the need for protection against chill is mitigated, hence the whole surface warms and convection increases; and perspiration ultimately brings evaporation to the rescue of a system whose (internal heat production+radiation received) cannot be otherwise disposed of.

Experiments, both direct calorimetric and also via the heat of oxidation of food-stuffs, indicate that a 10-stone man under the conditions of ordinary life loses from $2\frac{1}{2}$ to 3 million calories per 24 hours. Of this the warming of expired air accounts for 3·5 %, evaporation from lungs 7·5 %, evaporation from skin 14·5 %, and convective losses (including trifling pure radiation) 73 %. Hard muscular work means an all-round increase, as the whole surface becomes flushed and moist, and the less permeable articles of clothing are thrown off.

Birds having a relatively larger cooling surface give off heat faster—a sparrow a dozen times as fast per gramme. Their physiological activities must be intense—they are at 106° F.— and their appetites notoriously correspond.

Chapter XX

CONVECTION AND CONDUCTION OF HEAT

§ 177. **Convection.** By this is meant the conveying of heat from place to place in a fluid by the bodily movement of heated portions of it.

The motion may be mechanically forced, like the forced circulation of water in a surface condenser, or forced draught in a flue, or, simply, a wind ; but more usually the word suggests the natural motion of expanded heated portions in a fluid under the control of gravity.

When a fluid is heated locally neighbouring portions usually expand and therefore becoming less dense are lifted by the sinking of the colder denser portions around. The rising stream conveys its heat with it and constitutes the **convection current**. Meal thrown in shows these currents in a saucepan of water over a burner—up over the hot places and down all round ; flame and light ashes, smoke, or the well-known rippling appearance of 'hot air rising,' mark their track in air. In any case the warmth of the rising stream can be felt by the hand.

The convective circulation of heat evidently depends upon :—

(a) How much heat the fluid takes up per gramme (its specific heat).

(b) How much it expands, i.e. what lifting force begins to act on it.

(c) Its viscosity ; the less viscous the quicker it moves.

(d) The size and length of the pipes and channels through which the stream flows.

Water stands high in respect of (a), but in (b) at 4° it would fail altogether and generally in (b) and (c) is far excelled by air. Yet 'water-cooling' is quickest, for 1 c.c. of water will remove as much heat as 2500 c.c. of air, and it is so difficult to get this great bulk past a small hot surface. Hence the risk of melting out the seams of a tin can full of air only, and hence the need for extensive air-cooling surfaces, seen in the gilled cylinder of a

bicycle motor or the honeycombed miscalled 'radiator' of a water-cooled engine.

Fig. 78 shows the water-circulating system of a small motor. Hot water rises from the jacket surrounding the hot cylinder and then descends through gilled pipes, whence its heat is carried off by the wind.

Domestic hot-water heating systems are merely magnifications of essentially the same arrangement; all 'radiators' are on the down pipe.

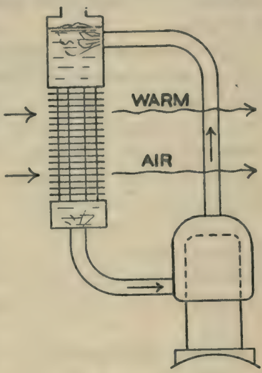


FIG. 78.

§ 178. **Conduction of heat.** For this process, as for convection, the presence of matter in the path is a necessity; unlike convection, there is no perceptible motion of that matter, and the process is at its worst in common fluids and reaches its best in those dense solids, the metals. It is a process perfectly independent of changes of density, and therefore of gravity. Difference of temperature is its sole and direct cause.

Substances differ very greatly as regards the facility with which heat travels through them, i.e. in conducting power. Special apparatus to show this is not worth while: take 2-in. pieces of stout copper wire, of iron nail, of solder, lead, brass, and electrical resistance wire, of slate pencil, chalk, and glass tubing, stick the end into a flame from which the fingers are shielded by a card, count seconds till you have to drop them, and you will have some notion of relative **conductivities**. Matches, paper, sealing-wax, and string you need not drop till flame itself reaches your fingers, they are bad conductors.

Many substances conduct so much worse than the metals that in common parlance they are 'non-conductors.' Chief among them is STILL AIR.

Wool, fur, and feather owe most of their value as clothing to the air they entangle and prevent from drifting off in convection currents. Hard-woven calico is a chilly integument compared with 'cellular' cloth of the same weight (but one's outer clothing had need be more or less wind-proof lest the air-retaining action of loose-woven stuff be violently overcome). Under the microscope Down is a most formidable entanglement of tiny barbs.

Asbestos, slag-wool, and light magnesia owe their value as steam-pipe laggings to their air-retaining porosity. To cool iron slowly the smith buries it in loose sand. Iron or even copper filings conduct very badly, the good contact essential for good conduction is lacking.

The hay and sacking wrapped round pipes and plants in winter act as air-retainers. These wrapped-up things, maintaining no vital heat of their own, must ultimately freeze in a long frost; but, like a mantle of snow, the wrapping will make the temperature changes more gradual, probably preventing local choking in pipes, and the sudden thaw in the morning sun so disastrous to an otherwise hardy plant.

In this question of clothing Conduction and Convection meet. Indeed, Convection can never be complete without Conduction, through thin adherent surface-layers and thin strata of fluid; precisely as mechanical mixing has to be completed by diffusion.

§ 179. **Conductivity.** Let heat be travelling straight through a plate of area A (Fig. 79, left) from a hot face at t'° to a cold one at t°. So long as the conditions remain everywhere the same it will be admitted that the same quantity of heat enters each square centimetre of plate, and the total is A times that of 1 sq. cm. It will also be admitted that the total quantity transmitted is proportional to the time T seconds of observation.

It is found by experiment that the flow is proportional to the difference of temperature $t'-t$ between the faces,

And it follows that it is inversely as the distance D it has to travel. For the plate can be supposed split into D successive plates 1 cm. thick, each with 1/D of the total temperature difference between its faces.

$$\therefore \text{ quantity transmitted, H calories} = \frac{\mathrm{CAT}(t'-t)^\circ}{\mathrm{D}}$$

where C is the constant depending on the material, its **Conductivity.** In this relation, putting all else $=1$, **the Thermal Conductivity of a material is the fraction of a calorie conducted from one face to the opposite face, 1° cooler, of a 1-cm. cube in 1 sec.,** Fig. 79, right.

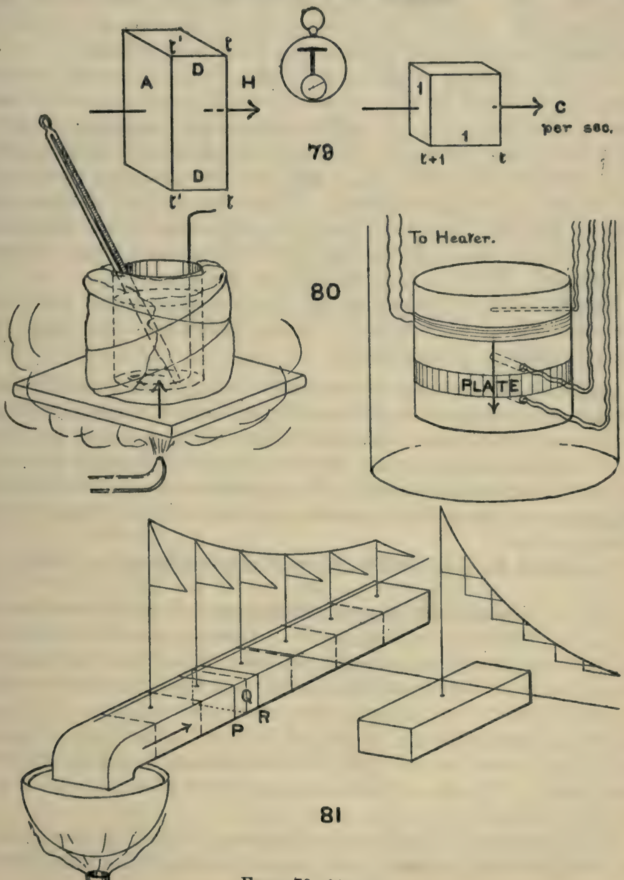
§ 180. **The Conductivity of poorly conducting substances,** Fig. 80. A simple method of carrying the foregoing into practice for a poor conductor appears in the following example.

Example 1, Fig. 80, left. The under side of a tile ·7 cm. thick is kept hot by a steam jet. On the tile stands a calorimeter with a flat bottom 20 sq. cm. area kept in good thermal contact with the tile by a smear of oil, and wrapped in wadding to hinder accidental access of heat. In 300 sec. it and its contents, 110 grm. of water, rise from 14° to 23·5°. Find C. of tile.

Here $H = 110 \times (23{\cdot}5 - 14)^\circ = 1045$ cals.

$AT(t'-t)/D = 20\text{sq.cm.} \times 300\text{ sec.} \times [100 - \frac{1}{2}(23{\cdot}5 + 14)]^\circ \div {\cdot}7 = 700{,}000.$

$\therefore\ C = 1045 \div 700{,}000 = \underline{{\cdot}0015}.$



FIGS. 79, 80, 81.

C. H. Lees has perfected this method of measuring the conductivity of poor conductors.—

A known supply of heat is electrically generated in a flat coil bedded between thick discs of copper [Fig. 80, right]. This is so good a conductor that the discs are at a nearly uniform temperature throughout, and the lower disc passes a uniformly spread heat to the plate of substance. Good thermal contact between disc and plate is obtained by a liberal smear of mercury on the amalgamated copper. H passing down through the plate is ultimately lost from the surface of a third copper block.

The temperatures of the coppers near the plate (and of the top disc) are obtained from electrical thermometers inserted in fine holes drilled in them. All outer surfaces are varnished so as to have the same emissivity E, which is calculated knowing the temperatures and areas of all and the total calories dissipated per second (=total generated). E thus found, the loss from the bottom disc and from the lower half of the edge of the plate can be calculated. This = H through middle horizontal plane of plate per second. Then t', t, A, and D give the required C.

Liquids were encircled by an ebonite ring which could be corrected for by working dry. Being heated from the top, convection currents were avoided.

Gases are measured in another way. A large thermometer bulb hangs in an enclosure and cools by conduction, convection, and radiation. Convection, usually the most effective, nearly vanishes below 10 cm. gas pressure, while conductivity remains constant (according to kinetic theory). Then radiation is separately found by repeating in a high vacuum, and subtracted, leaving conduction alone.

C = ·00003 for CO_2, ·00005 air, and 7 times as much for hydrogen.

§ 181 : **Conductivity of good conductors.** For good conductors the plate method gives bad results. Heat cannot be got into or out of the plates fast enough to keep them near the outside temperatures : the emissivity is much less than the conductivity. A boiler plate is far below the flame temperature and well above the water temperature. Possibly this is due to thin films of gases and water which on account of their viscosity almost stick to the plate. ·001 cm. of air conducts worse than several centimetres thickness of copper.

In one method this difficulty has been got over by measuring the temperature just *inside* the surfaces of the thick plate itself.

In the method introduced long ago by Forbes the plate is merely a selected centimetre in a long bar, kept hot at one end (say in melted lead). (a) After the bar has settled down to a steady condition its temperatures at equal distances are measured by (electrical) thermometers inserted in small holes in it, giving the curve from which the temperatures at P and R 1 cm. apart are obtained [Fig. 81, left].

(b) A foot length sawn off the same bar (and having a similar surface) is heated and allowed to cool, with a thermometer stuck in its middle. From its cooling curve the emission of calories per square centimetre per second at any temperature is found [Fig. 81, right].

The long bar beyond Q may now be supposed divided into equal blocks, each at the temperature of the thermometer in its middle and each of known surface area. Each therefore loses calories at a known rate, and adding all their losses together, their sum total is constantly made good by the heat flowing in through the section at Q. That is, we have obtained H that flows in through an area of cross-section A of bar per second when the temperatures 1 cm. apart at P and R are t' and t. Hence C.

In the figure the (a) curve runs down the bar and the little flags are the little bits of cooling curve (b) that the blocks would perform in say 1 min. if the supply of heat were suddenly arrested. The total loss is therefore equivalent to mass of any one block $\times$ specific heat $\times$ total height in degrees of all the little flags.

[Notice how Lees's method above described is a cutting and shortening of this bar, made possible by the smaller quantity of heat to be dissipated.]

Lees has used miniature long bars with electrical heater and thermometers, and combining modern experimental refinement with the most strict calculation has measured the thermal conductivities of many metals at temperatures down to $-185°$, finding that, utterly unlike electrical conductivities, most of them remain practically unaltered at all temperatures.

§ 182: It should be noted that the rate at which heat *first spreads or diffuses* depends not only on C, but also on the specific heat. For if the latter is small, so that only a small fraction of a calorie need be left in each cubic centimetre, a little heat can soon travel a long way. The first flush passes rapidly through bismuth (C = ·018, sp. ht. ·03; compare iron C ·17, sp. ht. ·11).

Other things being equal it can be shown that the *distance to which heat spreads is proportional to the square root of the time occupied.*

Hence even a poor conductor can momentarily snatch a little heat from a body suddenly coming into good contact with it—your bare foot on oilcloth.

The daily wave of warmth may penetrate a foot into the ground (not very strongly; work your hand down into the shingle): a water main a yard down is secure against a long (English) frost, and the annual wave is hardly felt below 50 ft.

And consequently, large masses which must necessarily scatter their heat far afield, take a long time to cool. The hot fibre of an Irwin oscillograph cools in about ·0002 sec.; the 1000-ton anvil, cast in situ, of a large hammer, was unapproachable for six months; and the earth, with its temperature gradient of 1° C. in about 100 feet of depth, emits less than half a calorie of its internal heat per square centimetre per day.

Table of Conductivities for Heat

Silver	1·09	Quartz (length).	·02	Ice	·0055
Copper, gold	·70	Marble	·005	Water	·00136
Aluminium..	·34	Glass	·0015	Mercury	·0154
Brass, zinc ..	·20	Gutta-percha ..	·0005	Many organic liquids	·0003 to ·0004
Iron (about)	·17	Vulcanized rubber	·0001	Gases	·00003 to ·00035
Lead	·08	Wood	·0002 to ·0005		

EXAMPLES.—CHAPTER XX

2. Describe various modes in which heat can be transmitted. Show how convection can be used in heating buildings by hot water and for ventilating purposes. [L]m.

3. Define conductivity and show how 'diffusivity of temperature' differs from it. [D]m.

4. Define conductivity for heat and explain how it can be measured for a good conductor like copper. [L.]

5. Describe a simple method of measuring conductivity of wood. What source of error is met with in applying the method to metals and how has it been avoided? [L.]

6. Three thermometers (a) next skin, (b) between vest and shirt, (c) between shirt and coat read 30·1°, 24·8°, and 21·4° C. Vest and shirt being equally thick, calculate their relative conductivities. [L]m.

7. Describe some method for conductivity of (a) copper, (b) glass. A plate 12×10 cm. $\times$ 1 mm. thick has one side in ice and the other in steam, $3\frac{1}{2}$ grm. of which are condensed per minute, calculate conductivity. [M.]

8. Calculate calories conducted per minute through a circular disc 2 cm. thick and 5 cm. radius when temperatures of the plane faces are 20° C. and 25°, conductivity of material being ·07. [L]m.

9. Calculate difference between temperatures of surfaces of a plate 1·5 cm. thick, of conductivity 0·1, when heat flowing through it per minute per sq. cm. evaporates 1 grm. of water of latent heat 536. [L.]

10. The walls of a cottage are 24 cm. thick and are of conductivity ·005. If the inside is 5° in excess of the outside, and area of the walls is 200 sq. m., how much heat is lost per hour through the walls ? [L.]

11. A steam-pipe at 170° C. is covered with badly conducting material 5 cm. thick. The outer surface is at 30° C. An hourly loss of 45,000 cals. is observed per 30 cm. length of pipe of mean circumference 50 cm. Calculate the conductivity of the material. [L.]

12. If a steam-pipe at 120° C. is covered with cement 2 cm. thick and of conductivity ·00018, how much steam will be condensed per sq. m. in a minute if outside is at 40° C. ? [L.]

13. Steam was blown through a rubber tube (1·2 cm. outside and ·6 cm. inside bore) of which 90 cm. was immersed in 1000 grm. of water. In 5 min. the water rose from 10·5 to 15·3°. Find conductivity of rubber.

14. [On Convection, etc.] A chimney 10 m. high contains gases at 60° C. ; the outer air is at 5° C. (Densities at 0°, gases ·0013, air ·00129.) Calculate (a) the reduction in pressure inside the room with doors, etc., shut ; (b) the volume of gases passing through a 9-in. (23-cm.) chimney-pot, door open ; (c) the coal consumed hourly in heating these gases (of sp. ht. ·33), if 1 grm. gives 6700 cals. ; (d) total work done per hour and the fraction this represents of total heat produced.

(a) Weight of 1 cm. square column in chimney $= 100 \times \cdot0013 \times 273/333$.
,, ,, ,, outside $= 100 \times \cdot00129 \times 273/278$.
Difference = driving pressure (or reduction in room)
$= 1000(\cdot001260 - \cdot001065) = \cdot195$ grm./cm.^{2}
$= 2$ mm. water $= \underline{192 \text{ dynes/cm.}^2}$

(b) Momentum imparted to chimney gases per sec. per cm.^{2} = driving pressure.
$\therefore$ mass entering per sec. $\times$ velocity = volume $\times$ density $\times$ velocity
$= \text{density} \times (\text{velocity})^2 = \cdot001065 \times v^2 = 192.$
$\therefore$ velocity = 425 cm./sec. at most.
$\therefore$ volume = velocity $\times$ area $= 425 \times \pi \times 11\cdot5^2 = \underline{176{,}000 \text{ c.c.}}$ per sec.

(c) Coal $= 176{,}000 \times \cdot001065 \times \cdot33 \times (60° - 5°) \times 3600$ sec. $\div 6700$
$= \underline{1840 \text{ grm.}}$

(d) Work done = lifting $176{,}000 \times \cdot001065$ grm. 1000 cm. high per sec.
$= \underline{6800 \text{ kg.-m. per hour.}}$

(e) = fraction $6\cdot8^8 \times 10 \times 981/1840 \times 6700 \times (4\cdot2 \times 10^7)$
$= \underline{\cdot00129}$ total heat energy produced.

Chapter XXI

THE MECHANICAL EQUIVALENT OF HEAT

§ 183. In the history of the Principle of the Conservation of Energy perhaps the most important chapter is that dealing with the determination of the Mechanical Equivalent of Heat.

Bacon and Locke had surmised that the well-known production of heat by friction, which checked visible motion, might be the conversion of that motion into an invisible commotion among the ultimate particles of substances, but the theory in favour even up to the middle of the nineteenth century was—as expounded by Black in the middle of the eighteenth—that heat was an *igneous fluid* or *caloric*, permeating the pores of all substances. It was admitted that caloric was *weightless*, for a balance bearing a bottle of water counterpoised by brass weights continued in equilibrium after a stay of many hours in a cold room had frozen the water and thus caused it to give up latent caloric amounting to more than a hundred times that lost by the brass weights.

Count Rumford, the English director of the arsenal at Munich, was struck by the heat developed in boring cannon—doubtless, like most of us, he had picked up borings fresh from the tool—and he made experiments to find out whether the current explanation, that caloric had been squeezed out of the solid metal, was probable. By the now familiar specific-heat experiment he could find no difference in the capacity for heat of solid metal and of borings, and in 1798 he set a horse to work a blunt boring tool on a cannon 'casting-head' immersed in water, and exultantly records his friends' astonishment when in $2\frac{1}{2}$ hours 2 gal. of water *boiled* while only a pound of chips had been produced.

Sir Humphry Davy in the following year rubbed together two pieces of ice in a frosty atmosphere (and even in vacuo) and showed that, with no possible access of heat from without, the friction continuously melted the ice, actually producing a liquid which, it was agreed, contained not less but *more* caloric than the ice.

§ 184. But it was not till 1840 that Joule of Manchester and others began to make extremely accurate experiments on the relation of work and heat, and to find that in whatever way they effected the conversion—by compressing air, by churning water, by grinding metal plates together, by hammering lead, by way of electro-magnetic induced currents, etc.—*a perfectly definite quantity of mechanical work completely converts into one unit of heat. This quantity is termed the* **Mechanical Equivalent of Heat** (*dynamical equivalent, Joule's equivalent,* J).

Heat is thus a 'mode of motion'—*a form of energy.*

Joule's favourite apparatus in his earlier experiments was one in which falling weights drove a paddle and churned water. It was a tedious experiment, demanding—and receiving—the most exquisite care. He found that 772 ft.-lb. produce one British thermal unit (pound °F.). Subsequent allowance for discrepancy between his sensitive mercury thermometers and the hydrogen scale, and for gravity at Manchester, has raised this to 777 ft.-lb.

Hirn went to work the opposite way; he found that there was a greater difference between the heat contained in the live and the exhaust steam from an engine when it was working hard than when running light. He measured this and found 1391 ft.-lb. =1 lb. °C.

§ 185. Notice that there is always an 'exhaust.' The definition of J given above cannot be read backwards. Different forms of energy are mutually convertible (with but small frictional loss) and all do naturally and completely convert into heat; but heat stands apart; no machine, practical or theoretical, can convert *all* the heat it is supplied with into any other form of energy. The remainder is not lost, it simply continues as heat, at a lower temperature and less useful. It has paid toll to the Principle of the Dissipation of Energy, § 26.

§ 186. In a *laboratory apparatus for measuring J* the brake-wheel of Fig. 4 is a brass drum filled with water and with a thermometer stuck in axially, the straps are silk ribbon. It is found that not more than ·5 % of the frictional heat produced escapes outwards through the silk, practically all going to warm the water. Then by § 28 the

Work done in ergs = circumference × net pull in dynes × no. of turns, *and this converts into*

calories = water equiv. of drum and water × rise of temperature

with the usual cooling correction. Hence the number J of ergs which = 1 calorie. Machines like this, or the common 'cone' friction-mill, or youthful attempts to imitate the fire sticks of the South Seas, serve at any rate to impress on the user how small a heat a great labour kindleth.

§ 187. Recent (1900) experiments at Manchester were carried out with a 'hydraulic' brake, a sort of reversed turbine. Two large cast-iron saucer-like wheels closely face each other, each is partitioned up inside into radial pockets slanting to meet those on the other wheel. Water run in gets caught up and flung violently from wheel to wheel, whereby one tends to drag the other round. The one was rotated by a 100-h.p. engine, and the other was prevented from following it by a load on a radial steelyard. Here the 'circumference' in the calculation is that of the circle on which the load hangs and in which it would have been hoisted. To avoid thermometer vagaries the water ran in from an ice tank and came out boiling into a weighing tank. All the inevitable heat leakages were most carefully gauged and allowed for, and the result is

$$\mathbf{J = 4{\cdot}184 \times 10^7 \text{ ergs per calorie}}$$
$$= 4{\cdot}184 \text{ joules per calorie}$$
$$= 1390 \text{ ft.-lb. per lb. °C.}$$
$$= 777 \text{ ft.-lb. per lb. °F.} = 1 \text{ British Thermal Unit.}$$

EXAMPLES.—CHAPTER XXI

1. Some shot, density 11·4, sp. ht. ·03, is contained in a cardboard tube 5 ft. long which is so manipulated that the shot falls from end to end 40 times. It is then poured round a thermometer and found 4·7° warmer than before. Calculate J.

Work spent among shot = 40×5 = 200 ft.-lb. (per lb. shot).
Heat obtained = $\cdot 03 \times 4{\cdot}7 = \cdot 141$ lb. °C. ,, ,,
Neglecting losses, these are equal. ∴ 1420 ft.-lb. = 1 lb. °C.

2. 200 grm. hangs from the rim of a brake wheel 80 cm. circumference and is just kept suspended by the friction between the cones of a slipping friction 'clutch' which forms part of a calorimeter of total water equivalent 40 grm. The calorimeter warms 9·0° during 1000 revs. and subsequently cools ·3° in half the time. Calculate J.

Work spent in friction = $200 \times 80 \times 981 \times 1000$ ergs = 1570 joules.
Heat obtained = $40 \times (9{\cdot}0 + \cdot 3)° = 372$ cals.

3. Water at 15° C. and 1000 atmos. pressure escapes through a porous plug into the atmosphere. Find its temperature.

Work per c.c. = $1000 \times 1{,}016{,}000 \times 1$ ergs = $4{\cdot}18 \times 10^7 \times 1 \times (t - 15)$.

4. How has it been proved that heat is a form of energy ? How can heat energy be converted into gravitational potential energy ? [L.]

5. When, how, and where is heat developed as the equivalent of work done when a man (a) jumps down on soft ground, (b) slides slowly down a rope, (c) walks downstairs ? [L]m.

6. Waterfall is 78 ft. high, water at top is at 40° F. What would be temperature of water (a) half-way down, (b) at bottom ? [M.]

7. What must be velocity of a 10-grm. body if its kinetic energy would raise 1 kg. of substance, sp. ht. ·1, through 10° ? [M.]

8. State the law of conservation of energy. Calculate the velocity with which a piece of lead must strike the ground in order to raise it from 16° C. to the melting point, 326° C., sp. ht. ·031. [L.]

9. 16-lb. hammer falls on 2-lb. iron at 20 ft./sec. 30 strokes per minute. If half energy goes in heating iron how much will it rise in 10 min. (sp. ht. $\frac{1}{8}$) ? [Ab.]

10. A $\frac{1}{2}$-lb. snowball thrown across the line is struck by the buffer-beam of an engine travelling 60 m.p.h. How much of the snow is melted by the shock ? [Ab]m.

11. A copper calorimeter weighing 100 grm. contains 990 grm. of water at 15° C. The water is stirred by a paddle which makes 1000 revs. The driving couple is 10^8 dyne cm. The water rises to 30° C. Calculate J. [L.]

12. Water under a head of 21 m. is drawn through a half-open tap into a pail. Calculate its rise of temperature.

13. A meteorite initially at 0° C. meets the earth's atmosphere and is vaporized by frictional heating. If its mean sp. ht. were 0·2, its boiling point 3000° C., and latent heat of vapour 50, and $\frac{9}{10}$ the heat developed is simultaneously lost by radiation, etc., find minimum speed at first contact.

14. A basin of water at 40° F. is warmed for washing the hands by pouring in a quart of boiling water. What addition of energy in foot-tons does this represent ?

15. A tramcar endeavours to start, but the wheels slip and 50 h.p. goes for 2 sec. in grinding up steam from the rails. How much moisture is evaporated ? [Total heat of steam 1090 B.Th. units.]

16. Calculate the practical efficiency of a very good steam engine which requires $1\frac{1}{2}$ lb. coal (giving 7000 B.Th. units per lb.) per horse-power hour.

17. Assuming that one-seventh the energy of the coal is utilizable by the engine, what weight of coal (giving 7000 B.Th. units per lb.) is wasted when a 200-ton train is stopped by the brakes from 27 m.p.h. ?

18. A tri-car engine averages 2 h.p. for 5 hours on 1 gal. (3500 grm.) of petrol of calorific value 10,000 cal. per grm. Calculate its efficiency.

19. A steam pile-driver burnt $\frac{3}{4}$ ton of coal (7000 B.Th. units per lb.) while delivering 2000 blows with a 2-ton monkey falling 3 ft. Calculate its efficiency. The concrete pile was driven 15 ft., it was specified to carry 30 tons. Estimate the theoretical efficiency of the whole process.

Chapter XXII

CHANGE OF STATE—MELTING OR FUSION

§ 188. If a thermometer is put into a vessel among fragments of a solid, such as naphthalene or sulphur, the whole steadily supplied with heat and the thermometer watched, its steady rise presently ceases. On inspection, what has happened is that the substance has begun to melt—to change its physical state from solid to liquid. And provided that it is kept well stirred up so as to expedite the sluggish spreading of heat through the mixture and prevent local overheating, the thermometer moves hardly at all until all the solid has melted, then resumes its steady rise. Repeating the experiment as often as you like with the same material the thermometer will always stick at this *same* **Melting Point** of temperature. Further, if the liquid is allowed to cool and congeal, the falling thermometer will stand steady for some time at this *same* temperature, now a **Freezing Point.**

Clearly the transition of any particular substance from its solid to its liquid condition—

(a) takes place reversibly at a definite temperature;

(b) involves the absorption and disappearance of a characteristic quantity of heat and conversely its reappearance during solidification. For on the way up heat is poured into the substance, without affecting its temperature, for a time proportional to the amount to be melted; and on the way down the body goes on giving out heat to its surroundings at the usual rate for some time without any diminution of temperature.

When the solid has been brought up to the melting point already, the number of calories then required to melt one gramme of it is called its **Latent Heat of Liquefaction,** *or the Latent Heat of the substance in its liquid state.*

The **Melting Point,** or more conveniently the **Solidifying Point** of any substance is determined in precisely the way suggested above, by finding where the heating or cooling curve (cf. Fig. 76) of a potful of it shows a horizontal 'flat.' For very low or very

high melting points of course special thermometers (preferably electric) must be used.

The measurement of the **Latent Heat of Fusion**—or the equal development of heat on solidification—has been described in Chapter XVIII.

§ 189. The process of Fusion is not always so simple as outlined above. Often the substance begins to soften long before it melts, from plastic solid it passes by slow stages into very viscous liquid, having all the time an increased specific heat, and the temperature at which it finally takes up the small remainder of its latent heat and satisfactorily liquefies may or may not be sharply marked, see § 195. Of pure crystalline substances Platinum and Iron are plastic and weldable 500° before melting but melt sharply at last; Silica (quartz) softens at 1500°, can presently be worked in the oxy-gas flame like glass, can later be shot or blown into threads, and has no well-defined melting point. The 'colloid' glass is at best a treacly liquid slowly hardening through working and annealing stages to its usual condition of not altogether perfect solidity, § 146.

Substances of mixed composition often give two or more flats on a slow cooling curve—solidifying points of fractions of definite composition crystallizing out of the fluid. This of course means a period of plasticity. The fusible alloys used as solders show this very well, the plumber's joints are 'wiped' when in a clay-like condition of solid grains and fluid metal. The solidification of paraffin wax may take place in three closely succeeding steps, and near the other end of the paraffin series the lightest petrol is such a mixture that it has only reached the viscous liquid stage at −190° C.

§ 190. Frequently a liquid cools below its freezing point without any signs of freezing, but this **undercooled condition** is of course unstable. Sooner or later rapid solidification begins, and setting free latent heat, raises the whole mass up to its true freezing point, and continuing more slowly keeps it there till all is solid. This undercooled condition is perhaps most easily induced if the liquid is dispersed in drops through another liquid, sulphur in zinc chloride solution has been cooled even to 0° C. without solidifying, and water in oil to −20° C.

Undercooling is often a convenience rather than not in finding freezing points, for the sudden rise of the thermometer

and its subsequent steadiness makes the determination very definite.

The condition is closely analogous to that of the 'Supersaturated Solutions' of the chemist, made by dissolving silver nitrate, say, in a minimum of hot water or by melting sodium sulphate, thiosulphate, etc., in their own 'water of crystallization' *plus* a very little more. These solutions habitually refuse to crystallize spontaneously, but do so when violently shaken up, and get warm from the liberated 'latent heat of solution.' [For to liquefy these substances in either way, whether by fusion or by solution, entails a large absorption of heat. Let the photographer recollect how intensely cold the bottle becomes when making up 'hypo,' or sulphocyanide solution.] The acetate of soda footwarmers at one time provided on the L.N.W. Railway could be restored to usefulness after their first time of cooling by a vigorous shake.

The surest provocative of crystallization in these supersaturated solutions is a crystal of the solid itself.

§ 191. It is well known that a distinct **Change of Bulk accompanies Fusion.** This change is easily measured by weighing out w of the substance into a dilatometer (say a specific-gravity bottle) filling up with water* and finding total weight b just below melting point and total weight a just above melting point when all has melted.

Then $b-a$ = weight of water* expelled.

$(b-a) \div$ density of water* at temp. of expt. = volume of water* expelled.

$w \div$ density of substance = volume of substance.

Vol. of water* expelled $\div$ vol. of substance = expansion per c.c. of substance on melting.

[If temperatures before and after differ appreciably, experiments are made at lower and higher temperatures to find the average loss of weight per degree as the contents expand without change of state, and this is applied as a small correction in reckoning the sudden change.]

Whichever is the bulkier of course rises to the top in a mixture.

§ 192. The change of bulk lays the process of fusion open to the influence of mechanical pressure. For evidently if an obstacle

* Or mercury.

is put in the way of the sudden free expansion of a body by imposing a heavy pressure which it must force back, it must be given the power to do this external work by increasing its molecular activity, i.e. by heating it hotter. Hence substances which expand on liquefying will have their melting points raised by pressure, while ice and other substances which *contract on liquefying* will have their *melting points lowered* by the pressure which is aiding their diminution in bulk.

This **Variation of Melting Point with Pressure** has been measured in an apparatus closely resembling Fig. 38. The space above the mercury in the strong bulb is filled with a mixture of the solid and liquid and the bulb sealed up. If the mixture expands it has to force mercury up the gauge tube, compressing the contained air to a pressure marked on the scale.

The bulb is immersed in a large water-bath, and kept very steadily at a little above the normal melting point. Some solid melts, and the mixture expanding forces up the pressure to a value which just prevents any more solid melting at that temperature. The latter is therefore the precise melting point at the final steady pressure. The temperature is stepped up another tenth of a degree, more solid melts, forcing the pressure up to the corresponding stop, and so on.

With ice and water the temperature is lowered step by step instead of raised.

§ 193: Theoretical calculation applied to the question gives a result which can be put in an *approximate* form—

$$\frac{\text{Rise of M. Pt.}}{\text{M. Pt. °Absolute}} = \frac{\text{expansion per grm. on melting} \times \text{pressure [dynes]}}{\text{Latent heat expressed in ergs}}$$

meaning that the necessary increase of molecular activity (taken as measured by temperature) is in the same proportion to the total molecular activity, as the extra work to be done is to the total work spent in freeing the molecules from solid bondage.

Putting 1 atmo. $=1{,}016{,}000$ dynes/cm.^{2} and $42{,}000{,}000$ ergs $=1$ cal., the formula becomes

$$\frac{\text{Rise of M. Pt. per atmos.}}{\text{M. Pt. °Absolute}} = \frac{\text{expansion per gramme on melting}}{41 \times \text{Latent heat in calories}}$$

and by this the bracketed figures in the table were calculated.

Seeing how small is the expansion and therefore how small an amount of external work is done even against heavy pressure it is evident that the effect of pressure can be only very small.

Thus the figures in the table at the end of this chapter show a large effect for naphthalene, where 30 atmos. would raise the melting point 1° C., while for water, with its great latent heat, it takes $1 \div \cdot 0072 = 139$ atmos. to lower the freezing point one degree.

§ 194. **Ice.** The rather exceptional properties of Ice have so profound an influence in nature that they demand special notice here.

The latent heat of water being so great makes its freezing a slow process, and even small quantities take a considerable time to freeze solid. Conversely the thawing of ice in mass takes a very long time, icebergs drift far into warmer seas and stronger sunshine before their dissolution, ice is won on Etna from quarries where snows of unknown age have become deeply thatched with volcanic ashes.

Ice floats, having in freezing expanded one-eleventh in bulk and gone down to a density of ·918, and averaging a good deal lighter when the multitudes of bubbles which give it its whiteness are taken into account—the bubbles of air dissolved in the water but thrown out of it in freezing, for air is insoluble in ice. [The estimate, based on this density, of the relative bulk of an iceberg submerged, is said to be frequently excessive on account of the berg being partly composed of imperfectly consolidated snow. A foot of fresh snow is equivalent to only an inch of water.]

Forming a firm floating layer, ice shields the water from the wind which was rippling and stirring up the surface strata. Hence, and also as it is a poor conductor of heat (·0054), the rate of loss of heat from a pond once well frozen over is much less than it was before freezing began. Thus the total formation of ice in a frost is a mere fraction of what it would be if ice sank.

In some clear-running rivers, however, for instance the Avon at Christchurch, **ground ice** is formed. The water of the river, kept mixed by the current, has cooled to the freezing point, or perhaps even undercooled a trifle ; the shallow river-bed itself has radiated its heat through the clear water and may have fallen even below zero. Little ice crystals are formed somewhere in the streaming water and getting entangled in vegetation at the bottom form nuclei of crystallization round which ice grows. Masses thus formed may often uproot their anchoring weeds when the thaw comes.

The expansion in freezing has an effect on domestic water-

pipes only too unpleasantly apparent when the subsequent thaw releases their contents. The thick lead service pipes ($\frac{1}{2}''$ bore 6 lb./yd., $\frac{3}{4}''$ 9 lb./yd.) now insisted on by the water companies are, however, a real protection against mishap, as any plumber will cheerlessly admit. They are uniformly strong, so that one particular spot does not readily bulge and weaken: water shut in between earlier-frozen parts in these pipes may rise to such a pressure as it freezes as to partially melt the ice plugs and escape back into the mains. But the stoutest hydraulic pipes exposed to *quick* hard frost soon split.

The investigation of the **lowering of the freezing point of water with pressure** has been carried much farther than in § 192. In 1785 Major Williams filled with water two cast-iron bombshells 13 in. diam. and 2 in. thick and exposed them to the arctic night. One shot its plug 140 yd. and 8 in. of ice protruded from the fuse-hole—a jet of mixed ice and water frozen instantaneously solid as it was driven forth—the other split and a thin frill of ice 2 in. wide exuded from the crack. Evidently a proportion of the water had not frozen at all until the pressure was relieved.

Mousson froze water in a narrow bore in a stout block of steel, and then forced down on to it a plug of soft copper by a steel screw cap. A little brass rod had been frozen in at the bottom of the ice and now the screwed-up apparatus was inverted and unscrewed, when the rod was found frozen in at the *other* end of the bore—the ice had been melted by pressure even at $-18°$ C.

Hopkins in 1854 repeated the experiment with a bronze tube. A little magnet remained frozen in at the top of the tube until hydrostatic pressure applied at the bottom (from a loaded oil piston, Fig. 40) melted the ice and a compass needle showed that the magnet had dropped.

Dewar in 1880 reversed the procedure of § 192. By pump and Bourdon gauge he fixed the pressure in a steel cylinder of ice and water, and an enclosed thermo-junction presently settled down to indicate the corresponding freezing point. At 700 atmos. this was just below $-5°$ C.

The action of freezing again as soon as the pressure is relieved is called **Regelation**. In a well-known experiment a block of ice is bridged between two stools and a heavy weight is hung in a loop of thin steel wire round the middle of the block. The wire slowly cuts through the ice but leaves it as solid as ever, with only a slight filmy appearance marking its track. The pressure under

the wire lowers the melting point, the ice melts, the water escapes past the wire and refreezes above it, its latent heat being conducted down through the wire (all below 0°) to the cutting side, which is a fraction of a degree colder. Catgut, a bad conductor, fails by not returning this heat fast enough ; mere pressure of course cannot go on liquefying indefinite quantities of ice, and the energy of fall of the weight is also quite inadequate : no regelation, no cutting.

The weight on a skate-blade, or that of a curling-stone, liquefies a surface film at the areas of contact, and the skater or the stone glides on a thin lubricant produced exactly when and where it is wanted, and the more freely the harder the pressure—an ideal system of lubrication scarcely attainable by the engineer, though occasionally imitated by orange-peel.

Regelation confers on Snow its binding power. Very cold snow is typically fine and will not bind ; in a less frigid atmosphere the flakes are larger—already clung together—and bind into admirable snowballs and miniature roof-glaciers. The pressure of crystal on crystal melts the points of contact and squeezes out water which immediately refreezes all round them and seals the grains together.

In this way the soft snow of the snow-fields gradually compresses and combines into the clear ice of the **Glacier**. The weight of the glacier on its sloping bed bears hard on projecting bosses of rock, crushes and partly liquefies the ice there, and squeezes it round them to refreeze again on the lee side. From this action, together with the existence of 'gliding planes' in the ice crystals, § 99, the whole glacier of hard elastic ice streams on like a river of very viscous liquid, at a speed averaging perhaps 18 in. a day in the Alps, but reaching as much as 80 ft. a day in the ice sheet of Greenland. Embedded in its under surface, by the same action, are the hard fragments of rock which so slowly grind its bed to the polish that may endure for unknown thousands of years after the glacier has disappeared.

Doubtless the warmth carried down the crevasses by falls of sun-melted water, and also the internal warmth of the earth itself, have a great deal to do with keeping the lower surface of the icy blanket near enough to 0° C. for pressure-melting to be practicable.

Ice is a very volatile solid, giving off even at $-10°$ C. as much as 2·4 grm. of vapour into a cubic metre of air, and at 0° twice this amount, see Fig. 83, a far greater volatility than that of

camphor, etc. Recollect how ice and snow disappear from the paths during a few days' windy frost, and how sheets from the wash, which went stiff as boards when first hung out to dry, become soft again in a few hours.

§ 195: **Iron** is another substance of great interest.

Wrightson attached 4-in. balls of grey Cleveland cast iron (density 6·95 cold) to a spring balance and plunged them into a bath of the same iron molten (density 6·88). Presently the ball pushed up on the balance, showing a density 6·50; if free it floated well out of the liquid. It was now so soft that a steel pin could be stuck right through it, ultimately it quite suddenly collapsed into liquid. [What the tabulated latent heat may mean is doubtful, in face of this.] Thus cast iron will expand some 6 % in solidifying to the plastic condition (water expands 9·3 %) and this result was confirmed by actually measuring the diameters of 15-in. balls of both grey and white iron as they solidified.

Doubtless this expansion, shared also by type metal, etc., assists in getting sharp castings.

Accordingly, iron should exhibit a Regelation. Wrightson electrically heated wrought-iron bars in a close-fitting porcelain tube to a 'welding heat,' 1400° C. Suddenly squeezing their ends together at 1200 lb. per sq. in. (80 atmos.) sent the temperature down 57°! and the bars *welded* together as the pressure was removed. It is the same process then that unites white-hot iron under the hammer of the smith and cakes snow into lumps under the feet of the wayfarer.

§ 196: **Freezing mixtures.**

We have already noticed that the liquefaction of a substance by solution in water usually demands a supply of heat [e.g. *per gramme* common salt 20·7 cals.; sodium thiosulphate 44; sodium sulphate cryst. 57; ammonium sulphocyanide 75; ammonium nitrate 79 cals.]

Hence a soluble salt rapidly dissolved in cold water and absorbing this 'latent heat of solution' will bring the temperature down very low for a time.

Half a pound of powdered ammonium nitrate stirred into half a pint of cold water may reduce it to −15° C., and equal parts of powdered sulphate of soda and diluted sulphuric acid, or hydrochloric acid, will have about the same effect. These are the

only ice-less freezing mixtures practical enough to be worth mention [unless one includes solid carbon-dioxide 'snow' dissolving in ether at $-79°$].

Snow or ice-shavings dissolving in about two-thirds their weight of fairly strong sulphuric acid also reach about $-15°$ C.

Doubly effective are mixtures of ice and a solid salt, where both liquefy. 1 part of coarse common salt and 3 of broken ice will reach $-22°$ C., and 3 parts of crystallized calcium chloride and 2 of ice reach $-55°$, easily freezing mercury. The action is that the salt continuously dissolves to a saturated solution in the liquefying ice, and the temperature reached is the melting point of ice in equilibrium with saturated solution of the salt. For how it comes about that this is so much lower than its melting point in equilibrium with pure water the reader should immediately consult § 273.

Substance.	Specific heat.		Density at M. Pt.		Expansion per grm. on melting.	Latent heat of fusion.	Melting point.	Rise per atmo.
	Solid.	Liquid.	Solid.	Liquid.				
					c.c.		C.	C.
Aluminum	·22	(·28)	2·55	2·43	·05	107	657°	—
Lead	·0345	·036	11·4	10·4	·008	5·4	327°	+·022°
Bismuth	·0325	—	9·8	10·0	—·002	12·6	266°	—
Zinc	·093	·064	7·2	6·48	·016	28·1	412°	—
Cast iron	·11	—	6·5	6·88	—	(30)	1200°	(—·03°)
Water	·503	1	·918	1·000	—·09	79·8	0°	—·0072°
Acetic acid	·62	·50	1·055	1·055	—	46·4	16·6°	—
Sulphur	(·184)	·234	1·96 & 2·05	1·81	—	9·4	114°	—
Naphthalene	·32	·4	1·054	·915	+·15	35·6	79·9°	+·035°
Paraffin wax	·6	·7	·88	—	—	35	54°	—

	Specific heat. Liquid.	Boiling point.	Density at B. Pt.		Expansion per grm. on evapn.	Latent heat of boiling cals. per grm.	Critical temperature.	Critical pressure.
			Liquid.	Vapour at 76 cm.				
		C.			c.c.		C.	atmos.
Alcohol	(·61)	78·1°	·74	·00152	660	206	235°	65
Chloroform	·23	61°	1·41	·0039	255	58	260°	55
Ether	·55	34·5°	·695	·00245	410	90	195°	37
Water	1	100°	·96	·00059	1700	536	365°	200
Toluene	·4	110·8°	·778	·00303	330	83·5	321°	42
Carbon disulphide	·24	46°	1·22	·0025	400	100	275°	78
Mercury	·033	357·25°	12·75	·0066	152	62	—	—
Sulphur	·23	444·7°	1·8	·0065	154	362	—	—

CHAPTER XXIII

CHANGE OF STATE—VAPORIZATION

QUITE differently to fusion, the Vaporization of a substance goes on at all temperatures up to a limiting 'boiling point,' when quiet Evaporation suddenly passes into turbulent Ebullition.

§ 197. That **Evaporation** is constantly going on is evidenced by the *smell* of aromatic substances, many of which disappear so slowly that their loss of weight in a week may be inappreciable; instance the famous grain of musk. Then there is the dryness of the soil by day and its dampness in the cool of evening, proving that plenty of moisture was all the while coming up from below, and will soon be hanging as a mist when the air has become too cool to retain it as vapour. And wisps of mist wreathe over a sheltered running brook in a frost.

That the Rate of Evaporation is hindered by vapour already present in the air we acknowledge by setting things to dry in a wind or in a draught, to blow the moisture-laden air away.

That the rate increases very rapidly as the temperature rises is a fact forced on our attention in the summer, and one we all make use of in drying or airing clothes, etc., before the fire.

§ 198. How rapid quiet evaporation can become is strikingly shown by the phenomenon of the **Spheroidal State.**

Drops of water thrown on a red-hot plate—e.g. the top of the kitchen stove when really hot—run about hastily, but only gradually shrink up and disappear, without the least noise. A bright-red-hot iron, plunged into water and held still, goes on glowing for many seconds without producing any very violent disturbance in the water. Hot molten metal can be harmlessly poured over damp hands, as can the volatile liquid air over dry hands, 220° warmer than itself. Carbonic-acid 'snow' can be fingered lightly almost as safely as cotton wool.

The explanation undoubtedly is that the vapour of the volatile substance (water, air, CO_2) is being produced so fast from the

surface that it blows it out of contact with the hot body which is providing the heat necessary for this evaporation—partly by radiation, mostly by conduction through the thin layer of vapour. For if one wire from a battery and bell be hooked on a red-hot metal basin and the other dipped in the spheroid of weak acid the bell does not ring; and drops of sodium-sulphide solution bounce off a red-hot half-crown without blackening it in the least.

The vapour escaping from beneath unequally in different directions drives the drop about, and sometimes sets a large drop into very pretty vibration.

The drop has been found to be always below its boiling point, in fact a small piece of ice thrown into a red-hot bowl runs round for three or four seconds before entirely melting.

When the hot surface cools, the rush of vapour slackens, and presently the drop 'comes in contact' with the plate, and there is the sudden splutter one has been expecting. It is astonishing, however, what a length of time a spherule of water will remain quiet in a clean bowl of thin platinum after the gas has been turned out. And relieved of the atmospheric pressure, water has been observed in the spheroidal state on a plate at only 80° C.

The Spheroidal State has been given the credit for boiler explosions, and with rather more probability for the occasional spasmodic refusal of a 'flash-boiler' to work up to power—the pipes have got too hot and the injected spray rebounds and passes off to the engine unvaporized. The superiority of mercury- or oil-quenching for hardening steel lies in the fact that, being so much less volatile than water, these liquids do not form this feebly conducting vapour layer, and they therefore chill the metal much more quickly. Moissan, in making diamonds, solidified cast iron in molten lead quicker than in water.

§ 199. **Sublimation**. It is not every substance that fuses. Ammonium salts, etc., volatilize or 'sublime' without showing any signs of melting; they do not pass through the usual intermediate liquid state. Further, the range of temperature during which substances remain liquid, varies greatly. The normal boiling point of argon is −186° C. and it freezes only 3° or 4° lower, water has the normal 100° range of liquidity, sulphur 330°, mercury 400°, iron 1000°, etc.

But we shall see presently that increased pressure so increases the difficulty of vaporization that the liquid range becomes much longer, and under pressure camphor melts and boils in the usual

way, though normally one might say its melting point is above its boiling point.

§ 200. **The increase of volume accompanying vaporization** is very great. It is found by measuring the density D of the liquid and that, d, of its vapour at the same temperature. This change of density means a D/d-fold expansion. Since d increases fast as the temperature of vaporization rises, this latter must be specified. See table, p. 185.

NOTE.—d is *not* the chemists' 'vapour density,' which refers to hydrogen as standard.

A great deal of external work must therefore be done by the evaporating liquid in lifting the atmosphere to make room for its vapour. This work, however, represents on the average only one-eleventh of the total energy-value of the latent heat of vaporization, the remainder is spent in disentangling the molecules from their mutual liquid bondage. But it shows that increased pressure will raise the boiling point, and greatly.

§ 201. **Determination of the density of a vapour.**—This is carried out as for a gas in § 124, but usually without employing an air-pump. Some liquid is put in and the 3-in. bulb plunged into a bath of water, oil, or other suitable liquid at a temperature well above the boiling point of the liquid whose vapour density is to be determined. The vapour washes out all air, and being sealed up when the outrush ceases the bulb contains the weight of substance which fills it as vapour at the temperature and pressure of experiment. This weight$=$the observed increase of weight $+$(calculated) weight of air which filled it originally. Then the bulb is opened under water which rushes into the place of the condensed vapour, and hence its volume is obtained.

Other methods are detailed in all the chemistry books. Here we are concerned more with the pressure of the vapour, which depends on the closeness of packing of the molecules and their average energy of motion, and not on their internal constitution.

§ 202. **The Pressure of a Vapour.**

The obvious way of finding the vapour pressure of a substance is to introduce it into the Torricellian space at the top of the barometer, where the vapour forms quickly, unhindered by air, and drives down the mercury for a distance which measures its pressure, now substituted for the dead weight per square centimetre of that depth of mercury.

The process is frequently demonstrated as in Fig. 82. Three

or four barometer tubes stand side by side. The first is kept as standard ; under the foot of the second a few drops of water are blown from a little glass 'filler' with a bent-up point ; and under the third some ether. The liquids float up the tubes, the water drives the mercury down only 1 or 2 cm., but the ether is much more effective, having evidently a much greater vapour pressure at ordinary temperatures, and serves better for demonstration and argument.

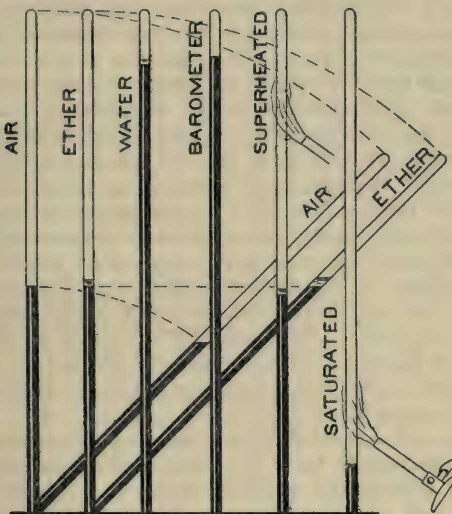


FIG. 82.

The first drop or two sent up break into long bubbles of vapour half-way up, and the mercury after being thrown about violently settles perhaps 10 to 20 cm. lower than it was, indicating this much pressure in the *perfectly dry* vapour above it. Another drop brings it down farther, having increased both the quantity and the pressure of the dry vapour.

But continuing drop by drop, some liquid presently remains unvaporized at the top of the mercury, and further supplies are now quite ineffective ; the vapour has evidently reached its maximum elastic pressure and can drive the mercury no lower.*

* Of course the dead weight of a *lot* of liquid will help press the mercury down, but only about 1/20th depth of ether.

§ 203. For distinction, the former vapour, into which more liquid could evaporate and increase its pressure, is spoken of as **unsaturated.** The latter, which can take up no more liquid, is a **Saturated Vapour,** *it remains unchanged when in contact with its liquid, all at the same temperature.* Vapours in these two conditions behave very differently.

The hasty evaporation of spilt liquid air shows that for the present purpose Air may be regarded as the unsaturated vapour of this liquid, and therefore, for comparison with the saturated vapour, some air can be blown into a fourth barometer tube until it brings the mercury down to the same level as in the other tube.

Now incline these two tubes, the mercury starts running along both towards their closed ends, for of course it is its *vertical* height that measures pressure. In the air (unsaturated vapour) tube, however, its *level* falls, for the compression of the imprisoned air by the advancing mercury has raised its pressure, according to Boyle's law. But in the saturated vapour tube the level falls only a trifle, the liquid above the mercury increases in quantity, and if the tube is left to itself for a minute or two so that the heat of liquefaction of this may be dissipated by cooling, the mercury returns exactly to the *level* it had originally in the vertical position. Now suddenly lifting to the vertical again the excess of liquid immediately boils off and (after a minute or two for warming after this loss of latent heat) the mercury stands again at the same level.

Evidently the saturated vapour has no characteristic volume of its own, so long as there is enough liquid present to keep it saturated. Reduce the available space and vapour liquefies; increase it and liquid evaporates. As soon as equilibrium is reached either way, there is the original pressure quite unaltered.

At a fixed temperature the Saturated Vapour has a characteristic pressure.

§ 204. Rise of temperature increases this pressure very rapidly. On the vapour tube, near the lower end, where there is liquid to evaporate and keep up the saturation, a touch of a flame will send the mercury down with a rush. Whereas heating the top of the tube, where there is no liquid to evaporate and accordingly the vapour becomes locally expanded ('superheated') and therefore unsaturated, causes only a very trifling motion of the mercury, no more than in the air tube after a similar treatment.

The rise of pressure of saturated vapour with rise of temperature is shown in Fig. 83, wherein the portion of the curve

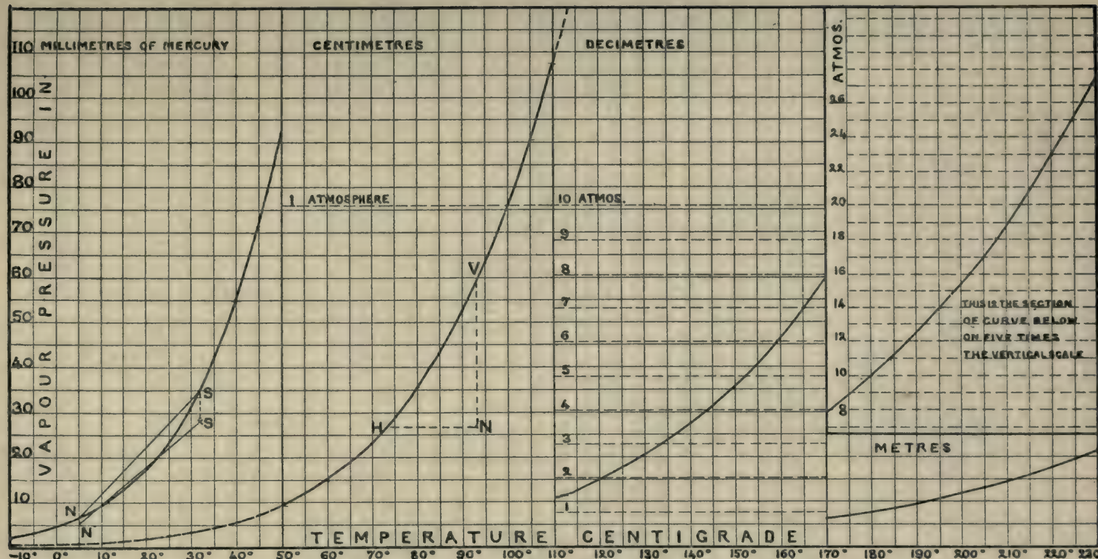

MILLIMETRES OF MERCURY
CENTIMETRES
DECIMETRES
ATMOS.
METRES
VAPOUR PRESSURE IN
TEMPERATURE CENTIGRADE
1 ATMOSPHERE
10 ATMOS.
THIS IS THE SECTION
OF CURVE BELOW
ON FIVE TIMES
THE VERTICAL SCALE
N
N
S
S
H
V
N
110
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
26
24
22
20
18
16
14
12
10
8
−10°
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
180°
190°
200°
210°
220°

FIG. 83.

from $-10°$ to $+100°$ has been obtained from a barometer tube, containing water as the volatile substance and jacketed by an outer tube through which a fluid at a known temperature was circulated. The height of a point on the curve shows the pressure at the corresponding temperature. The long curve (dotted from $-10°$ to $+50°$) is on a vertical scale graduated in *centimetres* of mercury. Its slope, i.e. the rate of rise of pressure with temperature, changes enormously; the rise between 95° and 100° is 65 times as great as between 0° and 5°. The lower part of this curve has therefore been shown on a ten times magnified vertical scale (millimetres of mercury).

§ 205. But we seldom go to the trouble of removing the air from the space in which a vapour is to be produced. Commonly we leave the air in and let the vapour mix with it, or blow it out as it can. Does a mixture of vapour and air obey Dalton's Law, that each gas in a mixture exerts its own partial pressure quite unchanged by the presence of the others? does the vapour attain the same 'partial' pressure as if there were no air present?

This can be tried by first admitting air to the Torricellian space, so as to permanently depress the mercury, and then finding if the further lowering on admitting liquid is the same as before. Or in another way (chemical hygrometer) by using a chemical to absorb all the saturated vapour which filled an otherwise vacuous space, and, secondly, all the vapour which formed in the same space already occupied by air, and comparing the two increases in weight. The result is that, as nearly as one can tell, a liquid evaporates to the same ultimate saturation pressure into the presence of a permanent gas as into a vacuum.

Thus when ether has been poured from its bottle, and its heavy vapour has visibly poured out with it, and a lot of air has entered in replacement, evaporation into this unsaturated air immediately begins and raises the total pressure until the stopper hops out. Soon an equilibrium is reached inside with perhaps ·4 atmos. due to ether vapour and ·6 to air, and the stopper put back shows no further tendency to lift (unless the room gets warmer).

And the barometric pressure is the total of the partial pressures of nitrogen, oxygen, aqueous vapour, argon, etc.

Example 1. Calculate the weight of hydrogen in 100 c.c. of electrolytic gas ($2H+O$) standing over water which rises 10 cm. into the graduated tube. The gas is saturated with moisture, at 17° C., barometer 75·5 cm. 1 c.c. dry hydrogen at 0° and 76 cm. weighs ·0000895 grm.

Of total pressure in tube, which $=75{\cdot}5-(10\div 13{\cdot}6)=74{\cdot}75$ cm., the

water vapour accounts for 1·45 cm. (Fig. 83), leaving 73·3 cm., of which the hydrogen causes $\frac{2}{3}$, $=48{\cdot}9$ cm. pressure.

$$\text{Hence } V_\circ = 100 \times \frac{48{\cdot}9}{76} \times \frac{273}{273+17} = 60{\cdot}5 \text{ c.c.}$$

$$\therefore \text{ weight} = 60{\cdot}5 \times {\cdot}0000895 = \underline{{\cdot}00541 \text{ grm.}}$$

But mixed vapours of mutually soluble substances obey no such rule, e.g. the saturation vapour pressure of dilute alcohol is far from being the sum of those of alcohol and water.

§ 206. **Evaporation and Boiling.** Observe what happens as water is heated. Bubbles soon begin to make their appearance: each consists mainly of dissolved air, but part of its elastic pressure is due to the vapour which has evaporated into it.

As the temperature rises the 'partial pressure' of vapour in the bubble (proportional to the percentage by volume of vapour in it) increases, e.g. at 50° about 9 cm. is vapour and $76-9=67$ air, at 90° 52·5 vapour and 23·5 air, at 99° 73 vapour and 3 air. Presently therefore it takes only a little air to form a large bubble at full atmospheric pressure. The small amount of air usually dissolved in the water therefore produces an increasing multitude of bubbles as the temperature rises, and these, as they gain in size and buoyancy, float up to the surface. All taken together, however, they have not carried off much vapour.

But when the temperature has risen so that the vapour pressure exceeds in the least the hydrostatic pressure in the liquid (made up of superjacent liquid+atmosphere, § 62), then bubbles *formed of vapour only* have sufficient strength to withstand this pressure, and the very smallest trace of air will suffice to start a bubble which can grow to any extent, instead of having to stop when the air percentage falls too low to make up a balance of partial pressure.

Bubbles therefore start in large numbers at the hottest parts, but rising into cooler liquid, collapse. For the cooling of the vapour lowers its pressure, and the hydrostatic pressure crushes in the walls of the bubble with an audible snap, in the absence of any residual 'air cushion' to soften the shock.

It is the noise of numbers of such collapses in its resonant interior that makes the kettle sing: the bottom layer of water is boiling hot, though the main bulk is far from it. Near the boil the song is softer, the bubbles are not so abruptly condensed by the warmer water.

The bubbles greatly aid the convection of heat, setting up a

rapid stream by their buoyancy, and giving up latent heat as they liquefy.

When the whole bulk of liquid has thus been warmed to this temperature at which the vapour pressure just exceeds the hydrostatic pressure, evaporation continuously goes on into the bubbles, they grow rapidly, rise and burst in abundance; the liquid **boils.** Vaporization suddenly becomes much more rapid because of the large increase of available evaporating surface afforded by the growing bubbles.

Now, any attempt to heat the liquid hotter means a greatly increased vapour pressure, much faster evaporation at any surface that presents itself, i.e. faster output of larger bubbles—furious boiling—taking away latent heat so rapidly that the liquid can never rise much above the temperature at which boiling began.

Hence a liquid begins to visibly boil when it reaches the temperature at which its saturated vapour pressure is equal to that of the atmosphere on its surface, and thereafter it scarcely rises in temperature.

§ 207. This statement requires a little qualification, for **sometimes a liquid can be 'overheated.'** It was suggested above that a minute amount of air was still acting as nucleus: certainly *Nuclei of bubble formation* of some sort have to be present for steady boiling.

Everyone has noticed how the bubbles in a beaker of boiling water stream up from invisible specks on the glass, or afloat. Very similarly, while the half-emptied bottle of aerated water is gassing quietly from a few nuclear points, the tumbler—up till then exposed to air, dust, cloth-fibres, etc.—is soon quite coated with hundreds of bubbles and effervesces briskly.

The long-continued boiling of water in a glass vessel gradually changes from a free continuous ebullition to a spasmodic **boiling with bumping**—and all the sooner if there is present a trace of caustic alkali, a substance which assists the water to dissolve adherent dirt and glass itself. In perfectly quiet intervals a thermometer in the liquid will rise 5° or 10° above the normal boiling point, to fall back to it when sullen explosions of vapour threaten to burst the vessel. Coke, pumice, porous potsherds, etc., thrown into the bumping liquid [and powdered sugar thrown into the aerated water] originate abundance of frothy bubbles, and steady boiling ensues for a long time. All are things on which air persistently clings. As in undercooling, this over-

heating is most noticeable in drops of liquid entirely surrounded by another liquid, e.g. air-free water can be heated in oil to 180° C. without vaporizing.

In Chapter XXVIII it will be shown that Surface Tension in the bubble walls causes an added pressure inside it which is greater the sharper the curvature, being something like 1 atmo. for a sphere ·00002 cm. diam., 2 for ·00001, and so on. How can a very minute spherical bubble start and grow against this overwhelming pressure? It cannot. But if there is a microscopic crack say in the surface of the glass, and air has got in and sticks there tenaciously, as it will, then the comparatively large and flat end of this air wedge will form the starting-place of bubble after bubble, never of excessively small radius and therefore never much affected by the surface tension. When this air has been dislodged, by gradual solution during long contact as in the soda-water bottle, by long boiling, or by pumping down the pressure above the hot liquid for a short time, then comes about the scarcity of possible jumping-off places which gives time for overheating, and overhasty evaporation into any bubble that does chance to form.

§ 208. The visible boiling of a liquid then is a useful indication that its saturated vapour pressure has become equal to that of the atmosphere of vapour, or air and vapour, above it.

This can be experimentally shown by steam jacketing the barometer tube in Fig. 82 which contains water; when steam is blowing freely through the jacket the mercury will be driven down just level with that outside.

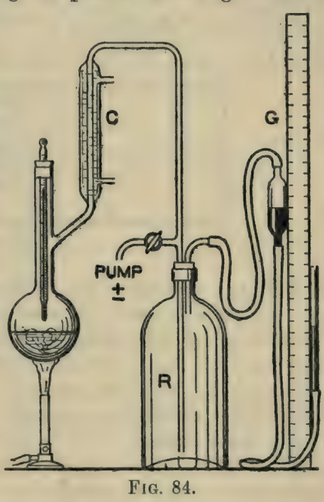


Fig. 84.

Hence the Temperature-Pressure of Saturated Vapour Curve may also be described as a Boiling Point—Pressure of superincumbent 'atmosphere' Curve, and we are relieved of the necessity of starting in a vacuum. Accordingly the curve of Fig. 83 has been continued by experiments in an apparatus of which Fig. 84 is a laboratory specimen.

Air is pumped into or out of the reservoir R to a pressure measured by the mercury gauge G and the liquid in the flask (containing a potsherd or two) boils steadily at the temperature corresponding to this pressure. The reflux condenser C returns the boiled-away liquid and keeps R and G comparatively free of vapour.

For high temperatures Regnault and others have built the whole apparatus of metal and used a high-pressure gauge.

The thermometer is put *in the Vapour* to avoid trouble from bumping or dissolved impurities. The action is that pure distilled liquid condenses on the thermometer bulb (which should be protected from splashes) and equilibrium is established between this liquid and the vapour near by.

§ 209: The general shape of the Saturated Vapour Pressure—Temperature Curve is the same for all substances. Ramsay and Young indeed discovered that when A and B are 'chemically similar'

$$\frac{\text{Boiling point °Absolute of A}}{\text{Boiling point °Absolute of B}} = \text{constant, whatever the pressure.}$$

That is, if O, A, W, M, S, Fig. 85, are the curves for substances

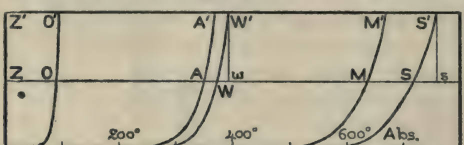


Fig. 85.

of different volatility, ZA : ZW : ZM, etc., as Z′A′ : Z′W′ : Z′M′, etc. Or, if the curve W were drawn on a sheet of india-rubber fastened along the Absolute Zero edge ZZ′ and stretched horizontally, the curve for the more volatile, lower boiling point, substance A would be obtained by letting the sheet relax till W reached A; or the curve for the less volatile M by stretching the elastic sheet further.

Even when one compares as in the diagram such widely different substances as oxygen, whose normal boiling point (i.e. boiling point at 76 cm. barometric pressure) is 91° A, alcohol 351° A, water 373° A, mercury 632° A, and sulphur 718° A, this rule still holds as a rough approximation. Perhaps its failure with hydrogen, boiling point 20·3°A, is excusable.

An immediate application is in correcting observed boiling points for barometric variations. The change of boiling point for a given change of pressure is proportional to the boiling point in °Absolute, for $Ww : Ss$, etc. $= ZW : ZS$, etc.

And for substances boiling within 20° or so of water—alcohol, carbon tetra-chloride, toluene, etc.—the pieces of curves AA′, WW′, etc., are practically equal, i.e. in all ordinary boiling-point determinations of such liquids, the same barometric correction as for water can be allowed, viz. 1° C. for 2·7 cm. (roughly 1 in.) barometric rise.

§ 210. Points on the curve Fig. 83 refer to Saturated Vapour. Points in the space below and to the right (convex side) of the curve refer to **Unsaturated Vapour.** For from such a point as N, one can travel back to the curve, the saturated state, in two typical ways, or in any combination of them :—

I. Along NH by reducing the temperature without change of pressure, as in a dew-point hygrometer, § 226. [Contraction of a gas cooling at constant pressure—Charles.]

II. Along NV by raising the pressure without change of temperature, as by slanting a barometer tube containing unsaturated vapour till condensation took place. [Compression of a gas at constant temperature—Boyle.]

Thus one can move about anywhere in this space, but the curve is an impenetrable boundary.

§ 211. Not quite impenetrable, however, for the following experiment will show that it is possible to break through into the **Supersaturation Space** to the left of it (concave side).

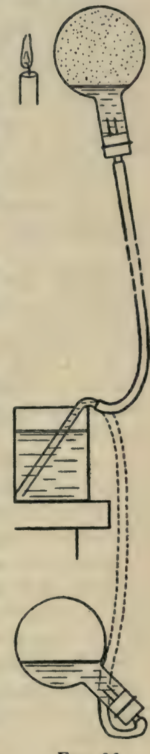


Fig. 86.

A flask containing a little lukewarm water is connected by a long flexible siphon to a further supply in a vessel on the table, Fig. 86. Lowering the flask to the floor, water siphons in and compresses the air a trifle. It is now well shaken to saturate the air and suddenly lifted high above the table ; water runs out, expanding and therefore cooling the air and hence condensing some of its contained vapour into a mist or cloud of tiny drops. A similar expansive cooling

accounts for the mist that clings in the neck of a bottle of Bass when the cork is drawn.

Violent splashing partially washes away the cloud, but it clears up completely as soon as the flask is lowered again to compress and warm the air. Repeating the process half a dozen times the cloud is fainter each time, and ultimately no cloud at all can be persuaded to form although the supply of vapour awaiting condensation is as great as ever.

If now the flask be held at table level and opened and a trace of smoke admitted, the lowering and raising will result in a regular fog. Evidently it was for want of nuclei of condensation that the vapour had remained **supersaturated**. The dust particles, of which the air of the room probably provided several thousand per cubic centimetre, had been gradually washed out while loaded with water. The smoke provided carbon particles in abundance.

In the absence of dust, and of electrified 'ions,' water vapour can be raised to an eight-fold supersaturation before visible precipitation of moisture ensues.

§ 212. **The Cooling effect of Evaporation.**

The measurement of the Latent Heat of Vapour has already been described in §§ 166, 167.

If a liquid is induced to evaporate without supplying it with heat the vapour carries off large quantities of latent heat and hence the liquid is rapidly cooled. Water coolers of thick porous earthenware (Spanish—*alcarrazas*) are widely used in hot countries; the water percolates and evaporates from the surface (kept of course in the shade), cooling the contents 10° or more.

The chill of damp clothes is due to removal of latent heat as the warmth of the body dries them.

Evaporation of this sort is promoted by removing the vapour as soon as formed, e.g. by **Wind.** Everyone knows the intensely chilling effect of wind on a wet skin, everyone blows on hot tea; in hot damp weather, when the air is nearly saturated, everyone longs for a breath of wind to blow away the vapour and relieve the insufferable closeness by once more permitting the natural evaporative cooling from skin and lungs.

With a more volatile liquid the cooling is exaggerated. Hence the use of eau de cologne to bathe an aching brow, hence the stinging cold of petrol spilt on the hands, hence the ease with which a tin box-lid can be frozen hard to a wet table by pouring a little ether in and blowing on it through a wide paper tube,

or with the bellows. Hence too the occasional freezing of a carburettor where petrol is evaporating rapidly, and the hoar-frost that forms on a steel bottle of nitrous oxide when it is freshly opened for use and the liquid is boiling away into anæsthetic vapour under 40 atmos. pressure.

Or the removal of the vapour may be effected by liquefying it elsewhere in a colder 'Condenser.' In *Daniell's hygrometer*, Fig. 92, the right-hand bulb is cooled by the evaporation of ether from its muslin-covered exterior, the ether vapour it contains is condensed (at a low point on its saturation curve), vapour flows over from the left-hand bulb, here more vapour forms to supply the deficiency, and the contained ether is cooled.

The *Cryophorus* (ice-carrier) is a similar double-bulb tube containing water instead of ether, one bulb is cooled in a freezing mixture and the vapour depositing as hoar-frost in this bulb draws off so much from the other bulb that presently the water in that begins to freeze.

Or the vapour is removed by an absorbent, strong sulphuric acid. The Pulsometer Co. make a quite practical *Freezing Machine* for producing a pound or two of ice on this principle. There is a small vessel for the water and a large one for the vitriol, and an air-pump, for in all three instruments of this paragraph there must be no air. Air makes their action hopelessly slow by simply getting in the way of the vapour molecules.

§ 213: **The Steam Engine, etc.** The steam engine works between a reservoir of hot saturated vapour*—the Boiler, and one of cold saturated vapour—the Condenser.

The earliest engines took steam at atmospheric pressure and then by admitting cold water reduced its pressure, and the atmosphere, or in a later form, fresh boiler steam, forced the piston down.

With the demand for steam locomotion came the high-pressure engine, utilizing only the upper part of the curve, Fig. 83, discharging its steam at or above atmospheric pressure and 100°, speedily to condense in the familiar clouds—it need hardly be

* And of hot water in equilibrium with it. What makes a boiler explosion so destructive is the enormous amount of steam suddenly evolved from the hot water. At 150 lb. per sq. in. above atmosphere water is boiling at 186° C. and therefore contains about 86 cals. per grm. which will produce ·16 grm. of steam when the pressure is relieved and the temperature falls to 100°. The final volume of released steam is therefore about 10 times that of the steam space in the boiler *plus* 250 *times that of the hot water.*

said that live steam itself is invisible. Evidently this is wasteful, for the rejected steam could still drive a low-pressure engine.

This it actually does, e.g. in 'multiple expansion' engines, where, having expanded and driven pistons in 'high-pressure' and 'intermediate' cylinders, it passes at about 100° into the large low-pressure cylinder. Having done further expansive work there, it passes on to the condenser and reaches equilibrium with water at about 40° C., 1 lb. per sq. in. An 'air-pump' removes the condensed water and any stray air.

Turbines are supplied with high-pressure steam superheated (as by passing through hot pipes in the flue) and therefore unsaturated. In the widening passages of the turbine this expands as a gas at first, § 210, until it meets the saturation curve, in accordance with which it then expands down to about ½ lb. per sq. in., leaving the turbine through wide casings to be condensed at about 27° C. by a special abundance of cold water. Economy is effected because all the additional 'superheat' heat is converted into work without any additional mass of steam being employed, and because the enormously expanded exhaust steam is scarcely warm, and carries off no *usable* heat.

The cooling towers of 'power stations' contain 'evaporative condensers'—stacks of closed pipes into which the exhaust steam passes to be condensed by water trickled on the outside. Many who have seen the clouds of 'steam' rising from them must have wondered why such expensive structures are erected instead of exhausting straight into the air. But steam blown from a pipe into the air is at 100° at least, whereas in the closed pipes it is liquefying at 40° under low pressure, and that difference of heat energy has been utilized.

This economy is strikingly exemplified in the Yaryan multiple-effect Evaporators, as employed say for distilling potable water at Suakim. High-pressure boiler steam liquefies under pressure, well above 100°, in the pipes of a first evaporative condenser. Its heat has boiled off water from the outside of these pipes to form steam of somewhat lower pressure. This passes to a second similar 'condenser-evaporator,' and so on, till in the sixth vapour is liquefying at hardly more than the temperature of the sea-water circulated outside it. Even the hot water from the earlier condensers is sent through pipes in the later to help evaporate more water. Whereas 1 lb. of coal seldom evaporates a gallon of water in a boiler, nearly 5 gal. is distilled per lb. at Suakim.

§ 214: It is shown in Thermodynamics that the rejection by any heat engine of a large quantity of low-temperature heat to a 'condenser' is inevitable, cf. § 185. At most the engine can convert into work only the difference between the heat-energies of the entering and the leaving fluid. This difference, divided by the heat-energy of the entering fluid, is a fraction called the *thermodynamic Efficiency* of the perfect engine [a very good steam engine is only half perfect] and this is shown to equal the difference of entering and leaving temperatures divided by absolute temperature of entering fluid. Thus gas engines, working from explosion temperatures, may be very efficient. Practically, the highest thermodynamic efficiency is that of the Diesel oil engine, about 29 %.

§ 215: **The Critical State.** If a volatile liquid, such as ether, or liquid sulphur dioxide, is sealed up with its vapour only in a little stout glass tube which it half fills, it may be heated high above its normal boiling point, and very remarkable changes presently take place, Fig. 87.

For a long time the liquid bubbles steadily, and a compensating trickling down is seen on the walls of the vapour-space. The liquid expands gradually at first, then rapidly to 60 % or more beyond its original bulk,* and bubbling becomes less active. The meniscus separating liquid and vapour becomes fainter and flatter, flickers, breaks up into a mist of visible drops in rapid motion, this melts away in wreathing striæ and—the tube's contents are perfectly clear and uniform. Looking through at the background the tube appears rather 'more refractive' than if empty, and that is all. [During cooling the same events occur in reverse order.]

FIG. 87.

The substance has ceased to exist as a liquid, it spreads uniformly over the whole volume, and if the experiment is conducted above mercury the volume may be varied without inducing any distinct liquid to reappear.

It has the properties of a vapour in that its pressure at the temperature of disappearance does not depend on the relative

* e.g. 1 c.c. CO_2 at 0° becomes 1·3 at 25°, 1·7 at 31°, 1·95 at 31·35° crit.

bulks of liquid and vapour just beforehand, and the temperature itself is quite fixed. It has, however, the power of retaining in solution solid matters, e.g. iodine, which were dissolved in the liquid, but are either insoluble in the vapour or of a different colour when mixed with it.

It is said to be in the **Critical State**, the temperature of disappearance is the *Critical Temperature* and the pressure the *Critical Pressure.*

Since even ten times the critical pressure has been tried in a vain attempt to obtain liquid above the critical temperature this may be called the 'ultimate boiling point'; beyond it the liquid cannot exist.

Thus it is possible to smoothe away the customary abrupt transition from liquid to gas, the two states can merge gradually into each other. In fact, if the tube is a little too full, the meniscus rises in plain view till it shrinks to nothing in the tapering top of the tube, while still a little below the critical temperature. The tube is still full of liquid, 20° higher we know this has ceased to be a liquid, hotter still it is an undoubted gas, but there has been no visible sign of change.

The impossibility of liquefying them by pressure and common freezing mixtures, which long ago earned for half a dozen gases the title of 'permanent gases,' is seen to be due to their critical temperatures being very low; see Table, page 211.

§ 216: **Isothermal curves.** The sequence of Volume-Pressure changes can be plotted by a family of curves as in Fig. 88. Starting at A as a gas and coming slowly backwards, keeping the temperature constant (hence the name Iso-thermal), reduction of volume is caused by an increase of pressure, and the curve AB rises in a hyperbola in accordance with Boyle's law, Fig. 52. Nearing B, the gas is approaching the condition of a saturated vapour, and the pressure-rise may show signs of failing.

At B it is saturated, and further reduction of volume causes liquefaction without any change of pressure along the horizontal BC.

At C all the vapour has liquefied and any attempt to squeeze a liquid into smaller bulk involves an enormous increase of pressure, CD is almost vertical.

Then assuming the substance to be one of the usual type, contracting on solidification, heavy pressure will crush it entirely into solid along DE, § 192. EF is the scarcely compressible solid.

For a higher temperature the curve is replaced by a similar

one lying wholly above the former, for the gas is more bulky than before, it is not saturated till a higher pressure and greater density (i.e. less volume B′), it liquefies at C′ into a warmer bulkier liquid and at a much higher D′ becomes a more expanded solid.

Any lower temperature Isothermal lies beneath and to the left. An extreme instance is dotted in (quite out of scale) along *ww*, it

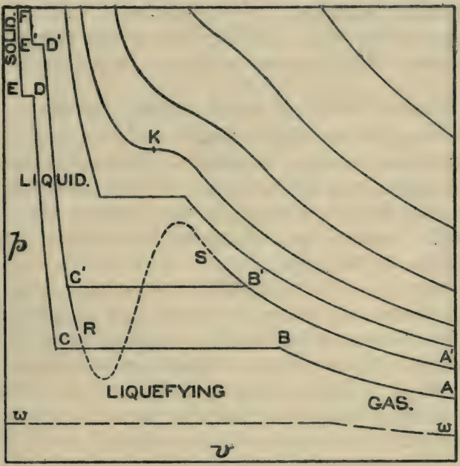


Fig. 88.

is the shape for water about 100°, i.e. 250° below its critical point, its liquid line lies only about 1/1700 of CB from the vertical axis of pressure.

Notice how the liquefying stage BC shortens at the higher temperature. This evidently accords with the smaller latent heats at higher boiling points under pressure, § 167. Ultimately the flat part shortens to nothing at K, the Critical Point, on the critical temperature isothermal. Above this temperature the curves show less and less inflexion, and are soon nearly gas hyperbolas throughout.

Note how *areas* in Fig. 88 represent *quantities of energy* PV.

§ 217: The Boyle-Charles equation PV=RT represents a family of hyperbolas (curves like Fig. 52), AB, A′B′, etc., as successive values are chosen for T. Van der Waals' remarkable modification of it $(P+a/v^2)(V-b)=RT$, § 104, plots into 'cubic'

curves of the dotted shapes, A′B′SRC′D′. It plainly represents gas, liquid, and critical point (horizontal tangent at point of inflexion), and it suggests also the curious continuity B′SRC′ instead of the flat break B′C′. And this is partly realizable, for the part B′S corresponds to the condition of supersaturation of a vapour, § 211, and C′R represents the superheating of a liquid described in § 207. Of the essentially unstable piece between R and S nothing is known.

The undercooling of a liquid, § 190, rather suggests that the flat DE may some day be replaceable by a similar continuous curve.

§ 218. **Kinetic Theory of liquid-vapour change.**

The following two premises must be made :—

A. It must be a mutual attraction of considerable magnitude that binds together a dense crowd of molecules in the liquid condition, with definite volume, surface-tension, etc. In the less densely packed state of gas or vapour this mutual attraction all but disappears, §§ 103, 104.

B. In a vast crowd (of molecules) possessing a definite average speed, individuals may at any moment have all sorts of speeds at random—the theory of probability suggests that of 1000 with average speed S there will be 95 with speeds below $\frac{1}{2}$ S ; 167, $\frac{1}{2}$ to $\frac{3}{4}$ S ; 417, $\frac{3}{4}$ to $1\frac{1}{4}$ S ; 153, $1\frac{1}{4}$ to $1\frac{1}{2}$ S ; and 168 above this. And if the average speed is reduced by removing the momentarily faster individuals, the speeds of the remainder will redistribute themselves ‘ by collision ’ in the same proportions.

Above the surface of a glass of effervescent liquid may be seen an active cloud, an inch or more thick, of droplets flung up from the bursting bubbles and falling back under the pull of gravity. The cloud has a fairly definite flat top, i.e. an average height of jump is fairly closely kept to (as above). Kinetically the surface of a liquid more or less resembles the top of this cloud. In the body of the liquid the mutual attraction acts in all directions on a molecule ; near the edge it of course pulls inwards only. The average molecule reaches a definite range before being pulled back, and the surface of the liquid is the ‘ envelope ’ of their paths. But some exceptionally fast molecules so far exceed this average range as to fly clear of the restraining attraction and become free molecules of vapour.

Since it is the faster molecules that escape, the average speed of those left behind in the liquid is diminished, i.e. if the energy of travel $\frac{1}{2}mv^2$ of molecules is taken as a measure of temperature,

the liquid has cooled. *The escaping molecules have taken latent heat with them and left the liquid colder*, cf. § 212.

In the liquid left to itself there will always be some molecules chancing to approach the surface exceptionally fast, and escaping, but the general falling-off of speed diminishes the number that come into possession of the requisite velocity. Thus evaporation always goes on, the liquid always getting colder, but slower and slower as the temperature falls.

Heat continuously supplied from without goes to increase speeds all round. If the average speed is maintained, so also is the number of molecules travelling faster and escaping, i.e. evaporation goes on at a constant rate.

As the temperature rises the increase in average activity of the liquid molecules probably makes their mutual attraction less effective, it relaxes their liquid bondage [certainly one of its indications, the surface tension, diminishes] and permits a larger *proportion* of the more rapid molecules to escape. Therefore the density and crowd-pressure of the vapour increases faster than in mere proportion to the molecular energy (absolute temperature), i.e. faster than that of a gas or unsaturated vapour.

What of the vapour-swarm of escaped molecules ? Molecules travelling near the liquid surface and coming within range of the attractive forces will be constantly falling in and replacing those that fly out. Thus at any temperature a state of ' statistical equilibrium ' is reached, when as many molecules are dropping back into the liquid as are escaping—*the saturated vapour* swarm-density, and therefore *pressure, is constant.*

Note that air molecules present can take no part in the interchange, therefore the saturation pressure of the vapour is reached quite independently of any other gas pressure present. But the neutral gas molecules of course get in the way of the vapour molecules ; the *rate* of evaporation into air is much slower than into vacuum.

Compressing a gas or unsaturated vapour packs the molecules closer, but their speed is too great and their stay in one another's proximity too short for mutual attraction to overcome the effects of ' collisions.' But at a lower temperature [speed] or a greater pressure [closeness together] this may happen, and the molecules quickly associate in twos and threes and companies and *drops of liquid* as soon as a sharp limit has been overstepped, i.e. *saturated vapour condenses freely as soon as a definite pressure is exceeded*, unless above the limiting critical temperature.

Molecules travelling in streams side by side, as they must above a small flat surface, are close together for a longer time than those flying in all directions past a point; hence one would expect condensation to begin on nuclei, such as dust particles, of comparatively extensive surface. In the absence of such nuclei it may indeed be practically impossible to gather together enough molecules close enough and for long enough to start condensation at all.

EXAMPLES.—CHAPTER XXIII

2. Represent graphically the volume changes of a gramme of H_2O between $-5°$ C. and 105° C. under atmospheric pressure. [Ab]m.

3. How can it be shown experimentally that the pressure of a saturated vapour is unaffected by the presence of a gas like air ? [L.]

4. A barometer tube contains mixed air and saturated vapour above a 70-cm. column of mercury (atmospheric 76). What is height of mercury when tube is depressed to halve volume above it, pressure of saturated vapour being 1·5 cm. ? [L.]

5. Draw curve indicating generally the change in maximum vapour pressure of water between 0° and 100° C. Show that the boiling point of a solution is higher than that of pure water.

6. A flask is partly filled with ice and is corked up. What is the pressure inside at 100° ? [L]m.

7. 1 litre of air at 100° C. and 77 cm. pressure is saturated with water vapour; find its increase in volume; temperature and pressure remaining unchanged.

8. What is the effect of pressure on boiling points and melting points? Describe illustrative experiments. [M.]

9. Why does a liquid vaporize much more slowly when only 1° below the boiling point ? [L]m.

10. Explain the working of a soda-water syphon. Why do the bubbles grow in size as they ascend ? [M.]

11. Define the vapour-pressure of a liquid and explain how it can be found for water between 75° C. and 120° C. [L.]

12. Describe and explain the apparent transfer of cold in the cryophorus. [L]m.

13. A current of dry air is blown through fresh water and then through salt water. All are initially at the same temperature. Explain all that may be observed, and state what would happen if the air went through the salt water first.

14. 4 litres of air at 17° and 76 cm. and dew-point 6·5° are bubbled through water and become saturated. How much water is taken up ? Water v.p. 17° = 1·44 cm., 6·5°, ·72 cm., 22·3 litres vapour at 0° and 76 cm. weigh 18 grm.

15. Would the amount taken up from salt water be more or less ? If the salt water b.pt. were 102°, how much would be taken up ?

Chapter XXIV

THE LIQUEFACTION OF GASES

§ 219 : **Gas Expansion and the Mechanical Equivalent.** Mayer, a physiologist, had in 1842 made an estimate of the mechanical equivalent of heat in the following way. The specific heat of air allowed to expand at atmospheric pressure as it is heated is ·239. According to thermodynamic theory, since confirmed by Joly's experiment of § 169, this is 1·4 times its specific heat when expansion is prevented (·171 near ordinary pressures). Now 1 grm. of air at 0° and 1 atmo. occupies 1/·001293 = 773 c.c. and expands 1/273 of this = 2·84 c.c. when heated 1°. It therefore *does work in lifting the atmosphere*

= pressure × expansion = 1,016,000 dynes × 2·84 = 2·88 million ergs.

Assuming that this work represents the additional heat energy absorbed by the expanding gas, ·068 cal. = 2·88 million ergs.

$$\therefore \underline{1 \text{ cal.} = 42{\cdot}4 \text{ million ergs.}}$$

§ 220 : **Question of internal work.** But is there no internal work done in the gas itself during this expansion ? Is there no energy absorbed in pulling the molecules farther apart against their mutual attraction a/v^2 of Van der Waals, § 104 ?

If there is any such absorption of energy, a gas which is expanding owing to fall of pressure, and is not compelled to make room for itself by pushing back pistons or the atmosphere, ought to *cool*, just as does a liquid when its molecules are torn apart by its evaporation in vacuo, though to a much less extent.

Joule and Lord Kelvin tested this in the **Porous Plug Experiment.** Gas under pressure escaped through a plug of cotton-wool squeezed between perforated diaphragms in a pipe ; there was a thermometer either side of the plug. They found a small cooling in the expanded gas, proportional to the fall of pressure, and amounting to about ¼° C. per atmo. fall for air and 1¼° for carbon dioxide. This proves that a little internal work is being done

in separating the molecules (and therefore invalidates the argument of the preceding paragraph, though only to a small extent). But hydrogen *warmed* about $\frac{1}{20}$° per atmo., as it expanded.

§ 221 : **Internal and external work.** We have now to reconcile with this the popular half-truth that an expanding gas cools strongly, cf. § 211.

Firstly, compressing a gas, as in your bicycle pump, undoubtedly makes it hot. Only part of the work done on the gas from without is stored as potential ' pressure ' energy, the rest is at once converted into heat. The old-fashioned philosophical toy, the ' fire syringe,' was a popgun containing tinder moistened with inflammable liquid. A particularly muscular operator driving in the piston could compress and heat the air sufficiently to ignite the tinder. The modern Diesel oil engine compresses air quickly to 700 lb. per sq. in., crude oil sprayed in immediately catches fire ; the engine uses no artificial ignition whatever. Air compressors are water-cooled to get rid of this heat, and by the time the air is stored cold in pressure tanks no small part of the work spent on it has been utterly thrown away.

If this air be used (without ' reheating ' in a stove) to drive an engine, rock-drill, etc., the working cylinders are cooled, and the cold air exhausted from them is doubtless welcome to the miner. Here evidently some of the heat has suffered conversion into external work. Even if the air merely blows from a perforated pipe the pipe cools, though no useful mechanical work has been done ; but here also the air in the pipe is doing work in setting the issuing air into rapid motion, carrying away kinetic energy (which could drive little windmills, for instance). And even when one laboriously pulls out the piston of an air-pump the expanding air cools (as a flash of mist will show in moist air) ; one is not doing work on the air, on the contrary, all its remaining pressure is assisting one to push away some of the surrounding atmosphere.

In all these cases there is considerable *local cooling*. But taking the whole system into account the *total cooling* is only of the trifling magnitude observed with the porous plug. In that experiment the air rushing through the narrow crevices may be cooling by its exertions in expanding but is being warmed again by friction on their walls. The air-jets leaving a perforated pipe are warmed again by friction as the surrounding air checks their turbulent motion. If they blew into a closed space they

would warm it by compression. Pipes such as these cool the air in a refrigerator, but they must discharge outside it. The heat developed by friction and percussion in the rock-drill very nearly makes up for the cooling in its exhaust air.

§ 222: **Liquefaction of the 'permanent' gases.** Small as is the porous-plug cooling effect, it is the basis of the modern 'regenerative' process of liquefying air.

The Liquefier, Fig. 89, is a coil of small copper tubing, about 2 mm. bore, wound round a hollow vertical valve rod (very much as the wire is wound round the leg of an electro-magnet) into a mass 10 in. long and $2\frac{3}{4}$ in. diameter, containing 560 turns but being really four tubes 'in parallel' (in case of choking). The whole fits in a thin metal tube closed at the bottom, and around this is a packing of non-conductor.

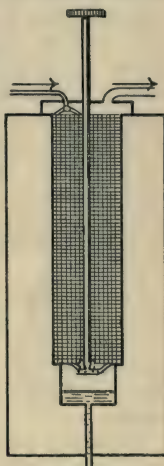


FIG. 89.

Air, freed from carbonic acid by passing over slaked lime, is compressed by a Whitehead-torpedo pump to 160–180 atmos. (rather over a ton to the square inch). Though saturated, it retains but little water per gramme, on account of its small bulk, and this little is removed in a caustic-soda cylinder. The air now enters the liquefying coil at the top, passes through, escapes at the bottom through a small regulating valve, then has to rise up among the interstices of the coil, and finally passes off at the top to a gas-holder ready for the pump again.

The expansion at the valve (to about atmospheric pressure) followed by the frictional checking of the violent outrush, causes the 'porous-plug' cooling, now perhaps 40° on account of the enormous fall of pressure. This cooled air rising past the coil cools the air flowing in it. When this escapes it cools still further and rising chills the coil still more, and so on, 'regeneratively.' Fortunately too the effect becomes greater as the air approaches the condition of a saturated vapour. Within four minutes about 5 % of the air is leaving the valve as spray. This is caught in a little spray separator and drips to the bottom of the enclosing tube, whence it is periodically run off into the now familiar

vacuum vessels. The machine produces about $1\frac{1}{2}$ litre per hour at merely the cost of attendance, running the 5-h.p. motor, and cooling water for the pump.

The air boils at atmospheric pressure at 78° A. (= −195° C.) at first, but as the more volatile nitrogen distils away the boiling point rises till the residue of Oxygen evaporates steadily at 91° A. (= −182° C.) and is received in the usual steel bottles.

All the ordinary 'permanent' gases are liquefied nowadays by passing them into test-tubes dipping in liquid air.

It appears at first sight hopeless to attempt Hydrogen by the regenerative process, since its porous-plug effect is a heating. But at 200 atmos. and −200° C. the effect changes sign, and it cools. Hence hydrogen is liquefied by pumping at $1\frac{1}{4}$ ton per sq. in. first through a coil cooled in carbonic-acid-snow and alcohol at −80° C.; second, through a coil in a vessel in which a regulated spray of liquid air is evaporating below −200° C. under only 10 cm. mercury pressure, exhausted by a $\frac{1}{2}$-h.p. pump; third, through the liquefier, which is enclosed in vacuum jackets. It is a very light liquid and somewhat difficult to manage, any air coming into contact with it solidifies forthwith, for it is boiling at 20° A.

§ 223: **Refrigerating machinery.** The liquefaction of nitrous oxide, carbon dioxide, ammonia gas, sulphur dioxide, etc., substances with high critical temperatures, is easy. They are unceremoniously pumped into coils cooled in cold water, and thence bottled off into steel or even, for sulphur dioxide, glass bottles.

Without a pump, the pressure of ammonia vapour rising from hot 'liquor ammoniae fort.' in a boiler is sufficient to liquefy the gas inside a water-cooled condenser.

The last three gases find employment in refrigerating machines, for the cold air pipes suggested in § 221 require bulky and inefficient machinery and the sulphuric-acid-absorption ice machine of § 212 is an expensive toy. Highly compressed by a pump they liquefy in water-cooled pipes, the liquid is admitted through small valves into larger pipes inside the refrigerator chambers, and there evaporates at perhaps atmospheric pressure. There is no doubt about the cooling effect of evaporation, § 212, which we now see might be described as a Joule-Kelvin effect in excelsis. The gas returns to the pump, half-a-crown's worth of fresh gas a year makes up for leakage. CO_2 machines are very

compact and efficient; SO_2 machines work at much lower pressure, and are no trouble to keep in order.

For ice-making, the cold pipes pass through strong brine, and this is circulated past the flat tanks of fresh water.

TABLE

Substance.	Melting point. °A.	Boiling point. (1 atmos.) °A.	Critical point. °A.	Critical pressure atmos.	Density at b. pt.	Vapour pressure at 15° C. atmos.
Helium	2 ?	4·5 ?	5 ?	2·3	·15	Above critical point.
Hydrogen	16	20	35	15	0·07	
Oxygen	55	91	154	58	1·13	
Nitrogen	60	77·5	124	27·5	0·79	
Argon	83	87	155·5	53	1·21	
Ethylene	104	169·5	9° C.	58	0·57	
Nitrous oxide .	170	183	37° C.	75	—	50
Carbon dioxide	above b.pt.	193*	31° C.	72	1·53*	52
Ammonia	197·5	234·5	131° C.	113	0·68	7·1
Sulphur dioxide	—	−10° C.	155° C.	79	1·4	2·7

°A. − 273 = °C. * Solid, subliming.

CHAPTER XXV

HYGROMETRY

§ 224. In accordance with Dalton's law water will evaporate till its vapour fills the space above it to the same partial pressure, whether any other gas be there or not. But the presence of another gas enormously hinders the *rate* of evaporation, for the escaping water molecules have to thread their way through a crowd of gas molecules. Hence the amount of water vapour present in the air above water or wet soil does not often reach its *saturation* value; even gentle atmospheric movements suffice to carry it away before this. Saturation may be reached on subsequent cooling and is usually overrun, and mist or cloud deposited, § 211 Thus **Hygrometry**, the study of the dryness or dampness of the atmosphere, will help in the forecasting of local weather.

The further the contained vapour is below its full saturation pressure the more water can the atmosphere still take up, the quicker wet things dry, and the drier the air feels. Since the maximum vapour pressure increases so rapidly with temperature, Fig. 83, summer air may feel very dry and yet contain more than enough water to saturate it in the cold of night. On a dry winter day there can be very little vapour present at all. And assuming a half-saturated state, it is evident that the vacant 10 mm. or so in summer will promote a faster drying-up than the vacant 2 or 3 mm. in winter.

Definition. The Hygrometric State, Saturation Fraction, Relative Humidity, or simply the **Humidity** is the ratio of the mass of water vapour actually present in the air to the mass that could be contained in the same bulk at the same temperature.

Or what comes to practically the same thing, since the vapour obeys Boyle's law almost up to saturation

$$Humidity = \frac{\textit{pressure of water vapour actually present in air}}{\textit{pressure of saturated vapour at same temperature}}$$

It is usually expressed as a percentage.

[In the daytime in this country it is very commonly 60 % to 70 %.]

§ 225. Of **Hygrometers** for measuring Humidity, the '*chemical*' is direct but slow. The air leaves its moisture in weighed 'drying tubes' as it is drawn through them to replace the water slowly flowing out of the aspirator, of known content. The observed increase of weight is then divided by the weight of the same volume of saturated vapour at the same temperature, obtained either from the tables or by a similar experiment in which the air would be first passed through tubes of soaked wool.

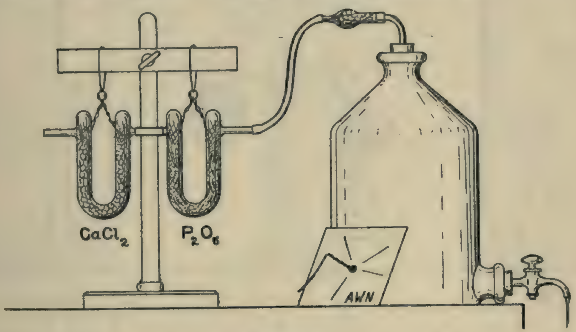


FIG. 90.

§ 226. In the more common **dew-point hygrometers** a cold surface cools the air near it down to a *temperature at which the amount of vapour present suffices to saturate it*, and thereafter begins to precipitate as a thin 'dew' on the cold bright surface. *Then the Humidity is the saturation pressure at this Dew-point divided by that at the actual air temperature* (read off from Fig. 83).

A clean glass of water kept stirred and gradually cooled by a lump of ice, Fig. 91, will serve the purpose in a way familiar enough on summer dinner-tables. Dines's hygrometer is a modification and shares the disadvantage of requiring ice.

The ancient Daniell's hygrometer is a bent double bulb tube containing ether and its vapour. More ether is poured on one muslined bulb, and evaporating, cools and condenses the vapour inside. More vapour comes over from the ether three parts filling the lower bulb, bringing its latent heat with it, and this bulb gradually cools until the dew appears on its surface (sometimes gilded). The instrument must be kept well shaken up to keep the bulb at the same temperature throughout, and *of course as with all hygrometers* neither the breath nor the warm perspiring hand

must come near the air-temperature-thermometer (on the stand) nor the cold surface ; nor need success be expected in the sun or in a draught [Fig. 92].

As dew enough to see means that the temperature is dropped a little too far, cooling is stopped and a rising reading taken when the dew just dries off, and the mean of both (with care only half a degree apart) is the dew-point.

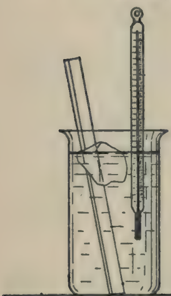


FIG. 91.

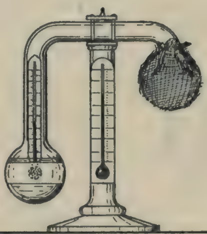


FIG. 92.

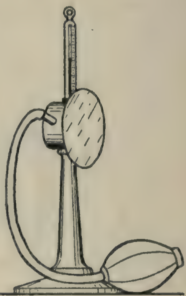
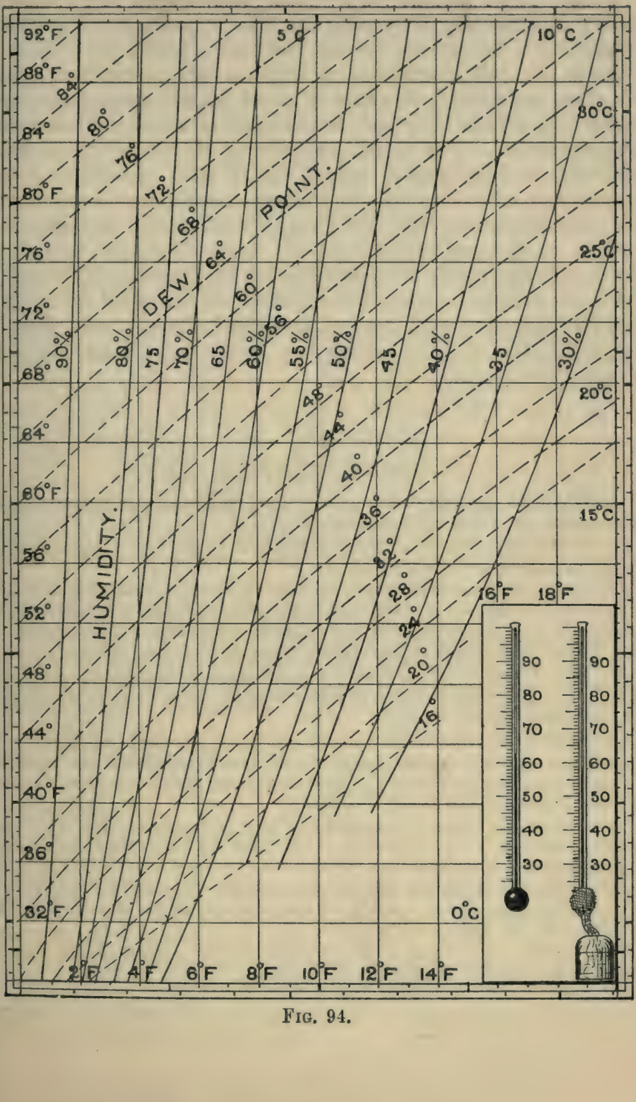


FIG. 93.

A small Daniell does very well, but the big, clumsy, overfilled shop instruments waste patience and floods of vile-smelling ether.

A more modern instrument has a little box, full of ether or petrol, glued to the back of a thin dark glass plate (better than polished ? metal). Evaporation is excited by a bulb bellows and cooling is quick [Fig. 93].

§ 227. In the **wet and dry bulb hygrometer** (self-acting, and sold under numberless names) one of a pair of thermometers should have its bulb wrapped in old washed *linen* kept wet, like a wick, by distilled water. The moisture evaporates faster the drier the air, and abstracts latent heat from the bulb, which therefore cools until the influx of heat by convection and radiation balances the rate of loss. [In the best practice, air is blown past the bulbs at a slow standard speed.] In Fig. 94 (based on comparative observations with other hygrometers) take the dry bulb temperature as ordinate and go along the horizontal to reach the difference between thermometers as abscissa. Your position on or between the continuous curves gives the Humidity, the dotted curves give the corresponding Dew-point. No difference of course means saturation ; a big difference, very dry air.



92°F
88°F
84°
80°F
76°
72°
68°
64°
60°F
56°
52°
48°
44°
40°F
36°
32°F
5°C
10°C
30°C
25°C
20°C
15°C
0°C
84°
80°
76°
72°
68°
64°
60°
56°
48°
44°
40°
36°
32°
28°
24°
20°
16°
DEW POINT.
HUMIDITY.
90%
80%
75
70%
65
60%
55%
50%
45
40%
35
30%
2°F
4°F
6°F
8°F
10°F
12°F
14°F
16°F
18°F
90
80
70
60
50
40
30

Fig. 94.

§ 228. There are many less reliable instruments more properly called **Hygroscopes** depending on the hygroscopic ('moisture-attracting') nature of fibrous materials (§ 255) or chemicals. Such is the hair hygroscope in which a hair (freed from grease by ether extraction) stretches in moist air and permits a spring to move a pointer (in watch form it sells as a traveller's detector of damp beds). The 'seed' (achene) of the feather grass (*stipa pennata*) or of the wild geranium, can be stuck upright on a card with a drop of wax and the hygroscopic twisted awn waves round its natural pointer (Fig. 90, leaning against the aspirator). A bundle of blotting-paper kicks the beam of a light balance as dampness increases. Twisted catgut is the secret of the weather-wise old couple in their hut perched on the shelf in many a country cottage, while every household has its salt seaweed trophy of the summer holiday, or its pet brick that 'gives up damp against rain.'

EXAMPLES.—CHAPTER XXV

1. Describe experiments to show that the aqueous vapour in the air cannot exceed a certain amount. [L]m.

2. Define the dew-point and state how to find it experimentally. Mention errors involved. [L.]

3. State the law regulating amount of water vapour in a closed vessel into which some water has been poured. Dew-point in open air being 15° and barometer 75 cm., if saturated vapour pressure at 15° is 1·27 cm. how much of barometric pressure is due to dry air ? [M.]

4. The dew-point of the air in a greenhouse rises from 9·5° C. to 20·2° C. ; calculate the proportion in which the water vapour present is increased. [L.]

5. Find mass of water in 1 cu. m. air at 20° C. when dew-point is 5°.

6. How is the vapour pressure of water measured at ordinary atmospheric temperatures ? How may such measurements be used to find humidity of the air ? [Ab.]

7. Define the relative humidity of the atmosphere, and show in a general way how the readings of the wet and dry bulb hygrometer are connected with it.

On a clear evening the dew-point is 42° F. Explain the effect of the moisture present upon the fall of temperature later, and consider the probability, or otherwise, of frost. [L.]

8. Explain use of any form of hygrometer. What is the relative humidity if air 40°, dew-point 12° ? [Ab.]

Chapter XXVI

ON METEOROLOGY

The heat of the sun-bath in which the earth is rolling is the great cause of atmospheric happenings.

§ 229. **The reception of heat from the sun.** The existence in the solar spectrum, § 417, of certain dark lines known to have their origin in the earth's atmosphere shows that air itself does absorb a certain amount of solar radiation, but probably only a little. The dazzling sunshine of the high Alps scarcely warms the clear air through which it passes. The greater part of the radiation that reaches the surface of the land or sea is absorbed to heat them, leaving a reflected residue by which we see them. But there is an intermediate absorbent, of enormous efficiency, and that is atmospheric **dust**—land-dust, dust of salt from the sea-spray, volcanic dust—dust and the moisture that so readily condenses upon it, Fig. 86.

July sunshine on the superabundant city street-dust heats it, and through it the air, to form that stifling mixture more insufferable than Indian heat. Sunshine bursting between shower-clouds in a cold north-wester, after a week of storm has washed the air, burns and tans with a direct heat beyond that of the average cloudless summer day. The absorption of radiation in summer air before reaching sea-level is now estimated to average 29 %.

In the morning the sun warms the more or less dusty air. The temperature rises, but the dew-point does not, the humidity falls low. As the soil warms evaporation increases and by the afternoon the humidity has risen to 70 % or so. At sunset the soil and the aerial dust radiate and cool rapidly—within a few minutes of September sunset one's summer flannels become very inadequate—the humidity passes saturation, and **Mist** forms on the dust particles, higher and higher as the cooling continues: "the mist rises in the meadows."

§ 230. **Dew** is formed in various ways.

I. The little water drops of the mist just mentioned slowly settle out of the air, a true 'fall' of dew.

II. Radiation from objects on the earth's surface goes on rapidly into a clear sky; the best radiators losing heat fastest gather a film of condensed moisture, quickly radiate away the latent heat it gave up, and so go on draining vapour from the adjacent air all night.

It is too often overlooked, however, that a good radiator may be in receipt of heat from beneath, e.g. the day's accumulation of warmth in the stone 'metal' of a road travels up to the surface by night and largely prevents the deposit of dew.

III. Grass blades radiate well, but for the abundant formation of dew on grass there is a more potent cause. It is not a fall at all, it is a rise through the vessels of the leaf of transpiration water—crude sap—from the roots. By day this evaporates from the stomata, but it cannot do so in the saturated air at night, and the root-pressure continuing forces it out in drops. [On many plants there are specially large water-pores through which this water can exude, there is one at the tip of each little tooth on the edge of a fuchsia leaf.] In late summer the earth is thoroughly warm and keeps the roots active, hence the grass dews are heavy at that season.

Hoar-frost. If the cooling is very rapid the vapour or the mist goes to build up solid crystals instead of liquid drops. Observation of a morning's hoar-frost ought to give a pretty good idea of the relative efficiency of various objects in usually condensing dew.

Everyone knows that clouds prevent dew, acting as blankets to check radiation from the earth into space. Clouds of smoke from green fires are utilized in Californian orchards to ward off frost when the fruit is 'setting.'

§ 231. **Altitude and temperature.** We live in the depths of a great ocean of air and on every square inch rests a column of that elastic fluid nearly 15 lb. in weight. Climbing a hill we climb above the lower layers and are relieved of their weight, the atmospheric pressure is less at the height. As an ordinary partly filled balloon rises the gas expands and fills it. Likewise if a quantity of air is rising it expands—a little square-inch column of it a foot high by the loch shore would be 14 in. high on the top of Ben Nevis, § 76. As it expands it does work, for imagine it enclosed in a tube, it would drive a sliding cork outwards

against the remaining atmospheric pressure—the little column would have forced back an average of $13\frac{1}{2}$ lb. through 2 in., $2\frac{1}{4}$ ft.-lb. of work. Hence it cools, § 221.

The ultimate result is that the atmosphere in 'convective equilibrium' shows a decrease in temperature upwards averaging 1° F. per 183 ft., or 1° C. per 100 m., of ascent. This result is theoretically calculable and is confirmed by recording instruments carried on balloons and kites.

Under a high sun evaporation goes on abundantly from sea and land, and as water vapour is only $\frac{5}{8}$ as dense as air the vapour-laden air streams upwards, cooling as it goes, till it reaches its dew-point. Then on the rising stream, perhaps a mile up, there forms a cap of mist, flat beneath and puffed out aloft, the small Cumulus cloud that floats so gaily through the noonday azure of true English June.

When moisture begins to condense latent heat is set free, hence the diminution of temperature upwards is interfered with. During ascent through a cloud the thermometer reading sinks more slowly, or stands still.

[Towards evening as the sun sinks and evaporation diminishes, the uprush of moist air is less active, and the evaporation of the cloud caps into the drier air around has time to clear them away, or else they flatten into thin layers at the dew-point in a quiet atmosphere; the sun sets in a sky either clear or streaked with Stratus cloud.]

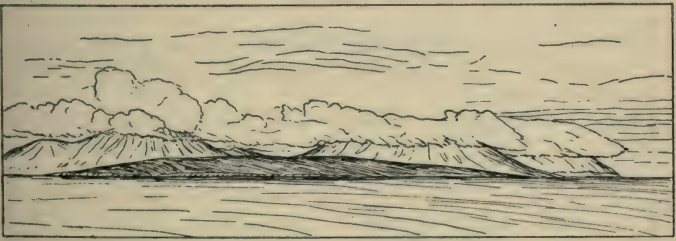


Fig. 95.

Fig. 95 is from a photograph, taken 6 p.m., August 1, of the hills of Hoy, in the Orkneys. The warm west wind blowing in from the Atlantic (right of sketch), and compelled to rise up the hill-side, formed the caps of cloud seen on the right, perfectly

fixed in shape though vapour was wreathing up through them at a great pace. In the gap the air could sink again and the clouds thinned out, to form again in rolling masses over the second hill. Later in the evening the air reached its dew-point at lower levels, and ultimately produced a wet sea-fog.

These hills catch the wind after an unbroken journey across the Atlantic, and though scarce 1500 ft. high show every sign of frequent drenching. And hills much lower and less favourably placed than these collect an astonishing amount of moisture. On the summits of the chalk downs are many 'dew ponds,' probably of early British origin, affording reliable supplies of nearly soft water, adequate to the needs of thousands of stock. Undoubtedly it is dew, formed chiefly as in (I) above, and very abundantly on account of the 2° or 3° F. coolness due to altitude, that drains off the neighbouring turf and fills these ponds. Yet mist may be so seldom noticeable on the hills that many more mysterious explanations have been given. Again, two of my acquaintance who built themselves houses, the one on a 400-ft. chalk down and the other on a 200-ft. gravel hill, have both been driven down again by exacerbations of the rheumatism they had planned to escape.

Having dropped its moisture as it rose over the mountains, and carrying with it all the latent heat of condensation, the wind may sweep down their farther slopes and be heated by compression as it sinks, until it blows as the hot Föhn of the Swiss valleys or the parching North-wester of Canterbury, N.Z.

[With all these considerations it will be evident that the change of temperature with altitude on mountains is seldom the normal convective change.]

Recent atmospheric research with free balloons (a rubber balloon 1 m. diam., full of hydrogen, rising steadily and ultimately bursting, when a smaller attached balloon brings the little meteorographs down in safety) has revealed an Isothermal Layer at an average height of 7 miles and temperature −55° C. The air there is attenuated (7 in. barom.), it is above the highest icy Cirrus cloud, and its temperature probably depends on its own absorption and emission of the solar radiation. It habitually moves from the west at 80 miles an hour (occasionally 200), and serene in the possession of such vast stores of kinetic energy it seems to have strangely little to do with the turbulent depths beneath.

§ 232. We have seen how mists can form by quiet radiation and clouds by the cooling of moist air as it floats upward. The meeting and mingling of hot and cold currents of moist air is another cause of cloud formation. Referring to Fig. 83, let equal parts of saturated air N at 5° (say a north wind) and saturated air S at 35° (say a south wind) be mixed. The resultant temperature is somewhere about the mean, 20°, the resultant amount of moisture per volume is the mean $\frac{1}{2}(7+43)=25$ mm. But the saturation curve is hollow, 17 mm. saturates air at 20° and the extra 8 mm. must form into a cloud. And even if the air is well removed from saturation (lower S, quite a common humidity) the cold air, containing really very little moisture, will still cause cloud so long as the mid point of (lower) NS lies above the curve.

In § 255 it will be shown that the smaller drops of a cloud evaporate and deposit on the larger ones; for this and probably also for electric reasons the cloud ultimately falls as Rain, or if the coalescence occurs below freezing, as Snow.

High cross currents, one above the other, are the probable cause of the rippled Cirro-cumulus clouds—mackerel sky, petits moutons, etc. A cold current drifting over the rising warmth of the city may form, not far above the chimney-tops, a thick blanket of dirty cloud—the sudden darkness that is the bête noire of the electric-light companies—while the streets remain free from mist.

Sea-fog is often due to the flowing of warm moist air over a cold current of water, instance the persistent fog of the Grand Banks of Newfoundland.

The Western Isles have a reputation for being enfolded in mist while the intervening Sounds are clear. In June, however, under the high sun, they enjoy the best of weather, for the quickly absorbing quickly radiating land is then warmer than the sea and melts the mists. With the lower sun and longer nights of August the land has begun to cool while the sea has attained its maximum temperature, its vapours then condense on the land.

A city fog is a mist deposited on the too-abundant nuclei, and probably lacking the usual incentive to evaporation of small drops, § 255, because foreign gases, bituminous matter, etc., are concentrated in solution over the surface and reduce the surface tension of the water. Its dry character is simply the flavour and aroma of these substances, especially sulphur dioxide, traces of which are likewise responsible for most of the 'dryness' in the air of a room heated by a gas-stove.

§ 233. **Winds.**

Land and Sea Breezes. The sea is disturbed and mixed by even small ripples to a depth of several feet, consequently the sun does not heat its surface by day as hot as it does the land, and by night the sea remains warmer than the rapidly radiating land. This gives rise to a Sea breeze by day, flowing in to supply the place of rising hot air over the land, and to a Land breeze by night, off the cool land on to the warmer sea. Though not often perceptible in this country these are of regular occurrence in the tropics, where radiation is intense and barometric changes usually trifling.

The **Trade Winds** are currents blowing from latitude 30° to supply the place of the uprising warm air of the equatorial calms. As the earth's rotation carries points in latitude 30° only $\frac{5}{6}$ as fast eastward as points on the equator these currents lag behind and blow from N.E. and S.E. instead of from N. and S.

The **Monsoons** are the 'resultants' of trades and of gigantic seasonal land and sea breezes from the land masses around the Indian Ocean as the sun moves from tropic to tropic.

§ 234. Weather in the British Isles is mainly due to stray curling eddies of the atmosphere, called Cyclones. The air over the Atlantic gets warmed and moistened, but is overlain by colder denser air and is consequently in unstable equilibrium, presently bursts through somewhere and starts a chimney-like updraught towards which all the surrounding air rushes. That from the north is coming towards a place of faster eastward motion and lags towards the west, while the south wind from still faster regions drifts to the east. Eventually curling round to reach the 'chimney' these give rise to a cyclonic circulation like Fig. 96, always going round *against the clock*.

The upward motion of the air of course relieves the earth of its pressure and the barometer falls low in the middle of the Cyclone, hence called a Depression (of the barometer). The system therefore appears on a barometric map as a series of concentric rings joining places of equal pressure, the lowest in the middle. The closer the rings, i.e. the steeper the barometric gradient, the greater the driving pressure-difference per mile of air and the harder the winds blow inward. Always they must blow harder as their courses narrow in towards the middle.

These cyclones vary in size; a good-sized one is drawn in Fig. 96, with typical barometer readings in inches.

They drift bodily over us, usually in an easterly direction, at perhaps 20 miles an hour; of course the wind velocity inside them may far exceed this. Observations of the change of direction and strength of the wind, and of the movements of the barometer, enable even a single observer to gain some notion of their course, and hence of the probable weather. British weather forecasts are made up from these data, together with temperature and humidity, supplied by a small selection of the 4000 meteorological stations.

Take the very typical case of a cyclone drifting E. with its centre passing N. of the observer. It is equivalent, and more convenient in the diagram, to keep the centre fixed and let the observer move W. along the arrow, Fig. 96. Recollecting that the air is rising and decreasing in pressure as it nears the centre of the depression, and is therefore cooling and probably forming

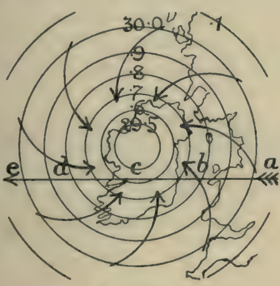


FIG. 96.

FIG. 97.

cloud and rain, especially if it comes from the warmer south, we can expect the following series of events :—

(a) Barometer beginning to fall, calm or light southerly airs; atmosphere often very clear, under cloud.

(b) Barometer falling rapidly, wind stronger S.E. or S., warm in winter, cold in summer; cloudy and wet.

(c) Wind veering towards S.W., strong; heavy rain.

(d) Veering rapidly towards N.W., strong; rain breaking into smart showers at increasing intervals. This is the weather that makes the townsman condemn the now rising barometer.

(e) Clear atmosphere, barometer rising briskly, cold N.W. wind gradually dying down, driving small clouds in a blue sky; hot sunshine through the washed air.

When a large depression passes centrally overhead the probability is that an easterly gale is succeeded by a calm day and then follows an equally strong westerly gale. The reader should work out for himself what happens at a place north of the centre.

The regular sequence is frequently complicated by a change of course (many small depressions are apparently thrown back to sea again by the precipitous cliffs of the western coasts), by the breaking-up of the centre, or by a succession of following depressions. Up the western margin of a great continental Anticyclone little depressions are apt to go curling one after another, giving us a fortnight's unsettled weather.

§ 235. The **Anticyclones** that give our fine settled weather are much larger areas of high barometer, from which winds blow 'outwards and clockwise,' gradually spreading the high-pressure area, Fig. 97, whose centre moves much more slowly than a cyclone's. The high pressure is due to the downrush of air from above; it is heated, dried, and cleared by compression as it comes down.

Typical anticyclonic weather is either calm or marked by a steady blow of dry wind from the same quarter for several days.

The centre of the anticyclone is on the left as you face the wind. As the sky is clear or thinly veiled, radiation goes on unchecked and gives extreme dry heat in summer or protracted frost in winter.

Chapter XXVII

VISCOSITY

§ 236. "If the paint be too thick, thin it by the addition of turps." So runs the amateur painter's instruction, making use of one of the many meanings of 'thick' and 'thin.' Physically we say ". . . too viscous, reduce its viscosity. . . ." In spreading paint the bottom layer of a thick smear adheres to the wood and the upper 'layers' are dragged over it by the brush. The force necessary for this, the drag felt by the brush, is due to the friction between 'layer and layer' of liquid. It is to this internal friction between contiguous portions of fluid moving at different speeds that the name **Viscosity** is applied.

Recent experiments have shown that wherever a liquid is flowing past a solid surface the two surfaces adhere without any slipping at all, so that viscosity always comes into play to hinder the flow of the upper layers. Instance the flow of treacle off a spoon; or the slow running of a detached thread of mercury down a slanted thermometer tube, not slipping down like a solid rod, but the outer sleeve for the time being sticks to the glass and its contents rush through it; inside out it is the motion of an umbrella ring over the handle. Instance also the flow of blood in capillaries, faster in the middle. Again, the 'skin friction' on a ship in water, friction really between the adherent water and the outer water; polish is useless, moderate smoothness suffices so long as there are no serious protuberances—rivet heads or barnacles—to cause eddies.

So with gases, long narrow pipes mean a defective supply of illuminant. Dust and mist settle very slowly through the air. A drop of water ·002 cm. diam. falls 1·2 cm. per sec. through the air, and the speed is proportional to the square of the diameter (so long as it is only a few centimetres per second). [But in gases at low pressures there is some slipping along the walls.]

In what follows, only smooth quiet motion at slow speeds is

intended. Motion producing eddies, turbulence, and noise is deferred to § 242.

§ 237. First take a case like that of honey flowing off a flat spoon. Magnify it, in Fig. 98 (A), and divide into strata of equal thickness and weight. One side of the first adheres to the spoon,

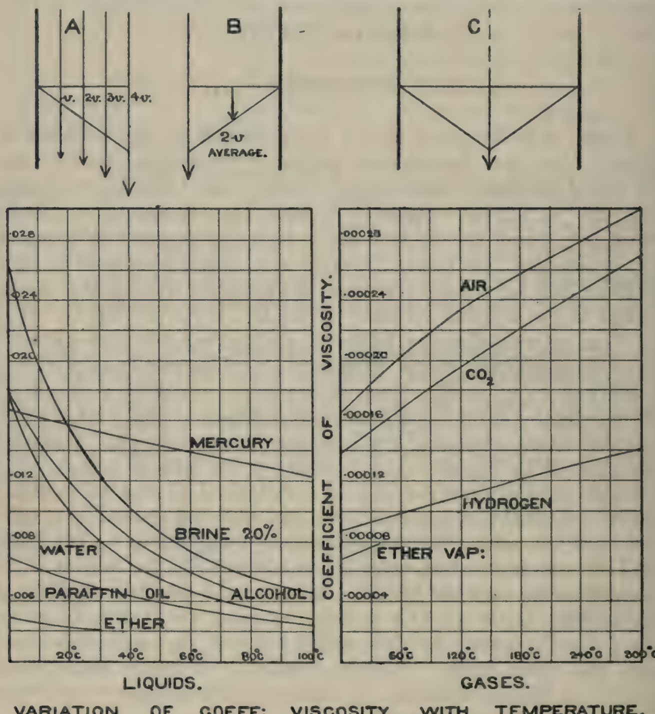


Fig. 98.

the outer side moves at speed v, the average speed of fall of the whole stratum is $\frac{1}{2}v$. The second layer moves on one side at v and on the other at $2v$, being subject to forces exactly like the first but attached to an already moving surface. And so on, the speed increasing proportionally to the distance from the solid and the average speed being half that of the outside layer.

The area of cross-section of the stream is its depth $\times$ its width. *The total flow* = average speed $\times$ area of cross-section *is therefore proportional to* $\frac{1}{2}$ (*square of depth*) of stream; the discharge from a tenth-inch layer is 100 times as fast as from a hundredth-inch film left to drain.

Next take an equal stream, but left-handed, and bring it up to the first, as in B. The two free surfaces are moving at the same speed, therefore nothing alters if we let them touch, and we then have a stream flowing between two parallel walls, C. The discharge is twice that of either, it is in the same proportion as before to

$2 \times \frac{1}{2}$ ($\frac{1}{2}$ *dist. apart of walls*)^2 i.e. to $\frac{1}{4}$ (*dist. apart of walls*)^2.

Finally suppose the stream confined by front and back walls as well as by the side walls. The outflow now

$\propto \frac{1}{4}$ (*depth between walls*)^2 *and also* $\propto \frac{1}{4}$ (*width between other walls*)^2
i.e. $\propto$ *their product* $\frac{1}{16}$ (*width* $\times$ *depth*)^2 $\propto \frac{1}{16}$ (*sectional area*)^2

and the same argument holds good for a circular tube.

Thus the carrying or discharging power of small tubes, e.g. the blood capillaries, increases very fast with increased size, viz. as the square of the area, or *the fourth power of the diameter*. It is only in larger pipes, when friction in the pipe itself is only a small part of the resistance to be overcome in pumping, that the carrying power becomes proportional to the area, as one expects. The water engineer pumping to a high reservoir ignores viscosity altogether, but in a particular petroleum pipe-line, when the oil proved too viscous to crawl 300 miles in a fortnight, a $\frac{1}{5}$ increase in bore would have been more effective than doubling the number of pumping stations.

§ 238: The Coefficient of Viscosity.

On the honey spoon the driving force is the weight of the honey, but if we suppose one centimetre cube to be set in motion by a force applied all over and in the plane of its outer face, causing that face to move at 1 cm. per sec. faster than the inner face, this force (in dynes) is equal to the **coefficient of viscosity** of the fluid.

Evidently the same force per square centimetre would produce a change of speed t per sec. over thickness t.

Doubled driving force means doubled velocity, and so on.

Double distance to go for the same driving force means only half the force available per centimetre length, and only half velocity, and so on.

Doubled viscosity means halved velocity, and so on.

Hence *the Quantity discharged through a narrow pipe per second*

$$\propto \frac{\textit{total fall of hydrostatic pressure}}{\textit{total length}} \times \frac{1}{\textit{coeff. of viscosity}} \times \frac{(\textit{area})^2}{16}$$

and the full mathematical discussion gives the actual efflux from a capillary tube as $2/\pi$ times this.

§ 239: The coefficient of viscosity is measured by running the fluid through a capillary tube of known dimensions, under known difference of level, collecting the discharge in a known time and applying the formula.

The efflux in cubic centimetres is inversely as the coefficient.

The coefficients for various liquids and gases are given in Fig. 98. Liquids become less viscous—more mobile—as temperature rises. Glycerine gets quite 'watery' at 100°. Substances like candle-wax and pitch can be called very viscous liquids, candles hardly bend in winter but collapse in summer, a particular pound square of pitch in the laboratory cupboard stands apparently changeless all the session but has subsided perceptibly more after each long vacation. 'Blood is thicker than water,' five times, at blood heat. Gases are far less viscous than liquids: their viscosity *increases* with temperature.

§ 240: **Lubrication.** Two surfaces of area A are separated by a thickness t of fluid. The force per square centimetre required to slide the upper over the lower at t per sec. has been defined as the coefficient of viscosity, the whole force is A times this. An increased speed necessitates a proportionally increased force (experimental, and corresponds to efflux). The dead weight on the upper surface makes no difference so long as it is not allowed to squeeze the layer of lubricant out thinner.

Thus **Fluid frictional resistance** is

proportional to viscosity of lubricant,
proportional to area in contact and to speed,
independent of weight carried;

while **Solid frictional resistance,** § 17, is

proportional to a coefficient of friction,
independent of area and speed,
proportional to weight carried.

The ample slow-moving bearing surfaces of the animal framework are constantly lubricated by the synovial fluid and probably obey fluid laws. The bearings of machinery lie somewhere between the two, depending on their oil supply. At low speeds the pressures are permissibly heavy, and squeeze the lubricant into thinner

layers as they increase. The rate of squeezing out $\propto$ (thickness)^2 $\div$ viscosity. The surfaces can never be quite true, and if the average thickness is too small the high places get into solid contact and make trouble. Hence the viscosity must be kept large, grease or graphite (which under great pressure behaves exactly as a fluid) is used, or hard tallow on 'launching ways'—temporary wooden slides not ideally plane.

At high speeds the oil is torn into shreds and patches which carry the weight like so many little flattened-out rubber balls, their total area and thickness depending on viscosity, surface tension, speed, pressure, and quantity supplied.

§ 241 : **Lubricants. Oiliness and stickiness.**

Why are honey and glue 'sticky' and grease and the paraffins 'oily'? Why not lubricate with treacle, or water—the film on an eel or under a skate-blade is slippery enough—or with oil of vitriol? Are they not all viscous fluids, having among them viscosities suitable for all sorts of bearings?

A lubricant must remain; it may neither disappear from its place, nor become too viscous, nor corrode the solid surfaces.

Water and the essential oils (lavender, etc.), are soon lost by evaporation, olive oil cools to a butter, glue to a firm elastic jelly, honey dries and crystallizes, linseed oil oxidizes to a hard mass, oil of vitriol corrodes, glycerine absorbs moisture and thins. On the other hand, a tiny drop of thin watch oil hardly alters in a year (most people seem to expect it to last for ever), common machine oils go many days and endure warmth, 'air-cooled engine oil' is viscous and almost vapourless at nearly a red heat.

The feeling of stickiness appears to be due to a rapid increase of viscous drag; there is no distinguishing it from oiliness till this change occurs. Try these domestic experiments:—A solutioned rubber patch feels greasy until nearly dry. Paste is slippery till its water disappears by evaporation and absorption into the paper. Syrup thickens as it gives up water to the absorbent skin of the fingers. Hygroscopic glycerine punishes a dry skin, but put on a wet one soaks in, leaving a surface film to collect dust.

§ 242 : **High speeds.** At higher speeds (depending on dimensions and viscosities) the fluid moves with *eddies* producing turbulence and, if air gets drawn in, noise. The water-tap begins to splutter, the gas-jet roars, the bullet sings, the boat leaves waves and busy little whirlpools in its wake.

The flow through a pipe is less than expected, the resistance

of the air far more (compare the statement in § 236 with the top speed of a ·5-cm. raindrop, 25 ft. per sec.), 'skin-friction' hardly counts with a cruiser.

Quantities of fluid are set into varying violent motions and the friction among them largely exceeds that at the measurable surfaces. Empirical laws are obtained suiting special cases. Resistances increase as the square of the speed, at least, and although in the end viscosity quiets it all, they often appear independent of it, diminished viscosity being counterbalanced by increased bulk of disturbance.

§ 243: **Viscosity of Gases. Kinetic theory.**

Consider a gas flowing over a surface. The marginal molecules of the forward drifting gas will, in their active swarming, frequently collide with the rough molecular banks representing the solid surface. They rush out again into the stream with their forward component motion lost, or even reversed; they hinder other molecules with which they happen to collide and so the drag spreads through the stream.

When the gas is hot the molecules rush about faster, more of them hit the rough surface, and the total loss of forward momentum is greater. Now the forward momentum of the gas was what we gave it by pushing on it, and having nothing whatever to do with the natural molecular speeds due to temperature, was no greater in the hot gas than in the cold. Hence a greater percentage is lost when hot; *gas viscosity increases with temperature.*

Alteration of pressure does not affect molecular speeds, at half-pressure there are only half as many molecules to hit the banks, the loss of momentum is halved, but only half the mass of gas is moving, and consequently *gas viscosity is independent of pressure.*

When the gas becomes so attenuated, however, that the reflected molecules usually hit the opposite bank before they have collided with other molecules the hindering effect disappears, for if the first collision sent them backward the second will probably send them forward again, taking the average among millions the two collisions cancel. Thus at very low pressures gaseous friction suddenly diminishes. The cold filament of a carbon glow-lamp vibrates for minutes, admit air by filing off the pip and it hasn't a quiver in it.

Chapter XXVIII

THE LIQUID SURFACE

§ 244. **Surface Tension.** Go to the pond in summer. Watch the ' pond-skaters ' darting over the surface which only their long legs touch. Less conspicuous, the ' water-boatmen ' resting or sculling on the underside, like a fly on the ceiling. To these small beasts the surface is a stretched sheet, smooth and tense, sustaining all the force they exert on it. Yet avoid the notion of ' water-skin ' that some speak of, there is no skin (unless there be one of scum), a skin would have two surfaces. There is, however, a boundary, a surface, with a stretch in it, a **surface tension.** Light weights depress the plane surface into little dimples, the skater rests in half a dozen miniature hammocks.

Let us first find a means of measuring this surface tension. The one we shall take is the phenomenon called **Capillarity,** the rising of liquid up narrow crevices and tubes (*capilla*, a hair)—of water through wood or brickwork, of oil up a wick, etc. Take two glass plates, wash them well and rinse in the liquid under investigation, keep them apart with a couple of darning-needles at the edges, strap an elastic band round, and stand them upright in a saucer of the liquid. It rises between the plates, and the higher, the closer they are (Fig. 99, in the dish).

§ 245. **The wetting of surfaces.** For this rise to occur, the liquid must wet the plates. Why do some liquids *wet*, i.e. adhere to and spread on some solids, and not all ? We do not know, but a familiar difficulty gives some clue ; melted solder will not stick to a tarnished copper bit. It gradually adheres if the bit be shielded from oxidation by melted rosin and be very hot, it adheres instantly if a corrosive chloride is present. Wiped off it contains traces of copper, i.e. the adhesion is probably due to the same molecular forces that are concerned in solution or in chemical action. " Adhesion," said Graham, " is an unsuccessful attempt at solution." Mercury readily adheres to zinc or gold,

which dissolve in it freely, but it is difficult to make it adhere to iron.

The wetting of most surfaces by ordinary liquids probably depends on their being already covered with an imperceptible film of moisture. Glass collects a particularly thick film out of the atmosphere, sodium amalgam kept hot on it develops a layer of hydrogen bubbles which I have found to correspond to ·00001 cm. thickness of water. Visible wetting will ensue with any liquid that can dissolve this film. Another common film, that of grease, which for a time hinders wetting by water, is an encouragement to the well-known ‘ creeping ’ of paraffin oils.

§ 246. **The measurement of surface tension by capillary rise.** To return to our plates ; if quite clean the liquid film spreads all over them as in Fig. 99 (A) and is continuous with the horizontal surface. Usually, however, it anchors itself somewhere fairly high up, as on (B), and on the not quite clean surface above it drops may hang. [An excellent test of the cleanness of a surface is that a film of water dries off it without collecting into drops.] Anyway, liquid is lifted up by the pull of the vertical parts of its surface.

The surface tension T is the pull in dynes exerted across each centimetre width of surface, Fig. 99 (T).

Taking a horizontal length l cm. of the plates the total upward pull exerted on the liquid between them is $2l\mathrm{T}$ (two plates).

The weight lifted is its volume × density × weight of 1 grm. in dynes $=(l \times b \times h) \times d \times g$.

Equating these, $2l\mathrm{T}=lbhdg$.

$$\therefore \mathrm{T}=\tfrac{1}{2}bhdg \text{ dynes or } h=2\mathrm{T}/bdg \text{ cm.}$$

In a circular **capillary tube** the peripheral pull $2\pi r.\mathrm{T}$ lifts the cylindrical column weighing $(\pi r^2 \,.\, h)d \,.\, g$ dynes.

$$\therefore \mathrm{T}=\pi r^2 hdg \div 2\pi r=\tfrac{1}{2}rhdg \text{ dynes/cm.}$$
$$\text{or conversely } h=2\pi r\mathrm{T} \div \pi r^2 dg = 2\mathrm{T}/r \,.\, dg \text{ cm.}$$

whence the rule that the height to which a liquid creeps in a capillary tube is inversely proportional to its diameter ($2r$).

The Surface Tension, at ordinary temperatures, of Water is 74 dynes per cm. width, of Mercury 547, Alcohol 23, Benzene 29, Ether 16, Paraffin oil 26.

§ 247. **Temperature and surface tension.** All surface tensions steadily diminish as the temperature rises and ultimately vanish

at the critical temperature, § 215, when the surface dividing liquid and vapour disappears (e.g. at 150°, T of ether is only 3).

The temperature decrease can be shown by touching a water surface (sprinkled with lycopodium or other dust to show its movements) with a bunsen flame. The heated spot suddenly

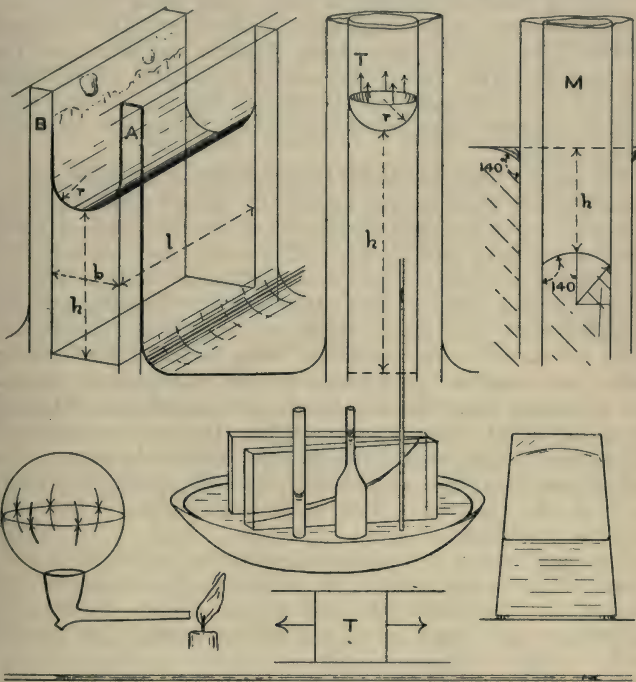


FIG. 99.

expands, for the tension there is lessened, it becomes a weak place and has to yield to the stronger pull of the cold surface all around. Similarly the film of rinsing water covering a clean glass plate shrinks hastily back from a heated spot.

§ 248. The high surface tension of water is often quickly reduced by dissolving small quantities of substances in the water. For instance, soapy water has only one-third the tension; from a

spot of shaving-soap froth dropped on to clean warm water there is a rushing outwards in all directions as the stronger surface tears at the weaker. In soft water this stops as the whole surface becomes soapy, but goes on longer on hard water which continuously destroys the soap.

Chips of Camphor dropped on clean warm water begin to perform little erratic movements rather reminding one of 'whirligig beetles.' As the chip very slowly dissolves one side is for the moment dissolving faster, the surface tension of the stronger solution there is weakest, and the chip is dragged the other way. The movements cease when the surface layers are saturated with camphor.

The surface tension of water is in fact greater than that of any ordinary pure liquid (except mercury). Alcohol sprinkled into a wet sink causes a violent commotion in the thin layer of water and each drop of spirit is left with a nearly dry halo round it. The strong water surface has shrivelled up and dragged out the weaker alcohol surface. The effect presently disappears as the liquids dissolve each other. Wine creeps up the side of the glass, there the alcohol evaporates the faster and a watery residue pulls itself together into 'tears' which trickle down through the spirituous film. The best known of these effects is the rapid spreading of oil dropped on water into the familiar iridescent films, the stronger water pulls out the weaker oil surface. The pull at each side of an oiled chisel 1 cm. wide would $=$ T of water $-$ T′ of oil. Conversely water cannot spread on oil, but pulls together into drops.

The high tension of a water surface makes it difficult to keep clean. It is constantly trying to pull sheets of every sort of contamination over itself. The best way is to keep it constantly overflowing the whole brim, and do not touch it or breathe on it.

The still greater surface tension of mercury, 500, acts in the same way. Water will flash instantly over the surface of mercury freshly redistilled into clean vessels, but so difficult is it to keep the metallic surface clean that this remained long undiscovered, and the usual standing of water in drops on mercury was regarded as a curious exception.

§ 249. **Films and froth.** Clean liquids do not form *persistent* films. Glistening bubbles on a pond are a hint not to drink, or even smell. On such a surface, spray drops from your paddle would not run yards before breaking in, as they will on the

strong clean surface of the stream. A hanging film, such as the wall of a bubble, must be a little stronger at top to bear its own weight, i.e. there must be a mixture of substances and a little automatic rearrangement in superficial concentrations.

§ 250. **Surface energy.** If under the tension of T dynes per cm. length crosswise, the surface moves back 1 cm., T ergs of work have been done, i.e. another way of regarding the surface is that it *possesses a potential energy of at least* T *ergs per sq. cm.*

Now the potential energy of a system always tends to diminish, § 26. In accord with this of course is the shrinking of the more energetic surface, already described. Hence also concentration of impurities in the surface, reducing its energy. If solution of permanganate be run through a tube packed with an inert powder (e.g. silica) colourless water drips from the end. The wetted surface of each grain represents so much energy, this is reduced if the salt concentrates there, and the vast surface of the myriad grains abstracts quite a significant amount. A big ink blot on blotting-paper has a lighter margin, the spreading fluid leaves its dissolved dye concentrated on the fibres.

§ 251: There is a way of representing the effect of **surface tension at curved surfaces** which is very useful.

Turn back to the parallel plates. The mass of liquid $lbhd$ grm. occupies a ' floor space ' $l \times b$ and if the surface let go would rest on this with a pressure $lbhd \div lb = hd$ grm. or hdg dynes per sq. cm. As this liquid stands above the level of the rest the hydrostatic pressure in it must be less than at that level, i.e. less than the atmospheric. We can say then that at a surface like this, straight one way and practically semicircular the other, i.e. with 'principal radii of curvature ' infinity, and $\frac{1}{2}b, = r$, there is a reduction of hydrostatic pressure on the bulging side of the curve of $P = hdg$ dynes per sq. cm. Now T was $\frac{1}{2}bhdg = r \,.\, hdg$.

$$\therefore P = \frac{T}{r} = T\left(\frac{1}{r} + \frac{1}{\infty}\right)$$

Again, the two hemispheres of a drop are bound together by a pull $2\pi r \,.\, T$ round their edges, producing a pressure $2\pi r T \div \pi r^2 = 2T/r$ on each square centimetre of their base. The drop has radius of curvature r in both N. and S. and E. and W. directions, and we can write its extra hydrostatic pressure inside,

$$P = T\left(\frac{1}{r} + \frac{1}{r}\right)$$

And in general the Difference in Hydrostatic Pressure on the two sides of a surface

$$P=T\left(\frac{1}{\pm r}+\frac{1}{\pm r'}\right)$$

where P is in dynes/cm.^{2}, T in dynes/cm., r and r' cm. are the principal radii of curvature in perpendicular planes and are + if the centre of curvature is on the side P is measured.

Thus in a cylindrical stream from a tap there is an increased pressure $P=T/r$. Inside a spherical drop $P=2T/r$ tending to compress it out of existence, i.e. help it evaporate. And inside a bubble in a liquid also $P=2T/r$, which may be very large if r is small, hence the great difficulty in starting small bubbles in a liquid referred to under Boiling. The gas in a soap bubble bears $4T/r$ (film has two surfaces); removing the pipe from your mouth and pointing it at a candle the flame is blown aside, Fig. 99, and the more violently the smaller the bubble shrinks. A soap film open to the air on both sides must be either flat or saddle-shaped, its curves equal and opposite.

A spot of wet binds two glass plates tightly together, for it flattens into a layer with strongly concave edges (Fig. 99, bottom), and the pressure in this is reduced by T÷half distance between the plates. The adhesion is tenacious, even in a vacuum. The thinner the film the tighter the hold. For the same reason well-fitting plates of artificial teeth cling to the palate, provision having been made for interfering air to be let out. Moistened sand is bound together by capillary pressure. The experiment of partly filling a tumbler with water, placing a card on it and inverting without spilling (Fig. 99, right), is similarly explained. It has nothing whatever to do with 'pressure of the atmosphere,' for the pressure of the air inside as it becomes saturated is greater than atmospheric, and there is the weight of the water (much or little makes no difference to success) to be sustained as well. The water between the outer edge of the tumbler and the card shrinks to a sharp concavity and the reduced pressure due to this holds up card, water, and all; in fact the more weight of water the closer the card pulls up. Drop ether on the card just before use so as to weaken the surface tension in parts, and the experiment fails.

We can see too that a capillary tube will not serve to pump water continuously to a level just below the natural rise, for the water would have to flow out of a side spout in drops and these

would bulge outwards, whereas the rise depends wholly on the inward bulge.

§ 252 : **Capillary depression of mercury** (or of any liquid in a tube it does not wet), Fig. 99 (M). Mercury has a bulged, not a hollow, ' meniscus ' in a tube, and is therefore pressed down instead of drawn up. The meniscus is not hemispherical, but meets the glass at an obtuse *angle of contact*, 140°, its radius is therefore r of the tube$\div$cos 140° and results in a downward $P=2T$ cosine $140°\div r$ dynes. A barometer with a narrow tube therefore reads too low by about cos $140°\div 13r$ cm., the evaluation of which is left as an exercise. The correction, however, is very uncertain because a trace of dirt alters the angle greatly. Safety lies in an inch-wide tube which has a negligible capillary error.

§ 253 : **Large drops.** Drops and bubbles at rest are spherical, for the pressure is uniform throughout and the sum of the curvatures must be constant [*or* because the sphere has the least surface and therefore least potential energy]. But gravity deforms large ones, as it introduces hydrostatic differences of pressure [or potential gravitational energy].

If, however, the weight is borne by floating in another fluid of about the same density, large spheres are obtainable and their vibrations when prodded are slow and easily watched. Large drops such as these are seen in ' sight feed lubricators.' Naphthalene melted in hot water breaks beautifully into these drops; aniline in warm water is even better.

§ 254 : **Instability of liquid cylinders. Drawn fibres.**

The quiet cylindrical stream from a water tap is in unstable equilibrium, for if a vibration cause a momentary thinning at one place—a smaller radius—an increased pressure arises there, pushes the liquid into the wider parts, thus corrugates the stream and speedily nips it into drops. Such jets are sometimes very sensitive and will magnify the ticking of a watch pressed against the tap into a succession of noisy splashes. The newly formed drops vibrate from egg-shaped to turnip-shaped, giving the jet its well-known bulbous appearance. Viscosity brings the vibrations to a standstill. Common shot are the drops into which slender streams of melted lead break up and solidify.

If the liquid be very **viscous** the small pressure differences will not succeed in breaking it up and it remains in long strings—

treacle, seccotine, or rubber solution drying 'tacky,' and the solidified products—glass tubing, silk, and spider threads. But water must break into drops on a wetted fibre, and a similar beading of sticky drops, easily seen with a pocket lens, gives the roundabout threads of the garden-spider's web their efficacy as fly-catchers.

§ 255. **Alteration of vapour pressure at curved surfaces.**

If in a closed vessel containing only liquid and its vapour (Fig. 100, left) the liquid rises a few centimetres in a capillary tube, it is obvious that the saturated vapour pressure at the concave surface in the tube is less than at the flat surface outside by the weight of the vapour above whose level the liquid has crept. There is approximately a diminution of vapour pressure on the hollow side of the surface of

$$p = \mathrm{P} \times \frac{\text{density of vapour}}{\text{density of liquid}} = \mathrm{T}\left(\frac{1}{r} + \frac{1}{r'}\right) \times \frac{\text{density of vapour}}{\text{density of liquid}}$$

It is easy to see from the Kinetic Theory how this comes about: its practical results are important :—

Suppose as in Fig. 100 a molecule, at d below the surface, whose next jump has a probable length = the radius shown and is

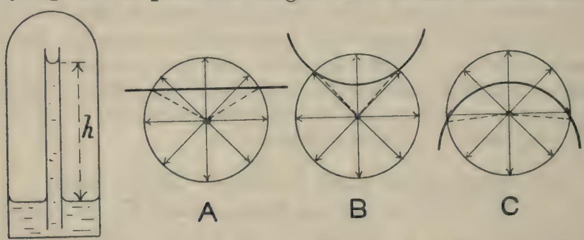


FIG. 100.

equally likely to take place in any direction. The three diagrams show that its chance of jumping through a flat surface A is greater than that of getting through one bulged towards it B, and less than through one bulged away from it C. Hence :—

From a *concave* liquid surface a molecule has less chance of escaping, while a molecule in the vapour above has a better chance of falling in; both effects *reduce* the number of vapour molecules per cubic centimetre, i.e. *the vapour pressure*. On to this hollow surface vapour may therefore condense from an unsaturated atmosphere.

Vegetable and animal substances—cotton, paper, wood, charcoal, wool, hair, catgut, etc.—abound in minute cells, and their cell walls are perforated by far more minute tubules through which ran the protoplasmic connecting threads, and are covered with pits, folds, and chinks. Hence perhaps their well-known hygroscopic character, every crevice holds a concave water molecule trap. All get too damp to electrify two minutes after taken away from the fire; a baked filter-paper gains weight too fast to follow on a fine balance; your clothes well dried before the fire lose half a pound of moisture; charcoal, cooled by liquid air, absorbs the last traces of gases from the highest vacua.

Similarly the vapour pressure in a small bubble must be far below the normal, hence the difficulty of starting boiling in a liquid freed from gaseous or solid nuclei. All the dodges for avoiding 'bumping' are directed to giving the bubble something comparatively flat to start on.

Conversely the vapour pressure over the *convex* surface of a little drop is abnormally high, for the molecules inside get a better chance of escaping and those outside a less chance of falling in. Little drops will therefore evaporate and *supersaturate* an atmosphere with vapour which must condense on the flatter larger drops. The big drops grow by the self-sacrifice of the little ones, a process always going on in clouds.

§ 256. Believing in molecules then, we may infer that surface tension is a molecular phenomenon just as vapour pressure is; the one due to some sort of mutual attraction among molecules closely packed and being noticeable when one-sided near the outskirts of the crowd, the other characteristic of molecules widely separated. Two or three molecules gathered together will not exhibit surface tension, it is a property of crowds. Verily a school-treat in a meadow exhibits it, the active swarm held together by social attractions is a drop from which few—the vapour molecules *pro tem.*—stray far afield.

Very thin films again cannot show the characteristic tension, one layer of oil molecules counts for little on the turbulent molecular sea of water beneath, 'camphor movements' go on till the surface has been quite measurably oiled. Little water ripples, themselves controlled by surface tension, are calmed by oil perhaps because their motion thins the oil on their crests (watch the play of colour in a patch on heaving water), the surface tension there increases nearer that of clean water; the surface

becomes unequally strong, it is as if a net were thrown upon it; the great waves remain, but their surfaces are polished.

The invisible 'black' top of a soap bubble about to break may be only 10^{-6} cm. thick (less than a tenth the thinnest adjoining coloured part), yet it is many molecules thick and exhibits a full surface tension.

EXAMPLES.—CHAPTER XXVIII

1. Describe and explain the effects observed when small pieces of camphor are floated on clean water. [M.]

2. Prove that water will rise in a vertical capillary tube to a height inversely as its diameter. How high will oil rise in a tube ·2 mm. diameter, if surface tension 3·8 and density ·75 ? [St. A]m.

3. By what fraction of an atmosphere do the pressures differ inside and outside a 4-cm. diameter soap bubble if T=25 ? [L.]

4. A drop of water placed between flat glasses spreads out to A sq. cm. and t cm. thick. Show that force between plates $= 2 \times$ surface tension $\times A \div t$. [L.]

5. Show that as in middle of Fig. 99 the height of a capillary column depends only on diameter of tube at the curved surface, provided that the liquid is once drawn up into the narrow part.

6. Calculate how high water, T=75, will rise in a capillary 1 mm. diameter, and in a piece of wood of which the vessels are ·0013 cm. diameter.

7. Work out the expression of § 252 for 1 cm. diameter.

8. Water from a depth of 4 cm. drips into the carbide chamber of a bicycle lamp through a nozzle ½ mm. diameter. Show that the lamp could produce, intermittently, a gas pressure = 10 cm. head of water, without blowing back.

Chapter XXIX

DIFFUSION

§ 257. If a few drops of bromine be poured into a tall glass jar which stands in a place free from all draughts and differences of temperature, their red vapour is seen to slowly spread up the jar and its odour is presently perceptible in the room. To spreading such as this, which has taken place without regard to gravity (for bromine vapour is six times as dense as air), and without any help from differences of pressure and temperature, the name of **Diffusion** is given.

If the jar were full of water the orange hue of dissolved bromine creeps upward in the same way, but far more slowly; it is a matter of days and weeks before it reaches the top. Or the stronger colour from permanganate crystals spreads equally slowly whether up through water or up or down through a jelly, showing that currents have nothing to do with it—such currents of denser solution as stream down from a lump of sugar, held in a spoon high up the side of the teacup.

§ 258. **Measurement of rate of diffusion. Diffusivity.** The rates of interdiffusion of pairs of Gases have been measured by enclosing them in the halves of a vertical cylinder with a diaphragm in the middle, the lighter gas being in the upper half. The diaphragm is cautiously slipped out and, after a definite interval, replaced, the contents of each half are analysed and the rate calculated.

It is found that lighter gases diffuse faster, hydrogen and marsh gas or hydrogen and air interdiffuse nearly five times as fast as air and carbon dioxide, the latter and nitrous oxide going one-third slower still.

Hydrogen travels through air about half as fast as heat through copper. Two inches of CO_2 at the bottom of a 2-ft. tall jar spreads uniformly through it in two hours.

Diffusion in Liquids is measurable by analysing the mixture at different levels in the tall diffusion jar of § 257 (though in

practice several more convenient experimental contrivances have been used). It has mostly been studied for salts in solution diffusing through water. A **Law of Diffusion** introduced by **Fick** in 1855 may be stated as follows :—

Taking two adjacent layers each 1 *cm. thick, the one containing* n *and the other* n' *mg. of dissolved substance per cubic centimetre, then* $F(n-n')$ *mg. pass from the one cubic centimetre to the other per second where* F *is the* **Diffusivity** (*or Coefficient of Diffusion*).

The transference in any actual case can be computed from this exactly as the quantity of heat conducted through a plate is calculated in § 180, F being the analogue of thermal conductivity and n of temperature.

§ 259. If n and n' were in layers x cm. apart the change of concentration in adjacent cubic centimetres (the motive force) would be only $1/x$ its previous value, and there being also x times as far to go, it would take x^2 times as long for the same quantity to flow across: hence the time taken to reach a certain concentration at a place is proportional to the square of the distance to be travelled (compare conduction, § 182).

Thus for example the CO_2 would have filled a little jar nearly uniformly in a minute of two, but it evidently requires some process far more violent than Diffusion to save us from stifling beneath a city atmosphere.

Stirring, which brings together portions of widely different concentrations in very thin streaks, hence ensures rapid and complete mixing by diffusion. Instance the streakiness on stirring together syrup (or whisky) and water, and its quick disappearance.

§ 260. Some approximate values of 86,400 F, i.e. the number of milligrams of dissolved substance travelling per square centimetre *per day* from a plane drawn in the water where it contains n mg. per c.c. to a parallel plane 1 cm. distant where it contains $n-1$ mg. per c.c., are as follows, at the temperatures stated, and increase rapidly with temperature :—

Hydrochloric and nitric acids	(20°) 86,400 F = 2·0 (50°) 4·0
Sulphuric acid	,, ,, 1·2
Acetic ,,	,, ,, ·64
Caustic potash	,, ,, 1·7
Potassium chloride	,, ,, 1·4
Common salt	(5°) ·76, (9°) ·91, (20°) 1·04
Alcohol	·5

Cane sugar	(10°)	·3
Albumen	,,	·06
Caramel	,,	·05
Gold in mercury	,,	·7
,, lead	(500°)	3·2
Benzene vapour in CO_2 at 20° . . .	F =	·06

§ 261. **Diffusion through porous diaphragms.**

Experiments on diffusion with fluids in open contact are so liable to be disturbed by currents that sheets of porous solids are commonly put between them to stop this wholesale mixing.

The spontaneous diffusion that goes on through these plates is to be carefully distinguished from the more familiar Filtration, Transpiration, or Effusion, § 89, in which any particles smaller than the pores are driven through pell-mell by a mechanical pressure, gas pressure for instance, or weight of liquid. This vitiating action has to be avoided by keeping the pressures on both sides of the plate equal throughout, if anything like accurate measurement is required.

It was with ' plates ' made from fine plaster, from well-baked china-clay, from compressed graphite, from meerschaum, etc., that Graham's classic experiments were made (ca. 1850). They show typically the greater speed of diffusion or ' Atmolysis ' of lighter gases. There is a striking lecture experiment in which an inverted porous battery-pot is sealed on the top of a long tube dipping in water. Over the pot is held an inverted bell-jar of hydrogen, a rapid stream of bubbles drives out of the tube. The bell-jar is removed and the water quickly climbs the tube as the hydrogen that has entered the pot diffuses out of it again faster than air can enter, even with the diminishing pressure inside in its favour. Again, sal-ammoniac vapour is passed through red-hot churchwarden pipe-stems, the pipes smell of ammonia while the gas that emerges at the far end reddens litmus ; the salt has split into ammonia (vap. density 8·5) and hydrochloric acid (vap. density 18·2) and the lighter gas has escaped more rapidly through the clay walls.

Graham's experiments led him to the **Law** that, other things being equal, *The rate of diffusion of a gas through the porous plate is inversely proportional to the square root of its density.*

This law tallies with Kinetic Theory. The molecules of gases at the same temperature possess, on the average, the same kinetic energy $\frac{1}{2}mv^2$, hence their speed $v \propto 1/\sqrt{m}$, see § 103. And since by Avogadro's law the number of molecules per cubic centimetre

of any gas at the same temperature and pressure is the same, m, the mass of one molecule, is proportional to the density of the gas; hence molecular speed varies inversely as square root of density. And everyone will admit that the rate of diffusion is proportional to the speed of the diffusing molecules. Hence the law.

Each gas present in a mixture on either side behaves independently of the others, simply on account of the characteristically different speed of its molecules. Each tends in time to reach the same partial pressure (i.e. molecular population density) on either side, but the lightest reaches equilibrium first.

§ 262. **Selective transmission of gases.** The diaphragms discussed above are porous in the ordinary sense, that a little pressure will drive any gas (cause it to effuse, § 89) through them. But there are several things, commonly regarded as quite 'air-tight,' which are permeable by particular gases. The red-hot iron walls of a stove are said to transmit poisonous carbon monoxide, they readily transmit hydrogen. Red-hot platinum is permeable to hydrogen only: a tiny blind-ended platinum tube is sealed to a 'vacuum tube,' § 669; heated in a spirit-lamp flame it at once admits traces of *pure* hydrogen. Thin india-rubber balloons blown with CO_2 soon collapse, and oxygen passes through them $2\frac{1}{2}$ times as fast as nitrogen, so that a toy balloon packed with sawdust to prevent collapse and put on a mercury pump slowly filters from the air a mixture much richer in oxygen.

Probably in these cases the gas actually *dissolves* in the solid, diffuses about in it and evaporates off from the other side. This must be how gases get through soap bubbles, as they quickly do. It will be recalled that the 'air' obtainable from solution in water is rich in oxygen, which is so much more soluble than nitrogen.

§ 263. **The diffusion of Liquids through membranes. Osmosis.** The passage of liquids through ordinary porous materials is a mere question of Filtration, which like Effusion is a gross mechanical process forced by pressure or induced by the capillary drag of surface tension. The chemist typically uses a filter-paper to retain undissolved and transmit dissolved substances; the porcelain tubes of the Pasteur or Berkefeld filters sterilize water because their pores are too small to admit bacteria, they have no power to deal with dissolved poisons.

We have seen that diffusion in liquids is a very slow proceeding

kinetically, on account of the dense crowd a molecule has to jostle through. It follows that 'porous' pots and papers are inefficient in studying it, the least alteration in pressure causing an infiltration that quite swamps its slow effects. Much less porous partitions must be used, 'water-tight' things like parchment, bladder, parchment-paper, etc. Liquid diffusion through these is designated **Osmosis**.

It is by osmosis that the living cell of plant or animal takes up its nutriment from, or gives out its elaborated or its waste products to, the watery fluids bathing its walls. Accordingly the most interesting part of the subject, and the most studied, to which we shall confine ourselves here, is that dealing with the diffusion of water and substances dissolved in it.

§ 264. Graham observed that parchment paper permits the passage of crystallizable substances ['**crystalloids**'] from solution on one side to weaker solution on the other, but does not transmit gum, albumen, starch, globulins, etc. ['**colloids**,' colla = glue]. On this he founded the process of **Dialysis**: a little drum, the 'dialyser,' containing mixed solutions, is floated on water; only the crystalloids pass through the parchmentized paper bottom. This process is useful in medico-legal work for separating traces of mineral poisons and alkaloids from the mass of colloids in the alimentary canal or tissues of the deceased, for colloids often mask chemical tests. And again, from ferric chloride solution the acid dialyses out leaving a solution of the colloid ferric hydroxide of greater therapeutic value than the original salt.

The ultimate particles of colloids in solution have in several instances been actually detected by the 'ultra'-microscope, and freezing-point determinations, § 273, show that their mass is never less than many hundred times that of a molecule of crystalloid. The natural explanation therefore appears to be that the colloid particles are too big to get through.

All the membranes used in the study of osmosis—parchment paper, copper ferrocyanide, etc.—are colloid in character. There is evidence that jellies consist of a sponge-like structure of colloid granules through which the liquid is dispersed. Crystalloids pass practically as readily through jellies as through water—instance the spreading of the red dye when 'raspberry' jelly lies on 'lemon' jelly—but colloids are much hindered.*

* The amœboid extravasation of leucocytes through the walls of the capillaries in the vicinity of a lesion is a proceeding on a far larger scale than that contemplated here.

The permanent suspension of the oil drops in an **Emulsion** is attributed to these granules of the colloid gum used in its preparation, these being large and numerous enough to greatly hinder the motion of the oil drops, though the latter are usually easy microscopic objects.

§ 265. It may be of interest to mention here the '**Brownian Motion,**' the ceaseless 'jiggling about' in liquids of solid particles which under the highest powers of the microscope are mere swarming dots. [Rub up a speck of umber in water and examine with high power.] According to kinetic theory the average energy of motion of all molecules in a mixture must be the same, whatever their sizes [and measures the temperature, § 218]. And this must hold good even for particles very many thousand times more massive, if set moving by the molecular hail. Now determinations of the mass and speed of Brownian particles of gamboge have been made recently under the microscope, and their $\frac{1}{2}mv^2$ averages 5×10^{-14} erg. That of the hydrogen molecule, calculated from kinetic theory, is 4×10^{-14} erg at the ordinary temperature.

So we may actually watch the movements of bulky partners in that dance the very existence of which has hitherto been a matter of pure faith with the physicist.

§ 266. **Osmotic Pressure.**

There is another way of studying diffusion through membranes, originating in an early observation made by the Abbé Nollet. He found that a bladder full of alcohol swelled and burst in water, while one of water collapsed when immersed in alcohol. The same happens with syrup instead of alcohol.

The diffusing water forces its way into the sugar solution even in spite of a pressure which increases till, if the membrane can sustain it, it reaches the maximum **Osmotic Pressure** characteristic of the solution and its concentration.

Domestic cookery affords excellent illustrations. Mushrooms sprinkled with salt slowly exude a dark juice which, boiled with spices, constitutes ketchup. The salt dissolves in their superficial moisture to a strong brine, the watery cell sap 'exosmoses' through the cell walls to dilute it. More salt dissolves and the process goes on till the cells are drained almost dry. Again, it is desired to stew some hard windfall apples. Cut up, covered with sugar and left overnight, there results a syrup on which float shrivelled pieces, tough as leather. On the contrary, cut up and

stewed in plain water the apples swell and their cells burst to a pulp which can now be sugared *ad libitum.* In the former case water passed from the unripe cell sap into the stronger syrup, in the latter case water 'endosmoses' into the acid sap until the cells burst.

The process can be followed under the microscope, using preferably cells with coloured contents, such as those of the filamentous algæ or of the beaded hairs on the stamens of the garden spider-wort (*Tradescantia virginica*). Examining under a high power, irrigate with strong brine or syrup. The protoplasmic lining of the cells, the 'primordial utricle'—the live cell itself—will be seen to leave the cell walls and contract as the water of the cell sap passes through it out into the strong solution. The cell is 'plasmolysed.' Irrigated now with fresh water it expands again, in fact the blue cells of the staminal hairs become more turgid and threaten to burst, like the apple cells.

Early experiments on Osmotic Pressure were those of Pfeiffer and de Vries. They soaked epidermal cells of the leaves of *Tradescantia discolor* in 1·2 % KNO_3 (saltpetre) solution; the cells reached a healthy equilibrium condition in an hour. They were then irrigated with various solutions and would show, by incipient plasmolysis, any variation corresponding to 0·1 % KNO_3. The following is an extract from their list :—

Equivalent to 1 % KNO_3 *solution* [*which is decinormal*] *are* :—		*Osmotic pressure of* 1 % *solution.*
5 per cent	cane sugar	·7 atmospheres
2·7 ,,	glucose	1·25 ,,
·58 ,,	common salt NaCl	6·1 ,,
1·4 ,,	glycerine	2·55 ,,
2·0 ,,	potassium citrate	1·75 ,,
1·8 ,,	magnesium sulphate $MgSO_4$	1·95 ,,
41 ,,	gum arabic	·085 ,,

§ 267. Solutions isotonic with the blood are of immense importance. 'Normal saline' must be used in the micro-examination of tissues. Lotions to be applied to inflamed surfaces should put no osmotic strain upon them. The sterile solution of salt or sugar transfused into the veins of a patient to stave off collapse from loss of blood must have a concentration equivalent to 1 % common salt. If stronger the corpuscles, etc., plasmolyse; if weaker, they may burst. Thirst is relieved by weak salt water, but nausea follows the plasmolysis of the gastric epithelium by sea-water.

And every swimmer knows how sea-water contracts the skin, leaving grime-retaining wrinkles on his hands ; while fresh water softens and swells the skin and dilutes the body fluids.

The crispness of a fresh green leaf is due to the turgidity of its cells, and this is maintained by the 'endosmotic' diffusion of the watery stem-sap into their more concentrated contents, up to an osmotic pressure of 20 atmos. or more. It takes a hard pinch to really damage the 'soft' tissue stiffened out with this pressure. Contrast the flagging leaf from which water vapour has transpired without renewal.

Loss of turgidity paralyses the cell. Hence it is that the micro-fungi—moulds, bacteria, etc.—although their capability of producing high osmotic pressures gives them enormous activity, can make no headway in well-boiled jam, for this represents a solution more concentrated than ('hypertonic' to) their cell contents, and plasmolyses them.

Further consideration of these actions in the organism discloses an apparent power of selection, for the living cell does not usually contain salts in the same relative proportions as the surrounding moisture contains them. Recent research, however, is finding a true physical explanation even of this ; one dependent on the formation of ionizable (§ 274) compounds between the colloids and crystalloids.

At present it may be stated generally that material travels as solutions of 'crystalloid' substances of small molecular mass, e.g. sugar 180, urea 60, etc., but is stored in the cells in the form of colloids (starch, etc.), of molecular mass many hundred times greater (cf. § 264).

§ 268. The cell mostly used for physical measurements of osmotic pressure is composed of a membrane of the colloid copper ferrocyanide, precipitated in the pores of a small porous battery pot to give it the needful mechanical strength. The ferrocyanide forms a thin layer in the middle of the wall as in the broken piece in Fig. 101. The jar is attached to a mercury gauge and is then filled with the solution, sealed up and plunged into water. The gauge rises hour by hour as the water slowly crowds in up to the high osmotic pressure of the solution.

A membrane such as this is called a **semi-permeable membrane,** for it will not transmit sugar and many other organic substances at all, while it is quite permeable to water. A well-made cell indeed transmits only small traces of the alkaline chlorides and nitrates, but one usually has to be content with less perfection

than this and to shun these *very* easily diffusible substances. [The exact mode of action of the membrane is obscure, it has commonly been regarded as a mechanical stopping of the larger molecules, but recent experiments have shown that membranes made with the use of alcohol are more permeable to alcohol than to water, so that the action appears selective, cf. § 262.]

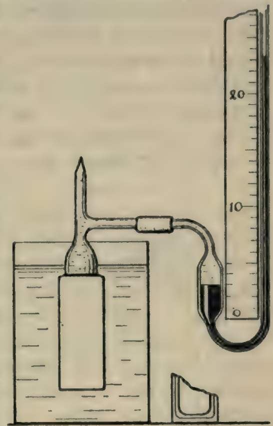


FIG. 101.

NOTE.—The rest of the chapter refers quantitatively only to DILUTE SOLUTIONS, very little has been made out about strong solutions.

With apparatus such as this it has been observed that—

(a) Solutions containing equal numbers of molecules of dissolved substance (i.e. weights proportional to Mol. Wts.) per litre have the same osmotic pressure, independently of the nature of this substance (e.g. borax and cane sugar).

(b) Osmotic pressure is proportional to number of molecules per litre.

e.g. sugar mols.	1/34	2/34	4/34	6/34
O.P. in cm. Hg	54	2×52	4×52	6×51

(c) Osmotic pressure is proportional to absolute temperature.

[(d) But solutions which conduct electricity have osmotic pressures higher than expected; and strong solutions always lower than expected.]

Quantitative experiments led **Van't Hoff** to point out that not only did (a), (b), and (c) resemble the gas laws of Avogadro, Boyle, and Charles, but that they were identical with them, and *the Osmotic Pressure exhibited by a substance in dilute solution is equal to the pressure which the vapour of the same quantity of substance, gasified, would exert in the same space as the solution occupies, and at the same temperature.*

e.g. Pfeiffer found that 1 % sugar solution at 7° C. gave a pressure of 50·5 cm. of mercury. For vapour ($C_{12}H_{22}O_{11}$) this would be 50·8 cm., reckoning that 1 mol. wt. (342) in 1 litre should give 22·32 atmos.

§ 269. Van't Hoff's Law suggests a Kinetic explanation of osmotic pressure.

It is perhaps illogical to suppose that the molecules of dissolved substance exert a pressure by impact in the same way as they would in gaseous form; for if so, why do not the vastly more numerous molecules of solvent exert a pressure great enough to burst any ordinary vessel?

We must argue in this way. The presence of molecules of dissolved substance leaves room for fewer molecules of solvent per cubic centimetre. There are therefore fewer solvent molecules striking the wall from the inside than from the pure solvent outside; on the whole there is a transference of solvent from without into the solution, while the dissolved substance cannot get out. This goes on until the pressure is crowded up enough to make simple ex-filtration equal to the osmotic influx. This pressure is simply equal to that characteristic of a swarm of molecules, equal in number to the molecules of substance, interspersed thinly enough (solution is dilute) to have no mutual attraction; i.e. to the gas pressure, see § 103.

§ 270. Let us therefore calculate the Osmotic Pressure of a solution containing n molecules of substance dissolved in 100 molecules of solvent (i.e. n times mol. wt. in grm. of substance in 100 mol. wt. in grm. solvent). n is in practice usually less than 1.

The volume of this mixture (neglecting the small addition caused by n) $=100\times$ volume of mol. wt. in grm. of solvent.*

1 grm.-mol. gasified at atmospheric pressure would occupy $22{,}300(1+t^\circ/273)$ c.c. Hence in the given volume n grm.-mol. exert, by Boyle's Law,

$$\frac{n\times 22{,}300(1+t/273)}{100\times \text{vol. of grm.-mol. of solvent}}\text{ atmos.}$$

$$=\frac{n}{100}\times\frac{\text{vol. of grm.-mol. of a gas}}{\text{vol. of grm.-mol. of liquid solvent}}\text{ atmos.}$$

$=$ **Osmotic Pressure of n per 100 molecular solution.**

As an illustration, taking water as solvent this works out to

$$n\times 22{,}300\times 76\div(100\times 18\div 1)=n\times 1000\text{ cm. Hg.}$$

10 *metres of mercury for a* 1 *per* 100 *molecular solution in water*, a pressure that no osmotic pot stands without leaking. Pfeiffer's 1 % sugar solution was a 1/342 in 100/18 $=\cdot 05$ molecular %.

* $=100\times$ mol. wt. $\div$ density.

§ 271. **Lowered vapour pressures of solutions.**

The surface of an evaporating solution of a non-volatile substance is an excellent sort of semi-permeable membrane. The substance cannot get out into the vapour (e.g. the distillation of pure water from sea-water), whereas all vapour molecules are able to pass back into the liquid as usual. Hence the concentration of molecules in the vapour over the solution is less than at the same temperature over pure solvent, wherefrom all molecules are able to escape. As the argument leads us to expect,

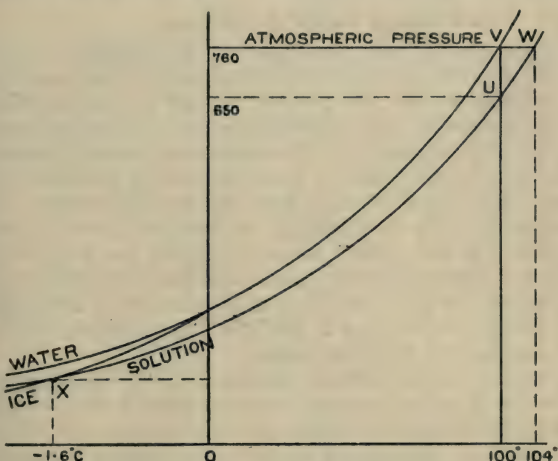


Fig. 102.

if among 100 molecules of solution there are n of dissolved substance, *the vapour pressure is diminished by n per cent of its normal saturation value.** Thus in Fig. 102 [which is purely diagrammatic and not to general scale] the upper curve represents the saturated-vapour-pressure temperature curve of pure solvent, Fig. 83, and the lower curve, everywhere 15 % below it, the vapour curve of a solution of $n=15$. [Experimentally this strength of solution is excessive.]

Ostwald verified this result of Raoult's by sending a slow stream of air first through the solution and then through pure

* Practically between ·96n and 1·1n according to solvent.

water. The loss of weight of the water bulb represents the additional water required to fully saturate the air, and expressed as a fraction of the weight of *all* the water vapour (collected in drying tubes) is equal of course to the fractional lowering of saturated vapour pressure due to the dissolved substance.

We should expect to find the atmosphere of the saltings less unpleasantly humid on the whole than that of the fresh-water marshes.

§ 272. **Raised boiling boints of solutions.**

To reach the full saturation pressure of the vapour the tendency of the (100—n) mol. to vaporize must be increased until equal to that of the 100 mol. of vapour to liquefy. This is done by increasing their molecular speed, i.e. by raising the temperature of the solution. The saturation pressure convenient in practice is 1 atmo., for that is the pressure of the vapour when a liquid visibly and steadily boils in an open vessel. Thus finding the rise in boiling point of the solution above that of the pure solvent is an indirect means of finding the solution's deficiency in vapour-producing power, and hence of finding n.

In Fig. 102 the 15 % deficiency of pressure VU at 100° is made up by heating the solution to 104°, its vapour pressure rising to the atmospheric at W, where it boils.

The precise relation between the deficiency of vapour pressure and the rise of temperature necessary to make it up is evidently settled by the slope of the curve near the boiling point. This of course has been determined by the experiments of § 208 on the solvent [the curve for dilute solutions, from which alone accurate results are obtainable, is closer and more parallel than solution curve in figure]. As explained in § 209, the curve is almost identical for all liquids, provided only that the temperature scale is stretched or compressed as a whole so as to bring the normal boiling point of the liquid under the 76-cm. pressure point on the curve. Measurement of the scale diagram, Fig. 83, will show that for 2° or 3° above the boiling point the vapour pressure increases 2·8 cm. Hg for 1° rise of temperature, so that *the* 1 % *molecular solution* with its 1 % deficiency of vapour pressure$=760\div100$ $=\cdot76$ cm. Hg *must be raised an additional* $\cdot76\div2\cdot8=\cdot27°$ C. *to boil it.* [The direct experimental figure for ether as solvent is ·284° and for carbon disulphide ·31°.]

In practice a very delicate thermometer is inserted in the

solvent, kept boiling as steadily as possible. Then the substance is put in and the rise observed. The thermometer must be *in* the liquid; the pure vapour leaves the solution superheated (unsaturated) but cools probably in the first centimetre of its path to its normal temperature of saturation, which it maintains by partially condensing, i.e. in the body of the flask it is no hotter than before, cf. § 145. A reflux condenser returns the boiled-away solvent to keep the strength of the solution constant.

§ 273. **Lowered freezing points of solutions.**

When a dilute solution freezes pure ice separates out [in fact freezing is the easiest way of preparing water of the utmost purity from ordinary distilled water]. Now the vapour pressure of the subliming solid, pure ice,* is less than that of 'undercooled' water* at the same temperature; the solid bondage of the molecules hinders them escaping more than does the liquid bondage, and the ice vapour-pressure curve slopes back from 0° more steeply than the water, Fig. 102. Thus it presently cuts the solution curve at X and this is at the freezing-point temperature of the solution.

For at temperatures to the right of X a piece of ice placed on the solution has a vapour pressure higher than the solution's, and will evaporate, and vapour will condense into the solution. The total rate of escape of the 100 mol. from the ice exceeds that of the $100-n$ from the solution. *Per contra*, below X, vapour from the solution would deposit on the ice which would therefore grow, i.e. the liquid continuously freezes. X is thus the only point where ice and liquid exist in equilibrium together, as many molecules leaving the liquid and re-precipitating on the ice as leave the ice and drop back into the liquid.

The same equilibrium holds for the submerged parts of the ice, for if it did not we might have ice and solution at a perfectly uniform temperature throughout, and the liquid evaporating and continuously 'snowing the ice under' from above while it continuously dissolved it from beneath, a Perpetual Motion.

The slope of the ice-vapour line having been found by experiment the result is that it cuts the vapour-pressure curve of a

* Taken throughout as types of the solid and liquid states of the solvent, whatever it may be.

1 mol. per 100 mol. solution at a temperature below the freezing point for various solvents as follows :—

Solvent.	Mol. Wt.	Freezing point.	Depression for 1 mol./100 mols.
Water	18	0°	1·05°
Benzene	78	5·4°	·65°
Acetic acid.........	60	16·5°	·65°
Formic acid	46	8°	·60°

§ 274. Summarizing, for a solution containing n grm.-mol. of substance dissolved in 100 grm.-mol. of any solvent :—

Osmotic Pressure at any temp. $= \dfrac{n}{100} \times \dfrac{\text{volume of grm.-mol. of a gas}}{\text{ditto of liquid solvent}}$ atmos.

Lowering of Vapour Pressure at any temp. $= \dfrac{n}{100} \times$ vap. press. of solvent.

Rise of Boiling Point at atmospheric pressure $= n \times \cdot 28°$ C.

Lowering of Freezing Point

$= n \times \cdot 65°$ C. for several organic solvents
or $n \times 1\cdot 05°$ C. for water.

The last two, which are experimentally easier, are used by the chemist to find molecular weights. Taking 100 mol.-wt. of solvent in centigrammes, say, and adding w cg. of substance, the change of temperature observed is divided by the ·28° or ·65° and gives n, the number of molecules added.

$\therefore$ Mol. Wt. of substance $= w \div n$.

§ 275. For reasons imperfectly known the above general laws are not widely obeyed with the first-rate accuracy that would give them high physical value.

A very great discrepancy is observed with solutions which can conduct electricity, solutions of electrolytes, § 646. They appear to contain when dilute, double the expected number of molecules. e.g. sea-water is a 3 % salt solution $= 3/59$ in $100/18 =$ roughly 1 mol. NaCl per 100 mol. H_2O and should freeze at $-1\cdot 05°$ C. It actually freezes just below $-2°$ C. And with calcium chloride, with 3 atoms in the molecule, nearly 3 times the expected change is observable.

From this has arisen the **Ionic Theory** that salts in dilute solution spontaneously split up into their atoms, electrically charged, called Ions, acting kinetically as molecules and electrically as the carriers of electricity through the solution, see § 647.

EXAMPLES.—CHAPTER XXIX

1. Describe experiments to test whether the diffusion of gases has any connection with their densities, and explain how the experiments do test the question. [L]m.

2. A closed porous pot full of air is provided with a manometer (pressure gauge). Describe its indications if the pot is suddenly plunged and kept in (a) coal gas, (b) carbonic acid. Explain on the kinetic theory. Suggest an application in mining.

3. On what does the rate of diffusion depend in (a) liquids, (b) gases? [L]m.

4. Define the coefficient of diffusion of a salt in a solution, and explain how it can be found experimentally. [L]m.

5. Show how diffusion between a solution of a substance and the pure solvent can give rise to osmotic pressure. Calculate it for 1 grm. glucose (mol. wt. 180) in 100 c.c. of solution just above 0° C.

WAVE MOTION

Chapter XXX

PERIODIC MOTION

§ 276. The periodic motion of a particle.

The Simple Harmonic periodic motion of a particle has been introduced in § 39 (after pendulum) and the results of that section must here be recapitulated.

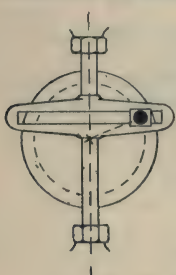


Fig. 103.

A particle which moves in a straight line under the action of a force attracting it to, and proportional to its distance from, a fixed point in the line, moves with **Simple Harmonic Motion**. *This is the motion, on any diameter of a circle, of the foot of the perpendicular dropped on the diameter from a point moving with uniform speed round the circle.*

In a common type of steam pump, Fig. 103, the vertical piston-pump-rod bears a long slotted cross-head in which a crank-pin works to drive a fly-wheel, etc. Assuming the fly-wheel speed constant the vertical motion of the pump is evidently a S.H.M., for the slot is the perpendicular to the vertical diameter. And the right and left motion of the pin in the slot is another S.H.M.

In the circular diagram marked like a clock-face in Fig. 104 the dots on the vertical diameter evidently show the positions of the point moving in a vertical S.H.M. at equal intervals of time. But it is more graphic to put each on a diameter of its own, spaced horizontally at equal *times* apart as shown, and so produce the **Sine Curve** [or Cosine Curve]. This curve would be obtained by carrying a card horizontally past a pencil on the pump-rod, or by sliding a plate at uniform speed below and at right angles to a pendulum whose bob is a can of sand with a hole in the

bottom. It is roughly attained by the small boy as he ambles beside a wall, chalk in hand.

The following particulars of a S.H.M. must be defined :—

The **Amplitude** is the maximum distance from the centre. It is the *radius* of the circle ; half the length of a pendulum swing ; the height or depth of the curve from the centre line.

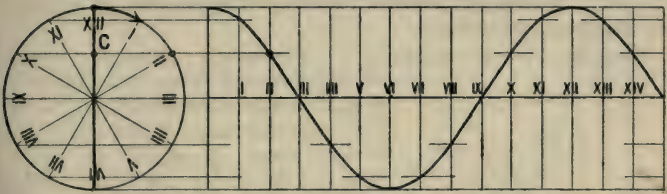


FIG. 104.

The **Phase** of the particle expresses its position at any moment. It is usually defined by the angle the corresponding point in the circle would have moved from its starting-point, e.g. the point C (Fig. 104) is either in phase 60° (II o'clock) when moving downwards or in phase 300° (X) when moving upwards.

The particle passes through all phases once in each completed motion.

The **Period**, *Periodic time, or time of vibration or oscillation,* is the time taken to complete one whole motion, i.e. the interval of time between two successive passages of the particle through the same phase.

The complete vibration or oscillation is the whole motion there and back, e.g. the ' time of oscillation ' of a ' seconds ' pendulum is 2 sec., each single ' swing ' or ' stroke ' occupying 1 sec. Never call a single swing a vibration or oscillation, for this has caused much confusion.

[The Frequency is the number of vibrations per second ; it is the reciprocal of the periodic time in seconds.]

The S.H.M. is of importance for these reasons :—

(1) Of all vibratory motions it is the most easily and naturally produced (e.g. pendulum, and elastic vibrations controlled by Hooke's law).

(2) It is by far the simplest to study scientifically.

(3) *Any periodic motion whatever can be analysed into, or built up as the resultant of, a series of S.H.M.'s.* For instance, the violent motion of a shuttle, or of a ball bounced on the pave-

ment, such as drawn out on a moving time sheet would give curves like Fig. 106 (H, K), or indeed any sort of wriggles, zigzags, saw-teeth or battlements (provided they do not overhang and require time to go back on itself).

The analysing process is too difficult for us here, but let us see how the building-up can be done.

§ 277. **Compound harmonic motion. Combination of S.H.M.'s in the same straight line.**

Suppose a body acted on by two periodically varying forces each of which would cause it to move in S.H.M. in the same straight line. At one moment say, under the first action alone it would be at A, Fig. 105, or under the second alone at B. Actually it will be at C, where OC$=\pm$OA$\pm$OB, the minus signs being taken for distances below the centre. The reader will see easily enough that representing A and B as the projections of *a* and *b* and completing the usual parallelogram, C is the projection of *c* and O*a*, O*b*, O*c* are the amplitudes and ZO*a*, ZO*b*, ZO*c* the phase angles of the two component and the resultant motions respectively.

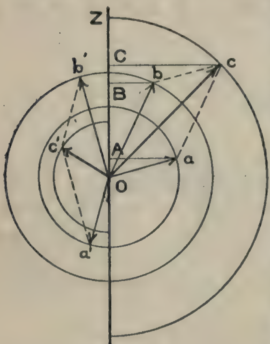


FIG. 105.

If the periods are the same O*abc* does not change its shape but goes round solid, *c* moving in a circle, i.e. the resultant of two S.H.M.'s of equal frequency of vibration is still a S.H.M. If O*a* and O*b* are in line the resultant amplitude is their sum, but as drawn is less than this for the two components do not agree in phase, and if O*a*, O*b* are 180° apart, i.e. in opposite phases, the resultant amplitude will be a minimum, their difference, or OC $=$O*b*$-$O*a*. On the left of the figure is shown their composition with 150° phase difference.

But if the two S.H.M.'s are *not of the same period* O*a* and O*b* go round at different angular speeds, one always gaining in phase on the other (e.g. minute and hour hands), and the shape of O*abc* continually changes, *c* does not move in a circle and the resultant motion of C is a compound harmonic motion, worked out as in Fig. 106 (A, B).

The curve C in the figure is got by adding the heights of the

two curves A B above the centre line, depths below are minus. Three or any number of S.H.M.'s may be similarly compounded.

Compound harmonic curves fall into two general types:—

(a) If one component is much the strongest and slowest the result is that this persists, merely battered by the others from the simple sine shape into some sort of regular zigzag. Such is the air motion produced by a musical instrument. In Fig. 106 F and G show resultant motions obtained by adding, to a

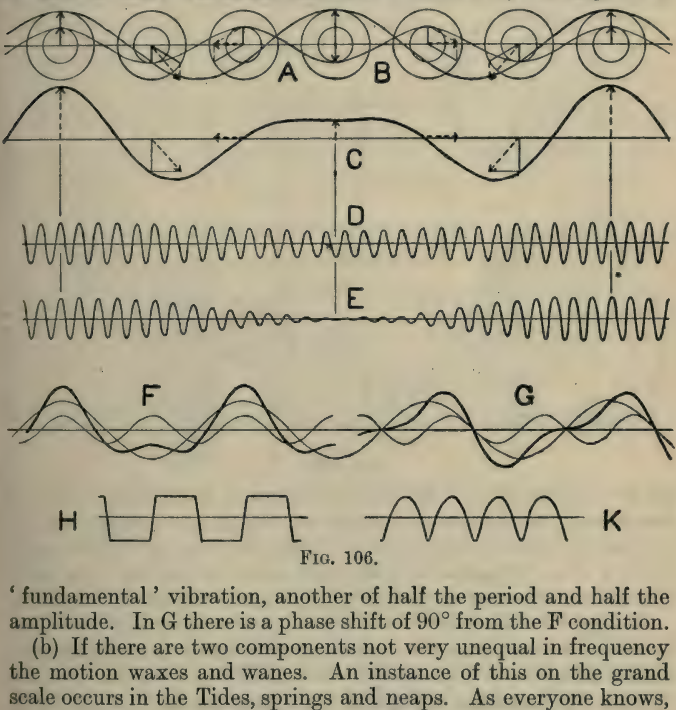


FIG. 106.

'fundamental' vibration, another of half the period and half the amplitude. In G there is a phase shift of 90° from the F condition.

(b) If there are two components not very unequal in frequency the motion waxes and wanes. An instance of this on the grand scale occurs in the Tides, springs and neaps. As everyone knows, these are due to a solar pull of period 12 hours and a lunar pull producing 5 times the amplitude and of period about $\frac{30}{29}$ of 12 hours.

In the Tide Machine, Fig. 107, two wheels rotating in times proportional to these carry cranks adjusted to the proper ratio of lengths (amplitudes) and set for starting in phases appropriate to

the particular day. Moving in slotted cross-heads the crank-pins give the proper vertical S.H.M.'s to two pulleys, and a wire over these adds up and writes the resultant curve on a moving roll of paper. Fig. 106 (D) represents it for a fortnight.

Astronomical irregularities in the motion of the sun and moon complicate matters, but the tidal rise and fall recorded at a port for many years can be analysed into about two dozen S.H. components, they are set upon a similar number of crank-wheels

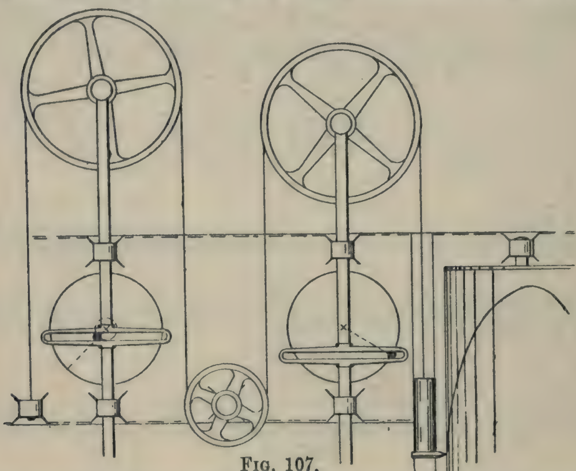


FIG. 107.

on the tide machine at the National Physical Laboratory, and the bewildering mechanism grinds out in a few hours a complete forecast of the shallow tides on some Indian harbour bar for the next three years. Unfortunately the tide machine that can cope with the additional vagaries of British wind and weather is not built.

In Fig. 106 (E) there is the same relative frequency but the amplitudes are equal, the vibration dies down to nothing and rises to double. See Beats, in Sound, § 313.

§ 278. **Composition of S.H.M.'s at right angles.**

In that delightful instrument, the Harmonograph, described in all works on popular science, a pencil is pushed north and south by a light rod connecting it to one heavy pendulum and east and west by another pendulum. Typical curves that it draws are shown in the Fig. 108. A shows the combination of two equal S.H.M.'s at right angles with the stated difference of phase.

Notice among the curves the straight lines for 0° or 180° and the circle for 90°. It is this circle that was resolved into the two equal S.H.M.'s at right angles in the donkey-pump, one just starting as the other is in mid-swing.

B shows a S.H.M. combined with one of twice its period and initially 60° phase difference. There are many other curves depending on the ratio of the two periods. With ratio 2 : 1, as drawn, the tracing point completes two cross-journeys during

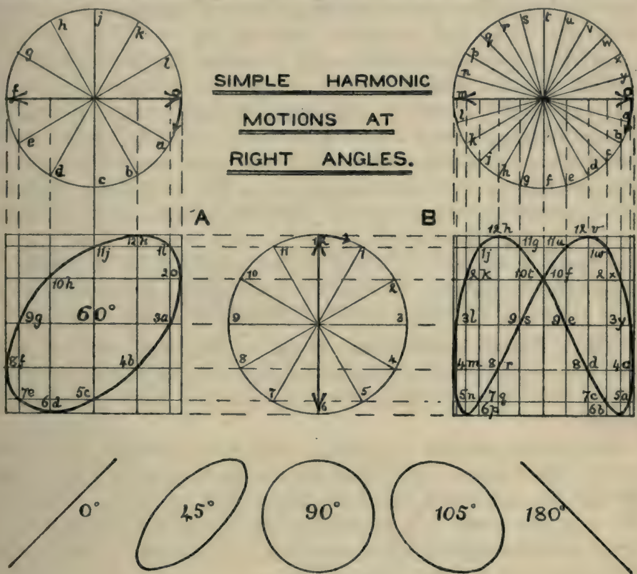


FIG. 108.

each vertical journey; with 3 : 2 it would make three during each two verticals, and so on. If the periods are not in exact ratio the figures go to and fro through all their changes every time that one motion gains a whole vibration over the other. It is this inexactness, together with the gradual dying down of the motion, that gives their interwoven beauty to the harmonograph curves. And see § 314.

§ 279: **Speed and energy of a particle vibrating in S.H.M.**

We have defined the force on m (hence its *acceleration*) in a

S.H.M. as proportional to the distance from the centre, i.e. to cosine of phase angle. By resolving the speed in the circle parallel to the diameter it will be found that the *velocity* in the S.H.M. varies proportionally to the length of the perpendicular, i.e. as the sine of the phase angle. It is greatest in the middle (when acceleration$=0$) and then$=$the full speed in the circle.

The energy of particles of mass m vibrating in the same periodic time will be proportional to the square of the amplitude, for doubling the latter means doubling the speed necessary to traverse it in the time, and therefore quadrupling v^2. The particle's energy is partly kinetic and partly potential due to displacement against the controlling force. In fact it changes to and fro between wholly kinetic in mid-swing when the speed is greatest and controlling force is inactive, to wholly potential at the end of swing when the speed is zero and the distance pushed out against the controlling force greatest [e.g. in the pendulum, distance lifted against gravity greatest]. The Total Energy then is always equal to $\frac{1}{2}m\times$ square of speed at mid-swing. Now this is the speed in the circle (which is there parallel to its diameter) and $=2\pi\times$ amplitude$\times$ revolutions per second. Hence the energy of a vibrating particle$=2m\pi^2\times(\text{amplitude})^2\times(\text{frequency})^2$.

§ 280. **Forced oscillations.** So far nothing has been said as to how the oscillations were originated, and they have gone on freely under their own natural controlling forces in their own natural period.

Experience assures us that it was some outside force that started the motion. It further assures us that using forces great enough we can make any body move how we like. A load on the rope of a crane is a pendulum, but the skilful driver slews it round and deposits it where required without much bother from oscillation and without undue delay.

Now what of the condition of affairs intermediate between this close artificial control and free natural oscillation ?

It is a state of oscillation more or less modified by external forces, a state of **forced oscillation.**

Push a child in a swing. Holding it, you can walk slowly backwards and forwards. Increase the speed, and you become aware that the thing has a tendency to swing of itself ; sometimes it moves easily, sometimes pulls you along, at other times it resists with unexpected force. With hard labour you have it swinging with the frequency you choose, but it will probably have you down before reaching the amplitude the youngster demands.

But be guided by the swing itself, give it push after push always at the right time, and with little effort you get an ample oscillation *practically in its natural periodicity.*

Try again to drive it with a higher frequency and your utmost exertions hardly shake it a yard.

Another experiment is this: hold up a simple pendulum, oscillate your hand horizontally with different frequencies, and

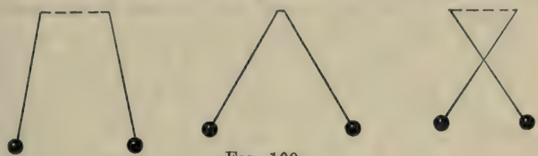


FIG. 109.

observe to what comparative extents the bob swings for each; see Fig. 109, notice the large motions of the hand when too slow (left fig.) or too fast (right fig.).

§ 281. These exemplify a perfectly General Principle:—

A body can be forced to oscillate in any period, but the forces required become less and less the nearer that period is to its natural one.

Or conversely, when a force of fixed magnitude is applied periodically the body will oscillate in that period, but the forced oscillations become large only when near the natural period of free oscillation.

There, they often increase enormously, and there is said to be **Resonance** between the vibration and the applied forces. The term is borrowed from Sound: acoustic instances of this mechanical action are given in §§ 323–5.

A familiar annoyance arising from mechanical resonance is the exaggerated jumping vibration of the railway carriage at one particular speed, that which happens to bring the rail-end jolts 'in step with' the natural frequency of bouncing of the carriage on its springs.

In a mechanical illustration of imperfect resonance two equal simple pendulums hang, a foot apart, from a short 'tight rope.' The first, set swinging crossways, pulls its point of support, and the whole rope, to and fro a little, and thereby gradually sets the second swinging strongly, adding up a succession of impulses always in the same phase as itself. But if the second pendulum is a little longer, the second impulse from the rope comes, say,

1 % too soon for it. On the next swing it is 2 % too soon and so on. The pendulum adds these impulses together as a succession of S.H.M.'s 3·6° apart in phase, and increases its swing up to the fiftieth. But the fifty-first is half a period too soon, it pulls in direct opposition to the first, and so on, the succeeding impulses up to the hundredth wiping out the effect of the first fifty. The pendulum therefore keeps on getting up a small swing and dying down again, and this imperfect resonance can never cause a strong movement.

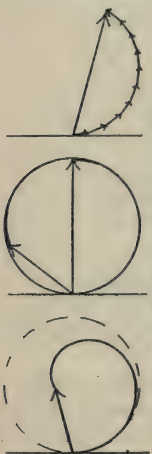


FIG. 110.

NOTE.—The adding up of successive equal impulses with a constant phase difference is easily effected graphically, as in Fig. 110. Little equal vectors representing the impulses are joined tail to head, each succeeding one turned through a small angle = common phase difference, and the straight closing side of the polygon thus formed gives the magnitude and phase angle of the resultant. With 100 small impulses the polygon becomes a practically continuous circle, with a maximum resultant 0–50, min. 0–100, max. 0–150, and so on. If the impulses gradually become weaker the polygon curls gradually closer into a spiral.

§ 282. Effect of 'Damping' on Resonance.

An oscillatory motion which gradually dies away owing to its energy being either spent in overcoming friction or 'radiated' out as vibration of the supports, sound, electro-magnetic waves, etc., is described as 'damped.'

Without air friction a clock pendulum would get up an indefinitely great amplitude as it continually added up the effects of impulses always in phase with its natural swing, and never lost anything. 1 part in 1000 away from this, only 500 impulses would be accumulated before they had drifted round into opposition and begun to destroy the motion. The difference due to this imperfect resonance is the difference between an indefinitely great number and 500, that is :—

With but slight damping, resonance is strong and its position very sharply marked.

But if after a dozen pushes or so a swing had been worked up which takes nearly all the applied force to keep up its vigour, constantly sapped by friction, etc., the difference between the

two previous cases quite disappears. In fact, a 'mis-tuning' of 1 in 50 would still supply enough impulses to work up the full resonance possible, only a small fraction of the maximum obtainable with good tuning and little friction.

With heavy damping, resonance is weak and its position not very definitely marked.

Fig. 111 shows the difference between the resonance of a tube to a tuning-fork, and the poor sort of 'tuning' obtainable in wireless spark-telegraphy, where the currents radiate or dissipate all their energy in two or three oscillations. (The abscissa represents frequency and the ordinate the intensity of resonance.)

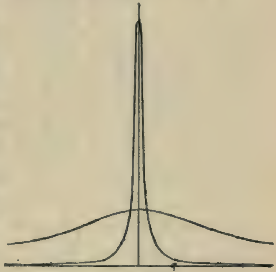


FIG. 111.

It is largely because a pendulum loses so little energy in friction that it so strictly regulates a clock to its natural period only. Another consideration, however, comes in :—

Extent of possible forcing. In any particular case the extent to which oscillations can be forced away from their natural period depends on the force available per unit inertia to be overcome.

A cheap clock wags its flimsy pendulum faster when just wound up (and try winding your watch up three times a day for a week) but a spring clock with a heavy pendulum is far less affected (may even lose from increased arc, § 38). By overblowing, one can falsely sharpen the pitch of a cornet, but not of a bulky organ-pipe.

EXAMPLES.—CHAPTER XXX

1. Define S.H.M. A particle describes S.H.M. amplitude 6 cm., period 8 sec. Draw to scale positions of particle at ends of successive seconds. [M.]

2. Define S.H.M. and show its period = square root of acceleration at unit distance from centre. [L.]

3. Prove that if P moves on a circle (centre O) with uniform speed, its projection N on a fixed diameter AOB moves so that its acceleration is always towards O and proportional to ON. Find what fraction of the periodic time of the motion is occupied by the passage of N from the middle point of OA to the middle point of OB. [L.]

4. Show how to combine two S.H.M.'s in different directions ; how would you experimentally illustrate such a combination ? [L.]

Chapter XXXI

WAVE MOTION

§ 283. **Wave Motion.** Suppose a long row of particles connected by some means which can transmit a force from one to the next, a long line of angler's split shot, for instance, strung an inch apart on a thread of the thinnest elastic, with an inch left at the beginning. Pull this in any way you like, and so displace the first shot. As it moves it gradually stretches the next inch of elastic, which begins to pull on the second shot, i.e. to impart momentum to it. It moves, and stretching the next inch of thread, begins to hand on momentum to the third, and so on, and soon every particle in turn is performing the same motion as its neighbour before it, but a little later. An alternating pull on the end sets up a typical *running wave motion, caused by every particle in a series performing exactly the same periodic oscillation; but each later, or lagging a little in phase, behind its neighbour on the side whence the motion arrives, while it equally leads the oscillation of its farther neighbour.*

The stronger the elastic links the less they stretch to transmit a given force, and the quicker and with the less phase difference the successive particles have to respond, but the heavier the particles the slower they get into motion, and the greater their phase lag. In fact the speed of travel of every sort of wave depends upon (is the square root of) the quotient of a quantity analogous to elastic force by a quantity analogous to mass.

To the definitions given concerning the motion of a single particle must now be added the following :—

The **velocity of travel** V of the wave is the speed with which any one selected wave form travels forward.

The **wave length** is the distance between two successive particles in the same phase of their motion, e.g. between two crests (0°) or between two points such as PQ (phase 315°), Fig. 112.

In order that a succession of waves of length L may continue to spread from a source vibrating n times per sec. [period $T = 1/n$th sec.] the first wave must travel away a distance nL in the second,

to leave room. $\therefore V = nL$ or the Speed of travel of waves = frequency × wave length.

A **wave 'front'** is a theoretical surface drawn through all adjacent particles which are in the same phase.

§ 284. **Water Waves.** Most familiar of all wave motions is **the deep-water wave.** It is a commonplace that floating weed only sways about while the wave form rolls on, but the oarsman or the swimmer has a much more definite impression. Swimming to

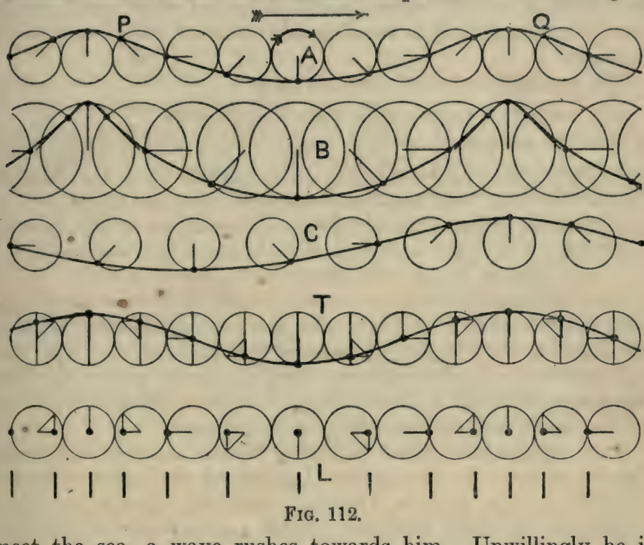


FIG. 112.

meet the sea, a wave rushes towards him. Unwillingly he is drawn forward to meet it, but just as its half-yard or so of dread altitude looms before him, blotting out the view and walling him in a vale of utter loneliness, he is lifted right up, apparently only condemned to receive a pleasant mouthful off the crest. But no; time is gained to lift him that last few inches by an unexpected retreat before the crest, he is already caught and borne back in the wave and suffers a loss of headway, until half-way down its back starts a swift swing through the shallow trough to meet the next comer.

All this is consistent with the water particles revolving in vertical circles with fixed centres, and moving at the top in the direction of travel of the wave. At A in Fig. 112 selected equi-

distant drops are represented by the dots moving in successive circles with a phase lag of 45°. B shows the effect of an increased amplitude without corresponding increase of wave length, the crest becomes more peaked till ultimately it is bound to break into white horses. Diminished amplitude gives low round tops such as characterize the ground swell into which storm waves die down as their violence abates. Wide but slow revolutions produce a heavy swell, adjacent circles would differ less in phase than in A (or the 45° circles would be spaced wider apart, as C): though the amplitude is considerable the wave length and speed are great.

The speed of travel of these waves is

$$\sqrt{(\text{gravity} \times \text{wave length} \div 2\pi)}$$

or *speed in miles per hour* $= 2{\cdot}7 \times$ *square root of wave length in yards* or in *knots* $= 3{\cdot}3\sqrt{\textit{fathoms length}}$. The amplitude of the disturbance has become very small at a depth of one wave length.

In **shallow water** the circles flatten into ellipses as the up-and-down supply of water is limited, and the speed decreases to

speed in m.p.h. $= 4 \times$ *square root of depth in feet.*

Presently the backward movement at the bottom of the ellipse is so much hindered by friction on the bottom that the front of the wave is starved for water, and the crest topples over the hollow face which shows almost the path of the particles. The last wave shows the flattened elliptic motion as the heaving surge up the beach and the subsiding backward scour.

Tiny ripples are called Capillary Waves and are controlled almost entirely by the surface tension of the water (Chapter XXVIII). The surface vibrates something like a stretched membrane or string and the ripples approximate to the type about to be described.

§ 285. Waves of transverse motion.

The next type of wave is seen in a jerked rope fast at the end, or a vibrating string. Here there is very little lengthwise motion possible, and all particles move simply *across* the direction of travel of the wave, up and down along the diameters as the vertical components only of the constructional small circles in Fig. 112, T. They need not be actually confined to these *lines*, but seen from the side must appear to be. They include not only the up-and-down waves of a shaken rope or tablecloth, or the straight-line vibrations of plane polarized light, but also circular ' skipping-rope ' motion or the irregular vibrations of ordinary light, which

are merely *confined to planes transverse* to the waves' travel. The typical wave form is now a sine curve like Fig. 104, but recollect that that was a diagram on a *Time* base, whereas now both co-ordinates represent lengths, and the whole might be obtained as an instantaneous photograph.

§ 286. **The speed of travel of waves along a stretched string** is found thus :—

Suppose a complete circular ring, Fig. 113, such as one can easily throw along a rope on the ground, and now suppose that the string is being hauled back just as fast as the ring runs forward. Then we have a ring of rope which maintains its position, in space, but whose circumference is travelling round at speed v. By § 40 there is tension in it, just as in the rim of a fly-wheel, of mv^2. This must $=$ T the pull along the string in dynes, or else one would overcome the other and upset the equilibrium.

Fig. 113.

$$\therefore\ v=\sqrt{\frac{T}{m}} \text{ or speed in cm. per sec.} = \sqrt{\frac{\text{pull on string in dynes}}{\text{mass of 1 cm. of string}}}$$

Now there is no need for the ring to be complete, for the tension is the same in every bit of it, and nothing has been said about the radius of the ring, which may therefore be anything and vary anyhow ; i.e. a distortion of any shape whatever travels on the string at the speed we have found.

§ 287. **Waves of 'longitudinal' motion.**

In the third type of wave the only motion of the particles is to and fro along the line of travel of the wave itself.

In Fig. 112 L, the particles perform their little harmonic movements along the *horizontal* diameters of the little circles, and become crowded together and scattered alternately, and pass on waves of compression and rarefaction at a speed far greater than their own motions. Such waves can be seen running up and down a long vertical wire helix (' spiral ' spring) when its end is pulled straight down and let go ; the spires close together and open apart periodically. They run on a piece of stretched rubber tubing slipping jerkily back through wet fingers : they produce a shrill sound when a glass rod is rubbed lengthwise with a wet leather : they travel in air or any other substance as the longitudinal waves of compression and rarefaction conveying **sound.**

It is a slow solitary wave of this type that passes along a checked goods train, and occasionally a few to-and-fro impulses of it can be felt by anyone standing in a long passenger train as it starts.

§ 288: **The speed of longitudinal waves** is calculated thus:—

A and B are two planes 1 sq. cm. in area moving at speed V and maintaining fixed positions in the wave (just as the fore-and-aft edges of a ship's rudder do in her stern wave: this by way of a rough illustration, but the wave now under consideration is of a very different kind). For this, the same mass m enters the space AB per second at A as leaves it at B or else AB's contents would vary in quantity, i.e. it would be moving about in the wave. Let u and w be the actual very small forward speeds of particles at A and B due to the compression somewhere behind B. A therefore catches up sparser particles at greater speed V—u and B closer ones at V—w. Divide the speed, which = the volume caught per second by the square-centimetre plane, by the volume that contains 1 grm. [$=a$ at A and b at B] and we get the mass caught $m=(V-u)\div a$ and lost $m=(V-w)\div b$.

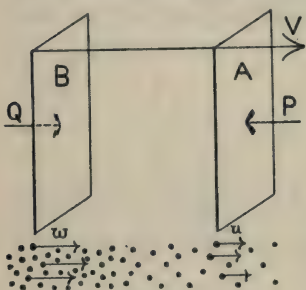


Fig. 114.

$$\therefore\ u=V-am. \qquad w=V-bm.$$

There is another condition of permanence: the resultant force constantly acting on AB is equal to the increase of momentum that takes place inside it per second.

The force is the forward difference of pressures Q—P on the square-centimetre planes, and during each second a total mass m which at A always moved at speed u has been increased in forward speed to w.

$$\therefore\ Q-P=m(w-u)$$
$$=m(V-bm-V+am)=-m^2(b-a)$$
$$\therefore\ -m^2=\frac{Q-P}{b-a}=\frac{\text{increase in pressure}}{\text{increase in volume of 1 grm.}}$$

But under Elasticity, § 96, the **modulus of elasticity** E was defined to be the ratio of the increase in pressure to the decrease

in volume it causes per cubic centimetre, i.e. per volume of D grammes [D=density].

$$\therefore -m^2 = -E \times D.$$

Now m=c.c. caught per sec.×mass of each=V×D (slight increases in density compensating the $-u$ and $-w$).

$$\therefore V^2D^2 = ED \qquad \therefore V = \sqrt{\frac{E}{D}}$$

or the speed of travel of a longitudinal wave is the square root of the quotient of the Elasticity of the medium by its Density.

This applies to anything, from rarefied hydrogen to a goods train. And see § 311.

§ 289: **Energy carried by waves.**

A wave train carries energy. One can do work at the far end of a rope or throw up water at a distance by setting up a wave motion. Elastic air waves carry sound, or sometimes the sudden energy of explosions. We saw, § 279, that the energy of a vibrating particle$=\frac{1}{2}mv^2=2m\pi^2a^2n^2$, and now in wave motion the mass of a single particle has to be increased to the whole mass of all the particles set into equal motion per second, giving

Power=energy conveyed by wave train per sec.
=total mass newly disturbed per sec.$\times 2\pi^2a^2n^2$.

Or the energy received by a surface per second from the waves of a train or column v in length and equal to the surface in area of cross-section, which fall upon it and are reduced to rest

=wave velocity×area of surface×density of medium
$\times 2\pi^2 \times$ (amplitude)$^2 \times$ (frequency)^2.

§ 290: **Pressure of a wave train on a surface.**

Let a continuous train of waves fall upon a surface which absorbs all the energy it brings, quieting the waves to rest as they strike it. Push 1 sq. cm. of the surface forward 1 cm., it acquires and stores the energy contained in an extra cubic centimetre which it now shields against the oncoming stream. It will give *you* that energy if you let it drop back 1 cm. [it will not give it back to the stream, for that is fed from the wave source] and it will do so by pushing on your hand with a pressure (which the stream exerts on it) which multiplied by the space pushed through, 1 cm., =energy per cubic centimetre. Hence an energy-conveying wave stream presses on an absorbing surface with a pressure in dynes per cm.^{2} equal to the energy in ergs per cm.^{3} in the stream. (A reflecting surface which flings all the motion back again suffers

double the pressure.) This pressure has actually been detected and measured in brilliant light, though exceedingly minute :—

Bright sunshine brings to a black square centimetre one-thirtieth of a calorie per second at a speed 3×10^{10} cm./sec. What pressure does it exert ?

Energy per c.c. $= \frac{1}{30} \times 4{\cdot}2 \times 10^7$ ergs $\div 3 \times 10^{10}$ c.c. (since the sq. cm. sunbeam fills (its speed) c.c. per sec.)
$=$ pressure $= {\cdot}000047$ dyne per sq. cm.

§ 291 : **Spread of energy.** Broad plane waves, except for a little diffractive fraying at the ends, § 295, travel on with undiminished energy per foot of ' front.' Such are the waves of sound in a pipe. But waves which broaden out as they travel forward and have to spread their energy over a wider front will then of course be carrying less energy per foot of front. Ripples in widening circles from a stone carry energy per foot width which is inversely as their radii. Light and sound waves spreading spherically carry amounts of energy per square centimetre of front inversely as the square of their distance from the source, since the areas of the growing spheres are 4π times the squares of their radii, and area $\times$ energy per unit area $=$ constant $=$ total energy contained in one wave, see also § 355. Here the amplitudes ($\propto \sqrt{\text{energy}}$) are inversely as the radii.

The energy carried through a square centimetre per second is the strict physical measure of the loudness of sound or the brightness of illumination.

EXAMPLES.—CHAPTER XXXI

1. Explain how energy may be transmitted by means of wave motion, with particular reference to sound waves in air. [D]m.

2. Define the amplitude, velocity, period, wave length, and frequency of a series of waves, and give relations between them. [L.]

Chapter XXXII

INTERFERENCE OF WAVES

§ 292. We saw in § 277 (b) that a particle disturbed by two harmonic forces will vibrate very differently at different times, its actual amplitude gradually alternating between the sum and difference of those due to the two forces independently.

So two wave systems spreading simultaneously will produce very different amplitudes at different places. Watch a steamboat in calm water, she makes the well-known V-shaped set of bow waves and she is followed by a broad 'swell' of nearly straight waves at right angles to her course and stretching across the river. The two systems overlap, crest is piled on crest and trough deepens trough and the V appears broken up into short sharp ridges arranged 'en echelon,' i.e. something like the broad treads of a step-ladder. Where crest falls into trough or trough beheads crest the surface is near its undisturbed level.

This is an instance of the **Interference** of two running wave systems. Another is the choppy water in the corner of a dock, where cross reflections from the walls produce a local bobbing up and down, a chequering which can be imitated by jarring an oblong dish of water.

In Fig. 115 let P and Q be two sources vibrating in the same phase and emitting equal wave systems. Any point on the bisecting axis CC is equidistant from both, therefore on this line crest arrives with crest and trough with trough, amplitudes are doubled and energy quadrupled. But along $\frac{1}{2}\frac{1}{2}$ which is (a hyperbola) such that any point on it is half a wave length farther from P than from Q, P's waves everywhere arrive half a wave length behind Q's, crests into troughs, the motion is destroyed, and no energy travels there. Along the next hyperbola 11 the difference of distance is a whole wave length and again crest coincides with crest: along the next there is $1\frac{1}{2}$ difference and no appreciable resultant motion. Hence there is a steady pattern of quiet rays and streams of short ripples as shown on

the right of the diagram, occupying the dotted and solid hyperbolas worked out on the left of the diagram from the intersections of the circular ripples that instantaneous illumination by a spark would disclose.

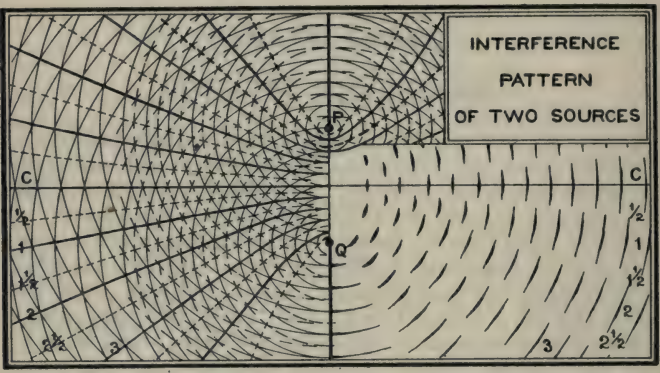


Fig. 115.

§ 293: To find at what points on a screen there will be most and least disturbance.

Fig. 116 represents the middle part of the right-hand side of Fig. 115, the problem is to find where the quiet and disturbed hyperbolic lines strike the wall. At C, where PCQ is isosceles both waves arrive in the same phase; at D, where PDR is isosceles one system has QR farther to go than the other. If this is $\frac{1}{2}$ wave length there is hardly any resultant motion at D, which is a point on the first hyperbola.

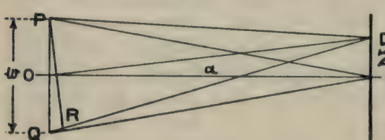


Fig. 116.

The triangles PQR and ODC are similar. $\therefore$ QR/PQ=CD/OD and keeping the angles small as in the figure this practically becomes $QR/w = Z/a$ or $Z = a \times \frac{1}{2}l \div w$ where Z is the distance between successive points at rest and in motion all at a perpendicular distance a from two sources w apart, sending out waves of length l.

§ 294: **Why a straight wave travels straight forward.**

Now let PQQ′ . . . (Fig. 117) be a straight or plane wave front, i.e. a plane passing through many particles PQQ′ . . . vibrating in the same phase. Each endeavours to send out its own circular ripples in all directions. P and Q together would produce the pattern already considered, but now Q′ Q″ . . . join in with their ripples, 'interfere,' and cause a general blur, and the only parts remaining definite and free from overlapping are the little arcs *p, q, q′* . . . of the outermost ripples, which of course have all travelled equal distances from their sources.

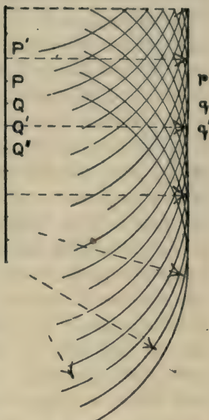


FIG. 117.

Together these coalesce into a new plane wave front and we see that a plane wave travels forward in a direction perpendicular to itself without alteration of shape. (A circular wave will spread radially into a larger circle.) Backward it cannot travel, for the particles there are already in motion; the most it can do is to reduce them to rest, and that, in the absence of freshly arriving disturbance, it does. Recollect how smooth a surface the ripples from a stone leave behind them.

§ 295: **Diffraction.** The resultant disturbance goes straight forward, mutual interference ensuring that none escapes obliquely, *except at the edges.* The constituent ripples behave like trees growing in a close plantation. These lose their natural spreading shape, and grow straight upward only, since that is the only direction in which they do not interfere and hinder one another's growth. But at the margin of the wood they bear spreading branches clad with foliage almost to the ground. So here we find that *at the edges* mutual interference fails to prevent the ripples spreading out sideways to some extent, Fig. 117, bottom.

This bending round the corner into the 'shadow' of the obstacle which has limited the breadth of the wave is a very important characteristic of wave motion, and is known as **Diffraction.**

It is easily seen behind a breakwater; the waves gradually spread into the calm water behind and only a triangular space is completely protected. Hiding behind a corner is not a complete protection from the waves of sound.

It is otherwise with Light, and the sharp shadows thrown by opaque objects were long a difficulty in developing the wave theory of light. But closer examination shows that light does spread into the shadow to a very small extent. If light coming from a pinhole in a card with a bright lamp behind it is passed through another pinhole a foot away and then received on a third card a foot beyond, the bright circular patch is *much* larger than the hole, and the smaller the holes the worse the discrepancy.

But this is not altogether a fair comparison. Standing on the breakwater we see the first dozen or two waves gradually curling round into the sheltered water, but the waves of ordinary light are only about a fifty-thousandth of an inch long. That means we ought to be inspecting with a microscope the space within a five-thousandth of an inch of the edge of the pinhole, instead of a foot away from it. A fiftieth-inch pinhole is a thousand wave-lengths broad, broader than the North Channel with regard to the Atlantic swell, and that does not diffract round into the Irish Sea to any extent. Again, sound waves are a few feet long: a train plunging into a deep cutting goes practically out of hearing, and hills or large buildings shut off the sound of distant bells almost as soon as the sight of the church tower. That is, when the observing spaces become large compared with wave lengths,

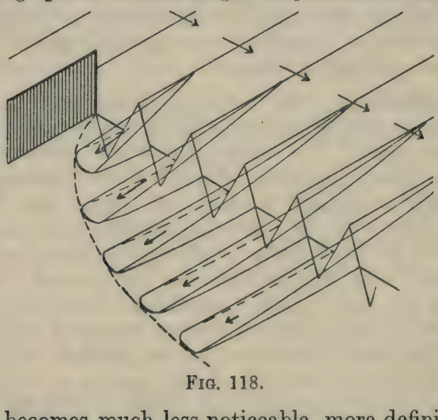


FIG. 118.

diffraction becomes much less noticeable, more definite shadows are cast, until in Light it requires special care to observe diffraction at all, and there again only half the spreading occurs with violet light as with the longer ripples of red.

The theory of all this, developed from the Principle of Interference, is too long to put in here. Return, however, to a sharp-ended breakwater for an illustration. The waves that escape past it ought to have cut-off vertical 'gable-ends,' Fig. 118. The 'gable' collapses as the water heaped up in it immediately flows out endways into the calm 'shadow.' The wave travels on with a sloping end, down which water continues to flow farther and farther out into the 'shadow.' This keeps on flattening the slope so that the flow down it, i.e. the endways extension of the waves, presently becomes very slow compared with its rate at first: diffraction several dozen waves beyond the obstacle is nothing like as noticeable as it was for the first few waves. There would be a return flow from the smooth water into the troughs, which has been omitted from Fig. 118 for clearness' sake. On the whole no water flows into the shadow, only the wave motion.

§ 296: **The Diffraction Grating.**

Let a single straight wave front strike the row of narrow equidistant obstacles in Fig. 119 (palings in a pond, for instance). A moment after, the state of affairs is as represented. Each gap has let through or transmitted, and each obstacle has reflected back, a separate little wave, and the *spaces being narrow* these spread in semicircular ripples. In any direction PL not one ripple is sent, but a succession of distant ones, their actual distance apart depending on the width of the grating spaces and on the direction of PL.

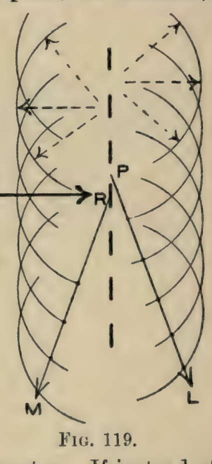


FIG. 119.

This can be heard in the musical sound which a paled fence echoes to a sharp footstep, the rapid string of little echoes blending into a note.

It is vastly important in optics, where a grating with perhaps 15,000 spaces to the inch will fling off light of different wave lengths (colours) in directions PL, PL′, etc., and so break up white light into spectra. If instead of one wave, a train of definite wave length falls on the grating, only waves of that length can exist anywhere, all others getting trampled out by interference, and these can pass off only in

certain directions, e.g. in the figure only a train of 6 mm. length can pass off along PL and obviously in no less-inclined direction; along such only shorter waves can pass out.

There is a reflected system, such as RM, precisely similar to the transmitted system.

If several definite periodicities can be analysed from the incident disturbance, several trains of diffracted waves will spread in definite directions, the longer waves being thrown off at greater angles: red is more diffracted than blue light. The grating has analysed a disturbance into its component S.H.M.'s (§ 276) and has spread them out to view as a ' spectrum.' We shall return to this under Light.

Chapter XXXIII

REFLECTION AND REFRACTION OF WAVES

§ 297. Waves beating on an unyielding surface are thrown back or *reflected.* If circular ripples from O fall on AB a point on the ripple which would naturally have arrived at C has had its motion reversed and has then travelled without change of speed to D ADB is an arc exactly equal to the original one ACB and the reflected ripples spread as if they came from a point I—a 'virtual image'—which is perpendicularly below O and as far behind the reflector as O is in front of it.

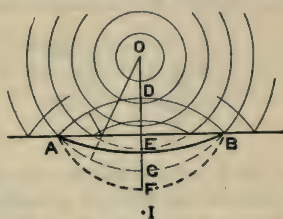


Fig. 120.

If the surface AB is not altogether impenetrable, but permits the wave motion to pass, in part, beyond it—say AB is the edge of a flat submerged rock or a shallower part of an experimental dish of water, or the surface of a wall through which sound is partly audible, or of glass transparent to light waves—then the reflected ripples carry back only part of the energy and more or less enfeebled direct ripples continue the original motion over the border, but *always with an alteration of speed.* They are **refracted.** We have stated that in shallow water, and have found that in media of greater density (§ 283), waves travel slower. E is less deep than C, the ripples are flattened as if they came from a centre at a greater distance (but are now not quite circular). Conversely, if the medium beyond AB transmitted waves faster, ACB would become AFB, and the ripples spread as if from a closer centre. For instance, an object under water appears nearer to the surface than it really is because the light waves have come out into the air, where they travel faster.

§ 298. The subject is pursued rather differently, but quantitatively, under Light. We shall here, however, work out the

Laws of Reflection and Refraction of wave motion from the simple case of a plane wave meeting a plane surface.

AC is the initial position of a plane wave front (§ 294) travelling at speed V in a direction perpendicular to itself and incident on the surface AB at an angle *i*.

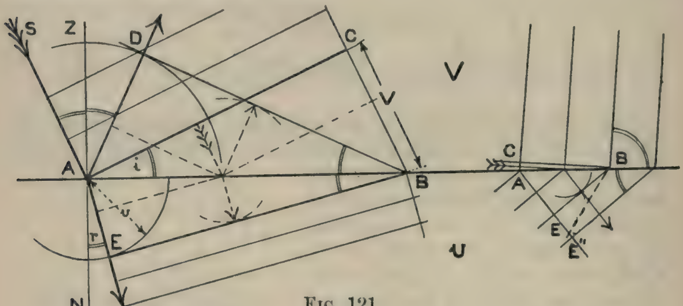


FIG. 121.

At A reflection is taking place. Shortly after, the broken wave occupies the dotted position. By the time C reaches B the reflected disturbance from A will have spread to D, where AD = CB, and **DB will be the reflected wave front** (built up as in § 294), which evidently leaves at the same angle as AC arrived, but on the other side of the perpendicular or ‘ normal ’ to the surface AB. And if the direction SA in which the wave came and the normal AZ lie in the plane of the paper AD will evidently do the same.

Hence the **Laws of Reflection :—**

I. *The directions of incidence and reflection and the normal to the surface lie in the same plane.*

II. *The angles of incidence and reflection are equal.*

The disturbance at A also spreads down into the lower medium but at speed v (slower as drawn) and arrives at E by the time C reaches B, and the *refracted wave front is EB*, inclined at the angle of refraction r to the surface and travelling along AE, which is at r to the normal AN.

Since CB and AE were covered in the same time they must be proportional to the speeds V and v in their respective media. $\therefore$ CB $\div$ AE $=$ V $\div v$, which of course is constant, and is called the Refractive Index of the second medium with respect to the first, and is usually written μ (Greek m ; mu).

In any right-angled triangle the length of a side divided by

the length of the hypotenuse is called the *sine* of the angle opposite to that side.

$$\text{In triangle BCA, } \frac{\text{BC}}{\text{BA}} = \text{sine } \text{B}\hat{\text{A}}\text{C} = \text{sine } i.$$

$$\text{,, \quad ,, \quad BEA, } \frac{\text{AE}}{\text{BA}} = \text{sine } \text{A}\hat{\text{B}}\text{E} = \text{sine } r.$$

$$\text{Divide, BA cancels out, } \frac{\text{BC}}{\text{AE}} = \frac{\text{sine } i}{\text{sine } r} = \text{also} \frac{\text{V}}{v} = \mu.$$

Hence the **Laws of Refraction** :—

I. *The directions of incidence and refraction and the normal lie in one plane.*

II. (*Snell's law.*) *The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant, and is called the Refractive Index of the second medium with respect to the first medium.*

§ 299. **Total reflection.**

When the incident waves are nearly perpendicular to the surface, Fig. 121, right, and sweep along it, CB nearly coincides with AB and the refracted wave front is BE. This is much longer and therefore weaker than AC, from which it derives its energy.

Conversely BE emerging into the faster medium would become CA, which cannot contain all the energy of BE, and much of this is therefore reflected back.

When the waves become strictly perpendicular AC=O and there is no energy to produce BE. Conversely BE cannot get out at all, but is *totally reflected*, and so are all waves beyond it, like BE″, according to the ordinary law of reflection.

$\text{AE}/\text{CB} = v/\text{V}$ and now putting CB in coincidence with AB, $\text{AE}/\text{CB} = \text{sine } r = v/\text{V} = 1/\mu$. Hence *when waves travelling at speed v make with the surface of a medium in which they would travel at greater speed V an angle greater than that Critical Angle whose sine is v/V* (*or* $1/\mu$ *of their own medium*) *they are totally reflected back into the slower medium.* See Light, § 373.

The reader who has appreciated § 295 will see that diffraction upsets the totality, but light leaks very little.

§ 300. **The deviation of waves passing through a 'thin prism.'**

Suppose plane waves fall flat on AB, Fig. 122, one face of a narrow-angled prismatic space ABC in which they must travel

more slowly (e.g. the tail of a sandbank). The point B of the wave does not reach C till the free part at A has reached E, where AE/BC=speed of travel outside prism/speed inside $=V/v = \mu$, the refractive index of the prism with respect to the outer space. EC is therefore the position of the waves as they leave. Draw CF parallel to BA, small angle FCE is the change of direction of wave front and therefore of travel—the **Deviation**—since the waves travel perpendicularly to their own fronts.

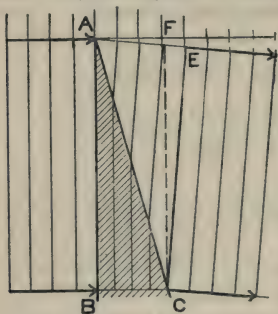


FIG. 122.

Angle A of prism=angle ACF =arc AF÷radius CF, since it is supposed so small that the difference between AF and the arc of a circle is negligible. Similarly angle D of deviation=angle FCE=FE÷CF.

Now $AE=BC\,.\,V/v=AF\,.\,V/v$. $\therefore AE-AF=FE=(V/v-1)AF$.

$$\therefore \frac{D}{A}=\frac{FE/CF}{AF/CF}=\frac{V}{v}-1=\mu-1.$$

$$\therefore \mathbf{D=(\mu-1)A.}$$

The reader can easily prove for himself that for any waves not very far from parallel to AB the same relation holds fairly true.

That is, provided all angles are small, the *Deviation produced by a thin prism is obtained by multiplying its angle by* (*the ratio of the speeds outside and inside it, less* 1), and does not depend on the particular angle at which the waves strike the prism.

§ 301 : **A refractive and absorbent medium : Resonance.** Making the water shallower has been suggested as a means of causing waves to travel slower, but this can be effected in another way. Suppose waves entering a fleet of fishing-smacks. Part of the energy goes towards setting them rocking. They will gently rise and fall to a long swell, which passes on scarcely impeded. Of little ripples, some would be stopped, while others get through the gaps unhindered ; the boats do not respond to their motion and either reflect or pass all the energy they bring. There is no change of speed in these two cases.

But if the periodic time of roll of the boats is near to that of the

waves they behave like swings kept going by properly timed pushes. They roll tremendously and absorb much of the energy of the waves, weakening and slowing them. That is, both **absorption** and **refraction** of waves occurs in a space like this, and the better the agreement of periodic times the more marked is this effect of resonance.

If the rolling boats were suddenly removed to calm water they would set up waves of the same period as those which most violently set them in motion.

We shall refer to this under Spectrum.

§ 302. **Stationary wave motion.**

The choppiness of water near reflecting walls, and 'interference patterns,' have been mentioned above, § 292. Let us see how

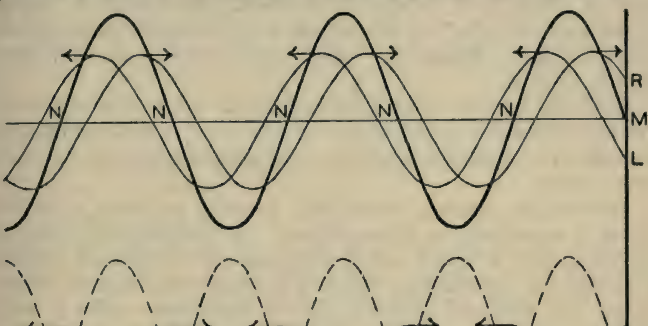


FIG. 123.

this so-called 'stationary wave motion' results from the interference of running waves. Take only one simple and important case, that of waves incident perpendicularly on a rigid obstacle; and take, as less complex in construction than those of water, waves on a rope or string arriving at the fixed end.

They are reflected just as if they came back with equal wave

length, amplitude, and speed from an 'image' source beyond the obstacle, § 297, and the direct and reflected trains interfere to produce a resultant shape obtained by adding both displacements together.

At the fixed obstacle there can be no resultant motion, therefore the train travelling to the left must there produce displacements always equal and opposite to those of the direct train moving to the right.

In the Fig. 123 MR=ML and both are increasing (with this clue the reflected wave train is drawn in the diagram) as the trains pass M opposite ways MR will always=ML and M remains at rest.

Adding the displacements all along the line one finds a succession of points N N at which the two displacements are *always* equal and opposite, i.e. no motion ever occurs at these points. They are called **Nodes** and remain fixed at successive *half wave lengths from* M.

Half-way between them equal and similar displacements always come to be added together, and the particles at these **Antinodes** vibrate with twice the amplitude they would have in the incident wave alone.

Whereas in running waves each particle performs a motion equal in amplitude to its neighbours' but progressively differing in phase, here is now a sort of undulation in which each particle performs its own motion, different in amplitude from its neighbours' but identical in phase (but at motionless nodes $\frac{1}{2}$ wave length apart the phase suddenly changes to the opposite). Running waves are imitated by a rotating corkscrew seen from the side, those 'stationary waves' by a rotating zigzag.

The lower figure of Fig. 123 shows the running waves and the resultant (thick) 'stationary wave' ·175 of the period after the upper figure, and just past its straight-line mid-position. The dotted lines are the extreme positions of the 'stationary waves.'

The argument holds for longitudinal waves, for the particles next the surface have to stop there, at rest. See later, § 320.

We see these nodes and vibrating segments on a long vibrating string, Fig. 138; quiet nodal lines and perturbed antinodal lines make up interference patterns on water: stationary light waves produce Lippmann's colour photographs. We see them in the longitudinal motions of a long wire helix made fast at the end—near nodes the coils are alternately squeezed up and expanded, but the middle one does not move, near antinodes the

coils are rushing to and fro—we can detect alternate quiet nodes and windy antinodes in organ-pipes resounding to a high harmonic.

'Reflection from a free end' is also competent to set up stationary wave motion, but there is a difference.—

Hang up two pendulums with their bobs touching, one of cork, the other of lead. Lift and drop the cork bob, it hits the lead and is reflected back *instantly*, that is like the reflection from a fixed end considered above. But lift and drop the lead bob, the cork flies off and comes back to return the blow *half a period later.*

Again, a shunting engine bumps into a train, sending a wave of compression clattering along the buffers. The last truck jerks out, immediately sending a wave of extension back along the couplings, and then under the pull of its stretched coupling crashes back and starts a compressive wave *half a period later.*

This 'reflection from a free end' can be studied in the wire helix, and it occurs at the open ends of sounding pipes. The reflecting place is one where the motion is most free, i.e. *an antinode* (left-hand end of Fig. 123 serves to show it). Reflection of light from the inside of the surface of water-air is similar.

§ 303. **Doppler's principle**, *dealing with motions of observer and of source of waves.*

A. Moving observer.

Sailing out against the waves, they pass the boat more frequently than when at anchor, and sailing with them they pass more slowly. If their speed is V and the boat's u, the speeds of passing in the three cases are the combined speed at which waves and boat rush to meet each other $V+u$; V, and $V-u$ the speed at which the waves overtake the boat. As the length of a wave remains quite unaffected by the boat's motion the numbers met in a given time are also in the same ratios, or

$$\frac{V+u}{V},\ 1,\ \text{and}\ \frac{V-u}{V}\ \text{times the normal number.}$$

B. Moving source (not precisely applicable to water waves).

The source of the waves may be moving at speed w through the medium which carries them, while the observer is at rest. From a source at rest waves spread in concentric circles, but if it moves the successive ripples start from centres farther and farther from the first, and Fig. 124 represents their distribution. Each ripple, once started, goes on spreading from its own centre at its natural speed V.

We cannot here deal with the last figure, which corresponds to a source moving faster than the ripples, e.g. a stick drawn through water or a rifle-bullet in air. But when w is less than V waves which normally occupy a space V get squeezed into $V-w$ ahead

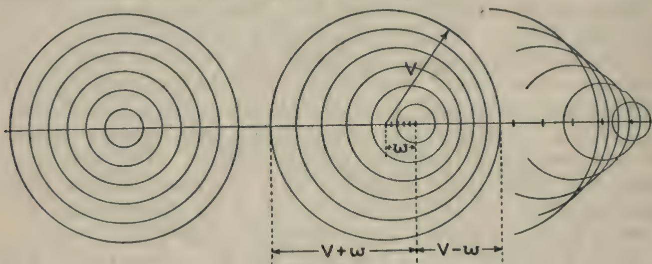


FIG. 124.

of the source and spread over $V+w$ astern, as in the middle figure. Their lengths alter in the same ratio, and as all are travelling at the natural speed V the number that pass an observer ahead is increased in the ratio $\frac{V}{V-w}$ and astern is decreased as $\frac{V}{V+w}$ [Speed÷wave length=frequency.] Hardly any change is noticed by an observer 'on the beam.'

C. If u and w are small compared with V the reader will see that it makes no appreciable difference whether the source or the observer moves. If they are approaching each other the frequency rises in the ratio (V+net speed of approach) : V, and if receding it falls in the ratio (V−net speed of recession) : V (which is the same as the former if we call recession a minus approach).

EXAMPLES.—CHAPTER XXXIII

1. Show from Fig. 121 how it is that waves gradually bend round when approaching a shallowing beach until they become nearly parallel to it.

2. Show that if two equal trains of waves are moving in opposite directions along a stretched string they will produce stationary waves. [L.]

3. Give an account of reflection of waves at a rigid wall and at a surface where there is no variation of pressure. Mention apparatus in which reflection of these two kinds is produced. [L.]

SOUND

Chapter XXXIV

SOUND, ITS NATURE AND SPEED

§ 304. **Production and propagation.**

An exploding cracker produces a sudden outrush of air straight away from it and a collapsing electric lamp-bulb induces a sudden inrush from all sides straight toward it to fill up the vacuum. Sharp short sounds result.

A big explosion produces a pulse in the air that can be felt as well as heard for miles ; a compression travels wave-like through the air, blowing in windows on its way ; its shadow has been watched speeding over a sunlit plain and the shock felt and roar heard as it passed.

The lower pipes of the pedal organ produce a vibration which is felt as much as heard.

The sudden compression of air in the outer ears on diving into water gives the sensation of a loud explosion.

One concludes that, physically speaking, the ear is only a part of the body surface specially sensitive to the shock of impinging air, and that **sounds** are heard when quick compressions or expansions reach the ear from some source of air disturbance.

The **medium of transmission** need not be air : with the ears under water in a bath drops falling or noises in the pipes are heard very distinctly, and the solid teeth and skull are sometimes employed to carry sound to the ' inner ear ' in deafness due to defect in the ' middle ear.'

That a resilient material medium of some sort is necessary is proved by the experiment of standing a cheap clock (in the tin case that lends ferocity to its tick) on some tow inside an air-pump receiver, and exhausting the air. The tick is no longer heard, nor hardly the ringing of the alarm ; the tow is an incoherent solid, and there is little air.

Noise and note.

Air pulses travel quickly, a single one is therefore difficult to study, and the jumble of irregular ones that we stigmatize as a '**noise**' does not interest us here. Fortunately it is easy to produce a long series of similar impulses by the use of some vibrating body—card pressed on a cog-wheel, fork, string, gong, whistle, etc.—and this steady succession produces a 'musical **note**' which can be studied more at leisure.

We know that this sets up a running wave motion in the medium, spreading spherically through it. The waves are alternate compressions and rarefactions. For the air particles blow only to and fro, like a wee changeable wind, in the direction of travel of the wave they hand on. Transverse motion they have none, for fluids have no 'elasticity of shear,' § 98; there is no force available to carry a sidewise motion forward to particles ahead.

§ 305. Reflection and refraction.

Echo is familiar to all. That it obeys the laws of Reflection of § 297 can be proved by screening off a watch a few feet away and then finding the positions in which a small drawing-board can be placed to reflect its sound to the ear. On a larger scale, notice where a distant train is when its roar is suddenly reflected from an isolated building.

The Refraction of sound is difficult to study, but a toy collodion balloon blown with CO_2 will act like a convex lens with a refractive index 1·25 (see Light) and can focus on the ear and make louder the ticking of a distant watch.

To-and-fro reflections account for the murmuring echoes of large buildings. Sharp corners and heavy projecting draperies break up the sound waves and smother the echo, while smooth vaults and domes are an abomination to the speaker, who requires the broad umbrella of a 'sounding-board' to keep his voice from reaching and rolling among them. Recollect St. Paul's, and its 'whispering gallery.'

On a larger scale 'irregular' reflections and refractions account for the roll of thunder, and probably for the fact that on a fine summer day, with 'light variable airs' of all sorts of temperatures and humidities, sound signals often carry very badly. An audience with its streams of rising hot air deadens the echoes of a hall. In homogeneous fog sounds carry well; but the many

hot chimneys of a city spoil the acoustic uniformity of the fog they foul, and sounds are deadened.

§ 306. Sound being a wave motion travelling among the 'particles' of air, it is only necessary to add to the various definitions in §§ 276 and 283, the statement that the frequency or number of complete vibrations per second of a note is the physical measure of its **Pitch**.

And turning to § 289 we may say that the physical measure of the **loudness** of a note of fixed pitch is the amount of energy the ear receives from it per second. In still air this varies inversely as the square of the distance from the source. It has been found to agree with the physiological estimate of loudness, within limits. No comparison of the loudness of *different* notes is physically attempted.

Some direct methods of measuring the speed of travel of sound, the frequency or pitch, and the lengths of waves, will now be described. From these the relation (cf. § 283) can be experimentally established—

Speed of sound in a medium = pitch of note × its wave length in the medium

and this relation will then be made use of.

§ 307. **Speed of travel of Sound. Direct methods.**

That sound travels in air at a speed which though high cannot be called instantaneous is familiar to everyone in the delay between the fall of a distant hammer and the sound of its blow, between the puff from a far-away engine whistle and its shriek, between noise and echo, between lightning and thunder. In a favourably placed railway cutting I know of, one can easily hear the guns of Shoeburyness three and a half minutes after firing, and it is said that guns have been heard three times as far.

About 1708 the earliest extensive experiments on the speed of sound took place not very far from the locality mentioned above, viz. between a cannon on Blackheath and Upminster church, $12\frac{1}{2}$ miles away across the Thames. The time the sight of the discharge takes to travel that distance is negligible, for light travels nearly a million times faster than sound. The report took from $55\frac{1}{2}$ to 63 sec. according to the wind. For the air moving as a whole of course carries all contained sound waves with it, and so modifies their velocity relative to the earth. Taking the mean of many results with winds of various strengths

from all points of the compass Derham obtained 1142 ft. per sec.

In 1738 and 1822 various French and Dutch observers, working in fairly calm weather, eliminated wind effect by firing almost simultaneously at both ends of about 11-mile distances. The wind accelerated one sound as much as it retarded the other. They obtained speeds (reduced to 0° C.) from 331 to 332·8 metres per sec.

In 1823 the experiment was repeated in this way in the Tyrol with one station 4000 ft. higher than the other, and in 1844 in Switzerland with the guns at 1800 and 8800 ft. respectively. The speeds up and down hill were identical, and were the same as at sea-level in Holland, showing that *the velocity does not depend on the pressure of the air.*

In 1822 and 1890 Arctic observations gave $(1050+1\times \text{temp. F.})$ ft. and $(333+\cdot 6 \text{ temp. C.})$ metres per sec. between $-40°$ and the freezing point, showing *how the velocity increases with temperature.*

In 1863 Regnault experimented in the newly laid water-pipes of Paris, using several hundred metres of air. A pistol-shot echoed from end to end, where were stretched membranes which ticked off the impacts on an electric chronograph. 330·5 m./s. was deduced as the speed at 0° C. and at all pressures from $\frac{1}{3}$ to 2 atmos. in dry air: in H_2 3·8, CO_2 and NO_2 ·801, and NH_3 1·23 times the air speed, see § 312.

He and others found that the speed is about 1 % less in a 1-in. pipe than in the open air.

In 1905 in a very large pipe 2 miles long it was proved that *difference of pitch has no effect whatever on the velocity.* Were it otherwise indeed a tune played by a distant band might become confused, and the characteristic quality of their instruments unrecognizable, § 343.

§ 308. **Simple echo method for speed.**

The reader can quickly get a fair result by a method described by Sedley Taylor, and doubtless suggested to him by the notorious ' knocker ' echo in the cloister of Trinity. The necessaries are an echoing wall, a bob on a bit of thread, and a foot rule. Step off 40 or 50 yards from the wall, stand and clap the hands sharply. The echo comes back at an interval too short to estimate. Multiply the interval. You cannot clap again at the instant the echo returns because that would drown it, therefore wait an equal

time and then clap, and so on. This is not so difficult as it sounds, because clap and echo will alternate like the tick-tock of a clock, and you know how these two sounds couple themselves either one way or the other when alternate intervals are not equal, and the clock is struggling on with lop-sided ticks. When after a little practice you have succeeded in this, shorten your simple pendulum until it beats exactly with your clapping, one single swing each time, and refer to Fig. 125. Or else count

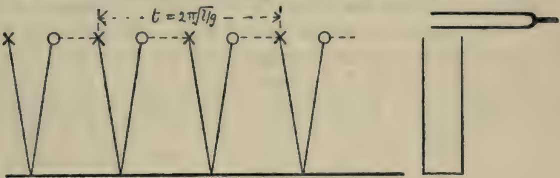


FIG. 125.

up strokes per minute by aid of a watch, then between any two, sound has travelled to the wall and back and might have done it again.

Then Speed $=4D\times$ claps per unit time.

§ 309. **The Resonance-Tube method.**

In the last experiment, as the striker approaches the echoing wall the speed of clapping must go up. Carrying this to an extreme, the echoing cloister shortens to a stoppered pipe and the motion of the arms is replaced by the rapid vibration of a piece of metal—say a tuning-fork prong—whose rate has been measured in some mechanical way, § 315. What happens ?

The prong starts at the top of its swing to drive air before it and therefore to send a compression down the pipe. The action increases up to mid-swing, when the prong is chasing the air fastest, and then gradually diminishes again. Accordingly the densest part, the '*crest*,' *of a compression leaves the fork at mid-swing*, travels to the stoppered end of the pipe, is instantly reflected, and returns. The prong has gone to the bottom of its swing and is now moving up. If the reflected 'crest' reaches the prong just at mid-swing up the two 'crests' are exactly added together : the fastest outrush of air particles from the pipe coincides exactly with the hardest pull upwards of the prong on the air. The two combine to drag air out of the pipe. The next down-swing of the prong therefore drives compressed air

into a partial vacuum waiting for it. Like a swing pushed always at the right moment the air in the pipe is gradually excited to more and more violent motion and *resounds* strongly to the note of the fork.

As it takes perhaps a hundred vibrations to work up strong resonance it will be seen that the reflected 'crest' and the mid-swing position of the upgoing prong must very closely coincide. For suppose the crest gets up 1 % too soon, i.e. the new push from the fork follows 1 % late, on the next pulsation it would be 2 % late, and so on, to 50 % late. Then the fork is in direct opposition to the vibrating air in the pipe and begins to wipe out its previous work for the next 50 swings,* then to rebuild it for 50, and so on. This imperfect resonance never gains much strength. Strong resonance then means that a sound wave travels down and up the pipe while the fork moves from mid-swing down to mid-swing up, i.e. makes half a complete vibration.

$\therefore$ S = twice no. of fork vibrations per sec. $\times$ twice length of pipe
= frequency $\times$ 4 times length of pipe.†

This is a very useful laboratory method. It enables different gases to be studied and different temperatures, by surrounding the pipe with a water or steam jacket. It will be returned to in § 325.

§ 310. Speed in water and in iron.

In **water** the speed was measured at night on the Lake of Geneva in 1826. The hammer of a submerged bell was let fall by a cord which simultaneously dropped a lighted match into powder. The flash was seen 9 miles away, and the sound listened for with a large ear-trumpet having a membrane stretched across its mouth under water. Speed 1435 metres/sec. at 8° C.

In 1889 gun-cotton was fired in Sydney harbour and electrically timed over 180 metres. The speed increased with the charge; with 4 lb. it exceeded 2000 m./s.

Half a mile of **cast-iron** pipe was struck at one end and at the other two sounds were heard through iron and air respectively. The interval was nine-tenths the calculated time through air, i.e. sound travelled ten times as fast in the iron as in air.

* It is adding together a series of S.H.M.'s 3·6° in phase behind one another, § 281.

† Actual length + $\frac{1}{3}$ diameter as mouth correction. See § 325.

§ 311. **Theoretical.**

In § 288 it has been proved that the speed of travel of a longitudinal wave motion in an elastic medium is the square root of its elasticity (dynes/cm.^{2}) by its density (grm./cm.^{3}) provided the particles themselves move but little. This is a wave of sound, not too loud.

For **water** modulus of compression is $2{\cdot}2 \times 10^{10}$, $D=1$.

$$\therefore\ S = \sqrt{E/D} = 148{,}300 \text{ cm./sec.}$$

For **cast iron** Young's modulus averages $1{\cdot}2 \times 10^{12}$, $D=7$.

$$\therefore\ S = 415{,}000 \text{ cm./sec.}$$

For **air** Newton employed in this formula (which he discovered) the result of his friend Boyle, that the elasticity = the pressure. For if PV is constant, 1% increase in P will cause 1% diminution in V, for $101 \times 99 = 100 \times 100$ very nearly. Therefore E which = increase in pressure ÷ contraction it causes in unit volume $= {\cdot}01P \div {\cdot}01 = P$. Taking atmospheric pressure $P = 1{,}016{,}000$ dynes/cm.^{2} and $D = {\cdot}00129$ grm./cm.^{3} $\therefore V = 28{,}000$ cm./sec., which is too low.

It was not until 1822 that Laplace pointed out that the compression in a sound wave is very quick whereas that in a Boyle tube is slow.

To obtain a correction for this, a large flask containing air at a pressure B, a little less than the atmospheric A, is suddenly opened and closed. Air rushes in to raise the pressure to A, but the sudden compression heats the air inside (§ 221) and after a few minutes cooling to its original temperature the pressure has fallen somewhat to C. That is, it took a sudden increase A—B to do what might have been coolly and quietly done by only C—B, viz. to drive a little air into the flask and slightly compress its contents ; or the sudden elasticity * is

$$\frac{A-B}{C-B} \text{ times the slow Boyle elasticity} * = \frac{A-B}{C-B} \times P.$$

The experiment gives this ratio 1·40 for air and

$$\therefore\ S = \sqrt{\frac{E \text{ sudden}}{D}} = \sqrt{\frac{1{\cdot}40 \times P}{D}} = \sqrt{\frac{1{\cdot}40 \times 1{,}016{,}000}{{\cdot}00129}}$$

$$= \underline{33{,}200 \text{ cm./sec.}}$$

* These are the adiabatic (heat not-passing-out) and isothermal (same-temperature) elasticities. That heat from the momentarily compressed parts in sound waves does not leak into the cooled

§ 312. From this relation

Speed $=\sqrt{}$(ratio of elasticities $\times$ pressure $\div$ density)

we see that in gases :—

A. **Change of gas pressure does not change the speed.**

For doubling the pressure would halve the volume and therefore double the density also.

B. **Speed is proportional to square root of absolute temperature.**

For if D is constant P will increase, or if P is constant (V will increase and) D will decrease proportionally to the absolute temperature.

$$\text{Thus speed at } 1^\circ = \sqrt{274/273} \text{ speed at } 0^\circ.$$
$$= (1 + \tfrac{1}{2} \cdot \tfrac{1}{273}) \times 330 \text{ m./sec.}$$

which fraction amounts to an increase of speed of 2 ft. or ·6 m. per sec. per 1° C. rise of temperature. Notice that this *approximation* holds good only *for air at ordinary temperatures.*

C. **In different gases the speeds are inversely as the square roots of the densities.**

For instance, 4 times faster in hydrogen than in oxygen. The densities of the gases used by Regnault (§ 307) for which he obtained speeds 1, 3·8, ·80, and 1·23 are in the ratio 1/1, 1/3·80, 1/·81, 1/1·3.

The last illustrates a discrepancy that must occur in this law when the ratio of elasticities changes. This ratio is less for complex molecules and rises to 5/3 for monatomic gases.

We shall see that the pitch of a wind instrument is proportional to the speed of sound in the gas which fills it. A whistle blown with hydrogen becomes therefore very shrill, but a very familiar instance is the sharpening of the hiss of an unlit gas-jet, which is the signal that the air has been blown out of the pipe and the lighter gas has arrived. Nitrous oxide is 1·5 times as dense as air and accordingly the 'laughter' induced by this anæsthetic when clumsily administered is in a pitch $\sqrt{1/1{\cdot}5}$ or 20 % lower than the natural voice, and is not pleasant to hear.

expanded parts is at first sight surprising, but such leakage would *very* quickly destroy the sound. And after all, even the primitive flask experiment is not grievously affected by heat leakage.

EXAMPLES.—CHAPTER XXXIV

1. How has the velocity of sound been determined, using the echo from a cliff? [L.]

2. Show that for a gas obeying Boyle's Law the elasticity at constant temperature = the pressure. [L.]

3. Taking the speed of sound in air as 1080 ft. per sec. at normal temperature and pressure, calculate the maximum speed of propagation of an impulse along a train brake-pipe which contains air at 6 atmos. and 10° C. [L.]

4. Explain how the velocity of sound in a gas depends upon the temperature. If a tube open at both ends has an effective length of 32 cm. and resounds most readily to a tuning-fork of frequency 520 at 15° C., what is the velocity of sound in air at 0° C.? [L.]

5. What conditions are requisite for (a) the production, and (b) the propagation of a sound? Why does the sudden closing of a book produce noise, while waving the book about produces none? [L.]

6. What experiments show that note of definite pitch corresponds to waves of definite length in air? [L]m.

7. Explain pitch and loudness. To what conditions in the air do these correspond? [L]m.

8. Explain the motion of air transmitting a musical note. Make diagrams showing (1) the variation of displacement with position at a given instant, and (2) the variation of displacement with time at a given point. [L]m.

Chapter XXXV

PITCH AND WAVE LENGTH OF SOUNDS

Frequency of Vibration, or Pitch

Two methods of **comparing** frequencies must come first, of them the former is very important.

§ 313. **Comparison of near frequencies. Beats.**

It was explained in § 277 (b) that a compound harmonic motion resulting from two S.H.M.'s not very unlike was characterized by its variable amplitude. When in phase, both pull together, and their amplitudes are added ; presently one gains half a period (180° of phase), they pull opposite ways and their amplitudes are subtracted, another half-period gain brings them into phase once more, and so on. The tides were given as an instance; every fortnight the solar tide gains one period (12 hours) on the lunar and there is one set of spring and one of neap tides.

Suppose now it is a particle of air near the ear, that is affected by the joint action of the air waves coming from two sources of sound not quite of the same pitch. Both combine in driving it in and out of the ear and its amplitude of motion increases and decreases *once, every time one source gains a whole vibration on the other.* Loudness being proportional to square of amplitude this means that the sound swells and softens once, or *one beat is heard.*

First acquaintance with beats is best made by sounding together two near notes on a harmonium ; they are heard as a tremolo varying from 2 or 3 per sec. to a rapid burr-r.

Beats enable the sustained notes of any musical instruments to be tuned together to within one vibration in 2 or 3 sec. Counting them gives the arithmetic difference between the number of vibrations during the time of listening, and reducing to 1 sec., between their frequencies. If one frequency is actually known adding or subtracting the rate of beating gives the other. The faster vibrator can be identified, because gradually loading it

with specks of wax (or slowing it in any appropriate manner) slows the beats down to zero when the notes come exactly into tune, and then increases them as the now overloaded spring gets farther and farther below the other in pitch. Loading the slower vibrator increases the rate of beating straightaway.

§ 314. **Comparison of frequencies nearly in simple ratios 1 : 1, 1 : 2, 1 : 3, etc. Lissajous' figures.**

In § 278 the curves obtained by combining S.H.M.'s at right angles are described. When the swinging harmonograph pendulums are replaced by vibrating tuning-forks the pen and link-

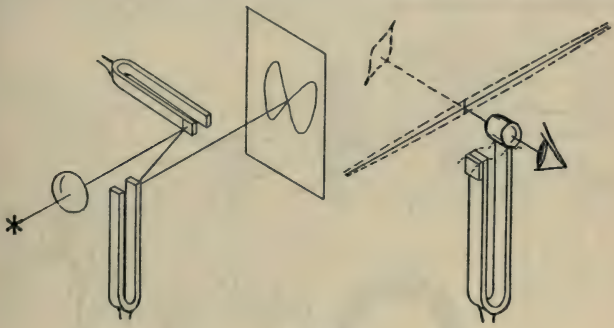


FIG. 126.

work have to be superseded by an inertia-less ray of light. To compare two forks one prong-end on each is ground flat and polished, one fork is fixed vertical and the other horizontal, and a ray from a pinhole with a lens in front is reflected from both in succession to a focus on a screen, Fig. 126, left. The vertical fork sounding alone draws the spot out into a vertical line of light, which the horizontal fork converts into a figure like those in § 278. If the ratio is not quite exact the figure slowly changes shape through one complete cycle of phases for each whole vibration gained, as in beats. Large standard forks are tuned in this way within one ' beat ' in two minutes.

A small lens mounted on a large fork constitutes the **Vibration Microscope.** Looking through it at a silvered speck on a string, say, little Lissajous' figures appear and enable vibrations to be studied, Fig. 126, right.

§ 315. **Measurement of frequency of vibration.**

The number of complete vibrations per second of a vibrating body, or the pitch of a rapid vibrator, can be found directly by chronographic methods, by comparison with the note of a syren, or stroboscopically.

Chronograph methods. The simplest way is that described in § 30, Fig. 5. Drop a smoked plate in front of a pointer attached to the vibrating body, then knowing g, n is calculated.

More elaborately in Fig. 127 (fork being tested) the plate is replaced by a rotating smoked-paper drum and close beside the marking point is another displaced electrically every second by a pendulum which touches a globule of mercury at the middle

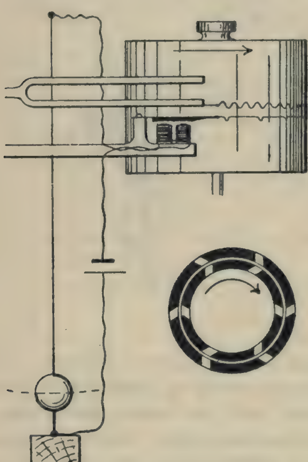


FIG. 127.

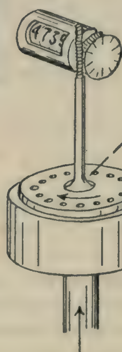


FIG. 128.

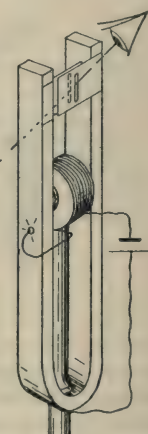


FIG. 129.

of every swing and completes an electric circuit. It is best to use alternate clock-marks only, count up the number of waves between them and divide by two.

To get the ratio of the frequencies of two vibrators let them mark side by side on a plate or drum moved at any speed, e.g. by hand.

A disadvantage is that the attached pointer loads and slows the vibrations.

§ 316. **Measurement of frequency. The Syren** is a machine which gives a measurable number of little taps or puffs per second and these blend into a note whose frequency is therefore known.

A card held against a toothed wheel (Savart's wheel) will do, but makes a sound of nasty quality, and a disc, perforated with a circle of holes revolving in front of an air-jet, is preferred. To get loudness the single air-jet is superseded by a fixed disc with as many holes as there are in the revolving wheel, so giving say a 16 times stronger puff 16 times per revolution. There is a revolution counter in gear with the wheel, Fig. 128. The holes in the two discs are often slanted in opposite directions [as in the figure on the left, a section through the outer fixed and inner spinning drums of a steamboat syren] so that the issuing air also drives the wheel, but for scientific purposes this is mock economy, for it demands an impracticable steadiness of wind pressure. A heavy wheel driven by a good-sized water motor, or electric motor off storage cells, is much better conditioned than the usual toy with its wildly varying speed.

The syren is sped up to the note to be tested, listening for beats, kept there as near as it will, and the time it takes for 500 or 1000 revolutions (say 8000 or 16,000 puffs) noted by stop-watch ; hence the puffs per second.

The syren sings when blown by water under water, hence its name. Air syrens driven by engines of 50 h.p. or more are effective coastal fog signals.

§ 317. **Measurement of frequency.**

The Stroboscope. Optical proof of Beat theory.

Attached near the ends of the large fork in use are two little overlapping plates, with a slit in each, so that both can be seen through only when in mid-swing. In each whole vibration therefore, two brief glimpses can be obtained, of the syren wheel, say. When in unison each hole moves forward one puff to its predecessor's position in one period of the fork. It is seen instantaneously twice per period. During the next period the hole behind it has moved up and is seen twice in exactly the same places as the former. Thus the whole ring of holes appears with double the actual number, and *motionless*. Now if the wheel begins to gain, the two glimpses of the second hole will show it a little ahead of where the first was, and the third will advance on the second, and so on, so that the wheel appears to slowly rotate forward. *Every time a real hole (omitting the optical*

duplicates) *passes, one beat is heard; the syren has gained one puff on the fork.* If the wheel lost the holes would lose ground and appear to run backwards, the sounds beating.

Driving the wheel faster the holes run out but presently reappear in proper number and running backwards; they slow, stop, and then run forwards. The syren has doubled its speed, and probably beats again with the octave in the fork's note.

A known fork fitted up with these plates, and usually maintained in motion precisely like an electric bell (or § 605) is a very exact 'stroboscopic' means of examining and controlling the speed of machinery, which must carry on an axle a disc with several rings of dots and a concentric square, pentagon, hexagon, etc. It is useful in the laboratory for watching equal-timed stages of rapid motions, etc. *Any vibration which keeps step with the fork is seen at rest.*

The whole affair is in a way the inverse of the cinematograph.

§ 318. **Standards of pitch.**

When once one fork has had its frequency accurately measured by one of these methods (best the stroboscope) a copy or an octave, twelfth, etc., can be tuned to it by Lissajous' figures with great exactness, and a whole accurate series built up.

Scheibler's tonometer, the best practical pitch instrument, consists of a row of such forks rising 2 or 3 vibrations each. They are compared with the note to be tested by counting beats.

Steel forks used fairly and kept clean are very constant in pitch. Rise of temperature slows them one ten-thousandth part per °C.

Lengths of Sound Waves

As it is quite impossible to apply a foot rule to a running sound wave the principle of interference has to be made use of, either in the 'interference tube' or by its action in setting up a stationary wave motion.

§ 319. **The Interference Tube**, Fig. 130, is a sort of double trombone 'slide.' The note is played in front of the upper T-piece and its sound travels round both ways to the lower T-piece, whence a rubber tube leads to the ear. Both slides are at first pushed home and it is the same distance round either way. As one is pulled out the sound heard weakens and ceases. This means that one path is now half a wave length longer than the other, so that crest and trough obliterate each other at the ear.

Thus the wave length is four times the distance the slide has been moved.

§ 320. **Stationary wave motion.**

It has been explained in § 302, Fig. 123, how running waves reflected from a rigid wall produce a 'stationary wave motion' which has nodes of no motion at half-wave-length intervals and antinodes of maximum motion half-way between them in the middle of the vibrating segments. This applies perfectly to the longitudinal waves of sound, for the rigid wall prevents the motion of the air particles in contact with it; they cannot be driven into it, and the atmospheric pressure prevents them rising from it and leaving a vacuum. In alternate antinodes there are little winds blowing opposite ways along the line of motion of the sound, and the nodes are places towards which and away from which these winds periodically blow; places where the air is alternately squeezed together and drawn out on both sides of the motionless nodal particle: *nodes are places of maximum pressure change.*

Then the length of the running wave is twice the average distance between the successive positions in which the instrument used to detect the vibrations shows signs of disturbance.

Means of examining stationary wave motion in the air.

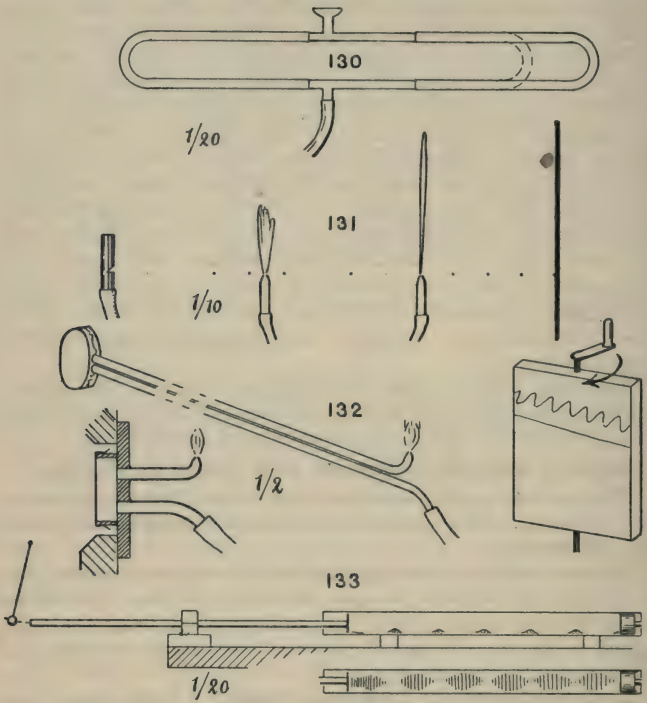
The sounding source therefore faces a flat reflecting surface a few feet away. Now to explore the line between them for nodes and antinodes.

The first method that suggests itself is to have a rubber tube to the ear, and to move its open end along the line. To-and-fro winds blowing across it will not send a sound along the tube, but at nodes the changes of pressure force air in and out of it and a sound reaches the ear.

The second means is a **Sensitive Flame.** A flame about 10 in. long, produced by supplying a pinhole jet from a heavily weighted gas-bag, is sensitive to a shrill sound, dropping to half its height, spreading, and roaring. See Fig. 131. Its quietness at the nodal points in spite of the noise of the whistle is of course what one notices. Such a flame will respond to pitches too high to be heard at all, and demonstrate them to a large audience.*

* A flame amusingly sensitive to ordinary pitches is obtained by shutting the slide of a bunsen which has one side hole only, and turning down the gas till the luminous flame just creeps below the lip of the tube on one side, becoming straight and narrow. It drops to the ordinary soft flickering shape at every word and footfall.

If the space between source and reflector is enclosed by a pipe, i.e. if the stationary waves are being produced inside a pipe at whose 'open' end is the source and whose far end is stopped up



FIGS. 130, 131, 132, 133.

flat (or else, it may be, is an open reflecting end, § 302) other detectors must be used.

§ 321. **The manometric flame**, Fig. 132. A 'manometric capsule'—a little tambour made by tying up a finger-ring of metal in the thinnest rubber tissue, so as to make it a flat box with elastic top and bottom—is mounted at the end of twin tubes long enough to reach down the pipe. One admits gas to the capsule and the other takes it away to a pinhole burner. At a

node the changes of pressure force the elastic drum-heads in and out, varying the drum's internal capacity and driving the gas out in puffs. The little flame sings and its reflection in a moving mirror is a drawn-out band of light with a jagged saw-toothed edge.

Manometric flames have been largely used in Sound. Several capsules and flames are sometimes mounted along an organ-pipe to demonstrate nodes and loops inside. Resonators (§ 323) can be fitted with them and the excitement of perhaps half a dozen observed at once, instead of listening to each singly. The capsule has been provided with a mouthpiece to receive spoken sounds, or it forms the reproducer of a phonograph, the shape of the saw-teeth in the drawn-out reflection of the flame serving more or less to analyse (?) the motion imparted to the drum-head.

§ 322. A **little paper tambourine with sand grains** scattered on it can be lowered on a thread down *vertical* pipes. The sand grains will dance and rattle except at the nodes.

In a *horizontal* glass tube Kundt scattered dry lycopodium and the apparatus is known as his **Dust Tube**, Fig. 133. The powder first forms large oval cross-striated patches occupying the loops and leaves the nodes clear [plan], but a long strong note blows it all away from the loops into little heaps at the nodes.

This dust tube has been used for measuring waves of half-length only ·175 cm. emitted from the perfectly inaudible vibration of a fork whose frequency was therefore 33,000 cm. $\div 2 \times \cdot 175 = 96{,}000$. It is often employed as an indirect means of finding the pitch of a rod in longitudinal vibration, and hence the speed of sound in wood, glass, etc. As in Fig. 133 the rod is held by its middle in a vice, and its end, with a card disc stuck on, projects into the dry dusted glass tube (1 in. $\times$ 4 ft.) which is closed at the far end by an adjustable plug. Rubbed lengthwise with a rosined cloth the rod emits a piercing note as it vibrates longitudinally, lengthening and shortening, with a node at the vice and antinodes of greatest motion at its free ends. From end to end the length of the rod is the half wave length *in it* of the note which produces *in air* the corresponding half wave length from antinode to antinode shown by the agitated dust in the tube.

Pitch of note = speed in air $\div$ 2 distance between dust heaps. Further, the ratio of lengths is that of the speed of longitudinal waves, i.e. of sound, in the material, to its speed in air.

Again, if the tube is filled with some other gas and the rod stroked, the dust figures will form at a new distance apart, and

$$\frac{\text{new distance apart}}{\text{air distance apart}} = \frac{\text{speed of sound in the gas}}{\text{speed in air}}$$

EXAMPLES.—CHAPTER XXXV

1. What are beats ? Prove that the frequency of the beats produced by two notes with a small interval between them is equal to the difference between their frequencies. How could you ascertain which of two vibrating strings producing beats had the higher pitch ? [L.]

2. A column of air and a tuning-fork produce 4 beats per second when sounding together, the fork giving the lower note, air at 15° C. At 10° C. they produce 3 beats per second. Find frequency of fork. [L.]

3. Calculate the velocity of sound in a gas in which two waves of lengths 1 and 1·01 m. produce 10 beats in 3 seconds. [L.]

4. Describe experiments to show interference and to verify laws of reflection in case of sound waves. When are (a) beats, (b) sound shadows, produced ? [Ab]m.

5. Describe the manometric-flame method of finding nodes and loops in an organ-pipe and give two examples of results. [L]m.

6. Give the theory of stationary sound waves in air and explain how they can be used to find the velocity of sound in a metal rod. [L.]

7. How can the wave length of a high-pitched whistle be determined ? How can it be shown that when plane sound waves are reflected at a plane surface the angles of incidence and reflection are equal ? [L.]

8. A vibrating tuning-fork is viewed through a rotating disc having a circle of holes. If the fork appears at rest what must be the relation between speed of rotation of disc and frequency of fork ? Explain how such an arrangement can be used to determine the frequency of a vibrating body. [L.]

9. Explain carefully some method of finding accurately the pitch of a tuning-fork, estimate the greatest percentage error likely to occur. [L.]

10. Describe the construction of a siren. An air siren is provided with a trumpet equivalent to a straight open tube 2 ft. long. At what speed should the machine be run for maximum effect ? [L.]

11. Show how ratios of notes on scale can be verified by siren. [Ab.]

Chapter XXXVI

RESONANCE. PIPES AND STRINGS

§ 323. **Acoustic Resonance.** We can now return to the subject of § 281 and find many illustrations. It was there pointed out—

(1) That anything can be compelled to vibrate at any rate and to any extent we please provided plenty of force is used. One can take hold of the prongs of a tuning-fork and move them in and out slowly, or the same fork when sounding will compel the table-top to vibrate when pressed on it, or sounding strongly will stir up the air inside any cavity whatever.

But (2) that when the periodicity of the force applied agrees nearly with the natural time of free swing, a quite small force will gradually work up a large motion.—A very small electro-magnet induces strong vibration in a large fork which itself makes and breaks the current at the proper moments. The air in the resonance tube of § 309 oscillates violently and emits a loud sound when excited by a small fork of its own natural time of swing. There is a different size of resonance box (like a cigar box with end knocked out) to mount each fork upon which will bring out its note far better than the table-top. And every jar, jug, bottle, box, lamp chimney, gas globe, etc. etc., has a note of its own to which it will resound most strongly when sung to. This note can sometimes be elicited by blowing across the mouth. Moreover, they will pick it out of any complex sound that contains it as a harmonic component, and resound to it. (A very irregular noise will provoke a feeble response from any resonator, e.g. breakers and sea shells). It is the note by which one guesses how the filling of a jug under the tap is progressing, for *partly filling so as to reduce the air space raises the note of a resonating cavity.*

On the other hand, *partly closing the mouth lowers the note.* One's own mouth cavity is the resonator to the vocal chords and the change of its note can be heard as one scratches the cheek with the finger-nail and slowly opens and shuts the mouth.

In a small room one particular low note will come out very loud as one sings or hums down the scale; it is a natural note of the resounding room.

Acoustic resonance is of course not peculiar to air cavities only. A fork pressed on the sound-box of a string instrument sets the whole box into slight vibration. If one of the strings is of the same frequency (or double, or treble, etc.) the tremor of the bridge will provide the necessary periodic push, and the string vibrates visibly and audibly.

Nor is solid contact necessary. Open the piano, depress the forte pedal to lift the dampers, sing any note, and the instrument answers *that same note*. The air waves you produced set the sound-board in vibration, but only the corresponding strings took up and stored energy from it, to be thereafter returned to it and thence to the air. (There may be, however, an improvement in musical quality, because the octave string also vibrates a little.)

§ 324. The piano contains only a few dozen definite notes, but intermediate notes are resounded to, though the frequencies of the strings either side of them will be probably a dozen per second wrong. Now it was pointed out in § 282 that when the resonator's motion was 'damped,' resonance was neither so strong nor so sharp, but occurred fairly well over a long range of frequencies. A broad sounding-board is of course intended and admirably adapted to give out quickly to the air the energy of the blow on its strings; therefore its motion quickly dies down, it is '*damped by radiation of energy*' and this explains why resonance was wider spread. [Perfectly sharp acoustic resonance is a paradox. It would imply no damping at all, therefore no radiation, no increased loudness, but on the contrary must be inaudible.] Since a broad board radiates sound-waves powerfully it ought (1) to pick them up easily (2) over a wide range of frequencies and (3) should therefore be able to emit many different notes when properly excited. A thin drawing-board carried along a city street trembles at every loud noise, while the immense variety of Chladni's figures, § 336, shows the truth of (3). An old violin owes some of its excellence to the equable response of the seasoned sounding-box to all notes.

Among air cavities, open boxes and broad-mouthed sea shells must emit their energy fast, and therefore resound broadly. Long narrow pipes, and the resonators with small mouths employed for analysing sounds, dissipate their contained energy much more slowly and therefore resound more precisely.

The only form of resonating air cavity that can be considered in detail in this book is a straight tube.

§ 325. **The Resonance tube.**

It will now be recognized that the fork of § 309 was simply calling forth the natural note of a resonating tube which had the same pitch as itself. The conclusion arrived at there can be restated in another form :—The length of the waves emitted by a plain pipe stopped at one end when sounding its natural note is four times the (corrected*) length of the pipe. For the distance sound travels in one period *is* the wave length.

Suppose now that the stopping is knocked out of this tube and an extension of double its length fitted on, and the far end stopped. Resonance will again occur to the same note, because the extension is a half wave length and the crest will just travel along it and back in one period of the note, and will catch up the fork on its *second* swing up. And another half-wave-length extension will produce a yet longer tube that can resound to the same note, the crest travelling in the tube for two extra periods and catching the fork on its *third* swing up, and so on.

The resonance tube in the laboratory is usually a long vertical glass tube an inch or more in bore, and the movable stopping is the surface of water which can be run in or out to any level, Fig. 134.

It is a very useful means of comparing the pitches of different forks, etc. : *pitches are inversely as the resounding length.*

And assuming the speed of sound S at the temperature of the air in the tube to be known the actual pitches follow from the equations

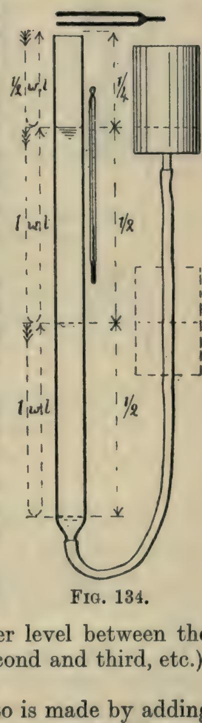


FIG. 134.

S = pitch × 4 times corrected* first length for resonance.

S = pitch × twice the lowering of the water level between the first and second lengths for resonance (or second and third, etc.), no correction being necessary.

* * The '*open-mouth correction*' referred to is made by adding one-third its diameter to the tube length, (or two-fifths if there

is a broad plate round the mouth). Air particles just outside the mouth cannot move so freely as in space open on all sides: this slows the motion and can be regarded in calculation as a lengthening of the tube.

PIPES

Musically speaking, most wind instruments are pipes, and from a physical point of view a pipe is a resonance tube provided with some means for producing a commotion in the air at one end of it.

§ 326. Pipes and how they are blown.

(1) The ancient Pan-pipe was a row of hollow reeds of graduated lengths, stopped by the stem 'knots' at their lower ends and

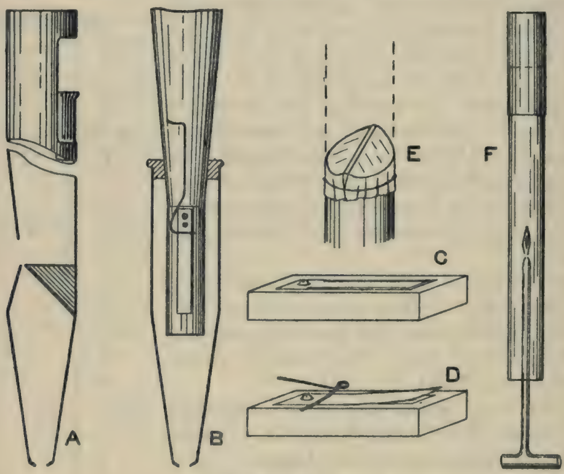


FIG. 135.

made to sound by blowing across the open tops. Nowadays one occasionally uses a key in the same way, and the winter wind uses a keyhole. Flutes and fifes are uniform tubes open at the far end and with a large side hole at the near end, merely blown across.

(2) In the 'flue' pipes of the organ, Fig. 135 (A), and in most whistles, there is the well-known 'mouth,' up across which blows a flat stream of air, from a narrow slit inside the lower lip, to impinge on the thin wood or metal edge of the upper lip. 'Stopped'

organ-pipes are closed at the top by an adjustable plug : 'open' pipes are open at the top. Steamboat whistles are stopped pipes with double mouths, railway whistles and factory bulls usually have mouth all round, to get most noise.

(3) In the 'reed' pipes of the organ, Fig. 135 (B), there is a **'reed'** consisting of a narrow elastic metal tongue almost closing the narrow slot through which the wind is supplied.

The tongue either swings in and out of a slot slightly larger than itself (free reed C) or flaps down on to a smaller slot (beating reed D), thus permitting the wind to issue in periodic puffs. For such reeds in miniature dissect a toy mouth-organ. The reed has a note of its own and the natural frequencies of tube and attached reed must be about the same, or resonance is defective and the pipe speaks badly.

Clarionets, oboes, and bassoons have 'reeds' of split cane. Stretched membranous 'vocal cords,' with the resonant pharynx and mouth, produce the human voice. E is a rough model larynx constructed of two pieces of thin sheet rubber tied over the cut end of a pipe so as to leave a narrow slit between them. A resonance tube (dotted) can be added.

(4) The lips are the vibrating reeds for brass instruments, and also in whistling, when the mouth cavity is resonator.

(5) Tubes can be sounded by a flame burning inside them ; F has a paper tuning-slide at the top. Recollect the musical efforts of an occasional incandescent gas burner, and listen to the deep booming of the chimney when you are 'drawing up' the fire with a newspaper.

Note.—A reed is practically a stopped end, it is only a small aperture and there is a wall of compressed air behind it.

One can understand metal reeds, but how is it that blowing contrivances which of themselves make only a feeble irregular noise—a very 'dry whistle'—can call forth loud musical notes from the tubes ?

Any fluid flowing through a narrow crack at more than a very slow speed sets up eddies. It is these that make the dry whistling sound : they are heard and seen when a flat gas flame is turned too high and flares. These eddies mean local variations of speed and pressure, § 87, and send little impulses fluttering into the pipe. The large mass of air begins to pulsate and soon alternately blows the thin stream of wind away or sucks it in in puffs * at times to suit itself, taking up the energy of the puffs to produce its

own note; just as a heavy pendulum takes energy when it pleases from the scape-wheel and keeps its own time.

How preponderant is the control of the resonator anyone can feel in whistling a tune. The lips remain fixed, while the tongue is busy all the time altering the size and shape of the resounding cavity.

* A badly aimed stream or misshapen upper lip of course enfeebles this action: steamboat whistles are often husky on this account even after clear of water.

§ 327. From what has been said in §§ 309 and 325 it will be clear that in a sounding pipe the air is acted on by waves running both up and down and is therefore in the state of stationary wave motion described in §§ 302 and 320. *Read these four paragraphs again.*

Stopped pipes, Fig. 136. Taking these first, the *stopped end is a Node* of no motion and the *open mouth an Antinode* of maxi-

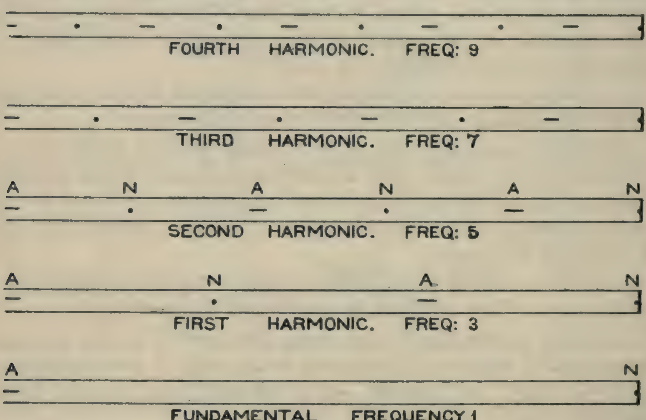


FIG. 136.

mum motion, the wind blowing to and fro most freely there. (·4 cm. motion has been observed by the aid of smoke at the end of a pipe 125 cm. long.)

If no other nodes are present the pipe is now sounding its lowest or **fundamental** note, and **the wave length of this, 4AN, is four times the length of the pipe.**

Then in § 325 it was pointed out that a tube 3 times as long could give the same note, having now an extra node and antinode at the thirds of its length. That is, it is giving *a note whose wave length is four-thirds the length of the stopped pipe. This is called its first* **overtone**, and its frequency is evidently 3 times that of the fundamental, since wave length × frequency = constant Speed. It is a **harmonic overtone**, for the ratio of frequencies is a small integral number and it lies in the harmonic scale (§ 346) containing the fundamental (G in the octave above the fundamental C). Indeed all the overtones of plain pipes are **harmonics.**

It was further pointed out that a tube 5 times as long could give the same note, having now 2 extra nodes and 2 extra antinodes at the fifths of its length. That is, it is giving a *note whose wave length is four-fifths the length of the stopped pipe, its second overtone,* 5 times the pitch of the fundamental.

So one can go on as in Fig. 136 dividing up the **stopped pipe** into any odd number of equal parts, keeping the stopped end a node and the open an antinode, putting in alternate nodes and antinodes along the pipe and producing successive harmonics of frequencies **1, 3, 5, 7, 9, and any odd number** of times that of the fundamental.

These notes can be brought out in succession by blowing the (narrow) pipe harder, but the full natural tone of the pipe results from a complex air motion which contains them all as its simple harmonic components or **Partials**, §§ 277, 343.

§ 328. **Open pipes**, Fig. 137, which are tubes open at both ends, must have antinodes at both ends, and the simplest stationary wave motion possible in them has therefore a node in the middle of the pipe. Such motion is possible, for as explained in § 302 reflection can take place from a loose or open end. The pipe acts like a couple of stopped pipes of half its length, put bottom to bottom. **The wave length of the fundamental of an open pipe, 4AN, is therefore twice the length of the pipe**, so that unstopping a pipe raises its pitch an octave, and vice versa. Blow across any bit of tubing, and try it.

The next possible motion, got by putting in one extra node and antinode, has an antinode in the middle and nodes at the quarters, *its wave length is twice half the length of the pipe* and its frequency twice the fundamental.

In the next there is again a node at the middle and the pipe is again like a stopped pipe standing on its own reflection; the frequency is three times the fundamental.

So one can go on as in Fig. 137 dividing up the **open** pipe into any even number of equal parts, keeping both ends antinodes, putting in alternate nodes and antinodes and producing successive harmonic overtones of frequencies **1, 2, 3, 4, 5, and any number** of times that of the fundamental. The presence of the even harmonics gives the open pipe a fuller musical tone (§ 343) than

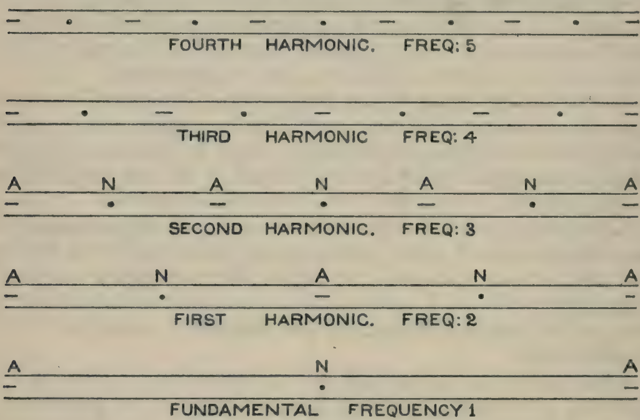


Fig. 137.

the stopped pipe: compare the 8-ft. open diapason with the 4-ft. stopped.

It can be shown that a **conical pipe**, whether open or stopped at the small end, has the full series of harmonics of an open pipe. Hence the tapering form of practically all reed instruments, for the reed is almost a stopped end.

§ 329. **Wind Instruments.** The production of several notes from one pipe in wind instruments is effected by altering the force of the blast (a), or by altering the length of the pipe (c), or both (b).

(a) The cheery simple compass of few notes of a bugle or post horn consists of the first five harmonics—into which the conical pipe breaks by harder blowing.

The most perfect example of this is the long French horn which gives the sixteen notes got by multiplying the fundamental

(herein called C for simplicity) by the natural numbers from 1 to 16 as follows :—

	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$		$\frac{15}{8}$
Diatonic scale . .	C	D	E	F	G	A		B
1st octave . .	1	–	–	–	–	–		–
2nd ,, . .	2	–	–	–	3	–		–
3rd ,, . .	4	–	5	–	6	–	7	–
4th ,, . .	8	9	10	11	12	13	14	15
5th ,, . .	16	–	–	–	–	–		–

11 is a trifle too sharp and 13 a trifle too flat, 7 and 14 are A sharps.

(b) Brass piston instruments have their tubes temporarily lengthened by crooks brought into circuit by pressing the piston valves. This enables the gaps in the natural trumpet scale to be filled up without going beyond the seventh harmonic.

(c) Fifes and flutes are virtually open tubes extending from the mouthpiece to the first hole that is opened.

Strings

Strings are supposed to be perfectly flexible, uniformly heavy throughout their length, and stretched with a force uniform throughout and quite unaffected by their vibration. Those mostly in use are catgut and thin steel, wrapped with wire for lower notes to increase mass without spoiling flexibility. Thick wires are very unmusical.

As everyone knows, **their musical vibrations are transverse ;** whether in one plane or like a skipping-rope does not matter in the least (cf. pendulum). They can be studied visually thus :—

§ 330. **Melde's experiment.** To a strongly vibrating prong* is attached a long horizontal thread of white crochet cotton stretched over a pulley at the far end by 50 grm. or more. The transverse waves sent running along the string are reflected at the pulley and the two equal wave trains running opposite ways set up a stationary wave motion, dividing the string into a succession of nodes and loops, as in Figs. 123 and 138. At first the motion is unsteady and dodges about, but after careful adjustment of the weights in the pan, shows well-defined segments and steady nodes, becomes more ample (resonance), and then the average length from node to node=half length of running wave.

* A small electric-bell mechanism can be pressed into service.

Now gradually increase the pull on the string and after an interval of unsteady quivering it will settle down to steady vibration with one less segment (B, Fig. 138, weight increased

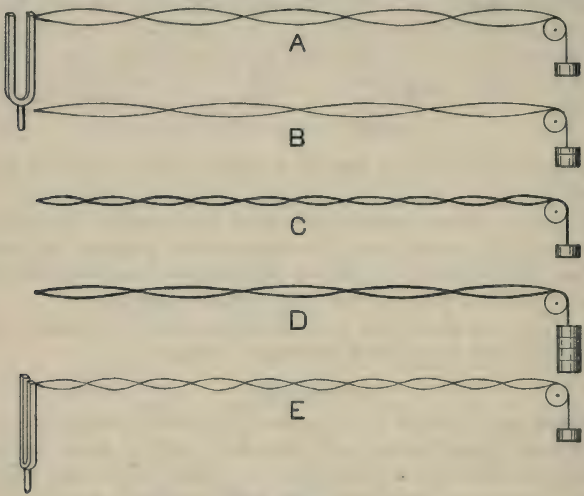


FIG. 138.

about one-half). Putting more weights on the pan causes this to be repeated, segments disappearing one by one.

Measurements of lengths and weights will have shown that

Length of segment varies as square root of stretching pull, e.g. to get segments of double length the pull is quadrupled.

Now loosely twist four threads together so as to get a string 4 times as massive, and hang on the same weight used to pull one thread. The segments shorten to half their length, C.

Length of segment varies inversely as square root of mass per cm. Hang on 4 times the weight and segments resume original length, D.

Finally, if the experiment is made with double the frequency* (n) the segments are halved in length E. Putting all together,

length of segment $\propto \frac{1}{n}\sqrt{\frac{\text{pull}}{\text{mass per cm.}}}$ and by weighing a

* Same fork serves, turned at right angles. In first position the reader can make out that frequency of string = half that of fork.

length of the thread to get its mass in grammes per centimetre, and reckoning pull in dynes, it will be found that the constant of proportionality is $\frac{1}{2}$ and

$$l=\frac{1}{2n}\sqrt{\frac{P}{m}} \quad \text{or} \quad n=\frac{1}{2l}\sqrt{\frac{P}{m}}$$

§ 331. **Theoretical deduction of this expression.** By § 286 the speed at which transverse waves run on a stretched string is $V=\sqrt{P/m}$.

Now V = waves per second × length of running wave
= waves per second × twice length from node to node.

$$\therefore V=2nl=\sqrt{P/m}. \qquad \therefore n=\frac{1}{2l}\sqrt{\frac{P}{m}}$$

The frequency of a string is the square root of [stretching pull in dynes ÷ mass of 1 *cm. of string] divided by twice the length of one vibrating segment.*

§ 332. **The Sonometer or Monochord** is the rudimentary string instrument employed in the laboratory in studying these laws of strings by ear. Over two sharp-edged bridges near the ends of a long sound box a thin wire is stretched by a spring balance or by weights. A third bridge, a little taller, can be placed under the wire to partition off any measured length of it. A second wire stretched on wrest pins is useful. Here the wire is plucked and becomes the driver and the sound box is the driven resonator which gives out the sound. For such a wire stretched between two heavy weights on the ground 'cuts through' the air almost noiselessly. The bridges which transmit the wire's motion to the board are therefore not exactly (though quite nearly enough for us) fixed nodes.

Only such vibrations can persist on a string as have the bridges for nodes. All others die out forthwith. Put the movable bridge at $\frac{1}{3}$, pluck the shorter section and the longer vibrates also, giving its octave. Put the bridge at say $1/\pi$ and the incommensurate longer piece will not take up any motion.

Provided with two or three forks of known frequencies the laws can be studied thus :—

(1) $\mathbf{n} \propto \mathbf{1}/l$, *frequency is inversely as length of vibrating segment.*

A. If the wire, plucked not far from one end, is touched *lightly* at the middle point this is induced to become a node and the fundamental is choked out, leaving the *octave* promi-

nent; the string vibrating in two halves. Touched at $\frac{1}{3}$ the *twelfth* sounds out (G in octave above C), at $\frac{1}{4}$ the double octave, and so on.

In all that follows the string is assumed to be vibrating in one piece, and l becomes its whole length.

B. Lengths in tune with the various forks will be found proportional to their vibration numbers.

Tuning is tested by slowing out of beats, or by a little paper jockey jumping off when the wire is exactly in tune and resounds to the fork pressed on the sound board.

(2) $\mathbf{n \propto \sqrt{P}}$, *frequency is proportional to square root of stretching force.* Tightening strings sharpens their pitch. The stretching weights necessary to tune the same length of the same wire to different forks will be found proportional to the squares of their vibration numbers.

(3) $\mathbf{n \propto \sqrt{1/m}}$, *frequency is inversely as square root of mass per centimetre.*

Different wires are stretched with the same force and the same length of each is used. Another bridge is moved under the additional wire and lengths on that found which are in unison with the notes of the different wires. Then these wires are cut and weighed, weight$\div$length$=$mass in grm. per cm. The tuned-up lengths on the side wire will be found $\propto \sqrt{m}$, hence by (1) B above, $n \propto \sqrt{1/m}$.

§ 333. What substance the string is made of does not matter in the least, nor how the mass is made up, nor whether it is round or square or a flat ribbon. It is only the mass per unit length that counts, and this should be computed first, in preference to inventing new formulæ for various cases.

Notice the use of the monochord in comparing the pitches of notes produced by any instruments. They are inversely as the lengths of wire in tune with them. And a knowledge of P and m will further enable them to be calculated absolutely, using the whole formula.

The **overtones** possible on a string are all those that have the bridges for nodes, i.e. the string may vibrate in any integral number of parts, giving frequencies 1, 2, 3, 4, 5 . . . times the fundamental.

§ 334. **The longitudinal vibrations of rods and wires** are the only others that lend themselves to simple theoretical treatment.

They have been referred to in § 322. The rod is held in the middle, or the rod or wire clamped firmly at one or both ends, and wiped lengthwise with a wet leather or rosined cloth, when without any visible vibration it emits an unmusical shriek. Like the air in a pipe, it is in lengthwise oscillation, for a pellet hung in contact with the flat free end dances off when it sounds, Fig. 133. The shuddering motion of rubber tubing pulled through wet fingers, and the wet and dry rings left on it, evidence a slower vibration of the same sort and glass tubing has been set into such violent motion that it shattered into rings.

The thickness of the wire or rod makes no difference to the pitch, each square millimetre of cross-section (of any shape) looks after itself, and a thick rod can be regarded merely as a bundle of thin ones each giving the same note.

Clamped points are nodes. Free ends, or the middle point when clamped at both ends, are antinodes. The wave length in the material is 4AN as usual, e.g. bar clamped at end has w.l. = 4 times length and the harmonics of a stopped pipe; wire clamped at ends has w.l. = twice its length and full series of harmonics. The speed of travel of the longitudinal disturbance = of course the speed of sound in the material

$$=\sqrt{\text{Young's modulus}\div\text{density}} \text{ and} = \text{frequency}\times\text{wave length.}$$

Torsional vibrations. Rods can also be set into shrill torsional oscillation by pulling a rosined string wound round them. The speed of a torsional wave is $\sqrt{\text{rigidity}\div\text{density}}$. The treatment is the same as above.

EXAMPLES.—CHAPTER XXXVI

1. If the handle of a vibrating tuning-fork is held against a wooden board the amount of sound produced is considerably increased. Explain why. Is the time during which the fork goes on vibrating affected, and if so, why? [L.]

2. Explain why a vibrating string is scarcely audible unless a sounding-box be employed. How do the shape and size of a sounding-box influence the audibility of the note? [L.]

3. When A sounds its fundamental B resounds, but A does not resound to B's fundamental. Which has the higher fundamental pitch, and why? [M.]

4. Describe the construction and mode of action of one form of organ-pipe. How has the motion of the air in such a pipe when working been investigated, and with what results ? [L]m.

5. How find frequency of fundamental of organ-pipe ? Pipe 5 in. long, at 15° C., find frequency and nearest note in musical scale if $c''=540$. [Ab.]

6. Forks of frequencies 125, 250, 500, 750, 1000 are successively held over the open end of a stopped pipe which resounds best to the 250. What happens with the others ? [L]m.

7. Two straight tubes of effective lengths 48 and 64 cm. resound to the same note. The shorter tube is closed at one end, the longer is open. Show the nodes in the tubes, and find the frequency of the note, the speed of sound being 333 m. per sec. [L.]

8. Given a stretched wire of variable length and a tuning-fork, how would you find the note emitted on plucking a thin bicycle-spoke ? How does this note change as the spoke is tightened ? [L.]

9. State concisely the laws of transverse vibration of a stretched string. Bowed transversely near its middle point, a wire emits a note of frequency 256 vibrations per second when loaded with a weight of 100 lb. ; what is frequency of note of double the length loaded with 225 lb. ? [L.]

10. A bridge is placed under the string of a monochord at a point near the middle, and on plucking the two parts of the string 3 beats per second are produced when the load is 8 kg. Determine the rate of beating of the two parts of the string at 11 kg. [L.]

11. Two strings otherwise equal have densities 1·3 and 21·8. Find ratio of frequencies. [L]m.

12. Compare frequencies of two strings of same length and diameter stretched with 10 and 1 kg. respectively, and densities 7·8 and 1. [L]m.

13. Calculate the frequency of vibration of a 55·9-cm. length of wire, of total weight ·3324 grm., stretched with 11·2-lb. wt.

14. Give an account of Kundt's dust-tube method of comparing velocities of sound in different gases. Trace effects of changing length section and material of the rod. [L.]

15. Find Young's modulus for a rod 1·72 m. long, density 8·5, which held at the middle point and stroked lengthwise gives 1000 vibrations per second.

16. Show how periodic movements of a stretched string can be represented by the passage along it of waves in opposite directions. [M.]

17. Distinguish clearly between the motions of the air particles in an open tube (a) when a sound wave simply passes along it and (b) when the tube is blown as an organ-pipe. On what condition can motion (a) set up motion (b) ? [L.]

Chapter XXXVII

VIBRATORS PRODUCING MORE COMPLEX TONES

§ 335. **Transverse vibrations of bars.**

From the days when we essayed tunes on a row of pins driven to different depths in the table we have all been familiar with the sonorous transverse vibration of 'bars.' In the little clockwork musical-box there was a whole row of them in a 'comb' plucked by pegs in a revolving barrel. One is the mainspring of that curious instrument, the jew's harp. Worked by wind, thin 'bars' form the reeds of the mouth-organ, harmonium, concertina, etc., and with the addition of resonance boxes or pipes of the American organ, the organ, and—the motor horn, voiceless without its trumpet. The old American clock hammered out the hours nasally on a wire gong and the modern drawing-room clock chimes quite tunefully on what are really long curled-up steel bars, struck near the fixed end with soft hammers. All these are bars clamped at one end and free at the other.

In the tuning-fork two equal bars balance each other's motion and clamping is unnecessary: a bit of wax stuck on one leg destroys the balance and the fork spends its energy in shaking the hand, and soon stops. Unclamped also are the straight bars of the harmonicon, supported (not too rigidly) at the nodes of their fundamental vibration about one-fifth length from either end.

Additional nodes are present in bars sounding overtones, and can be demonstrated by scattering sand on the horizontal bar. When sounded the sand gathers at the quiet nodes. In this way a node can be found about one-third way down a tuning-fork prong, when the shrill first overtone, more than 6 times faster than the fundamental, is ringing. The clock gong is struck near its root and overtones ripple along it; its fundamental vibrations when the free end is plucked are slow enough to count.

The Overtones of bars are not Harmonics, for they are not in the simple ratios 2, 3, 4, 5, etc. times the fundamental, § 327.

It is easy to see that the thicker and stiffer [Young's modulus] the bar is at the fixed part the greater will be the elastic forces called into play by a slight bend. This, and lightness in the free moving parts, means rapid vibration. Filing a fork near the tip raises its pitch ; near the base, lowers it. On a large scale all this is of interest to engineers, bridges and ships under 'live' forces are vibrating bars.

§ 336. **Plates.**

The vibrations of plates are very complex and numerous, patterns of nodal lines can be obtained by scattering sand on them when vibrating. These **Chladni's figures** are usually demonstrated on a square metal plate clamped in the centre and bowed somewhere on the edge, while another point is touched by the finger

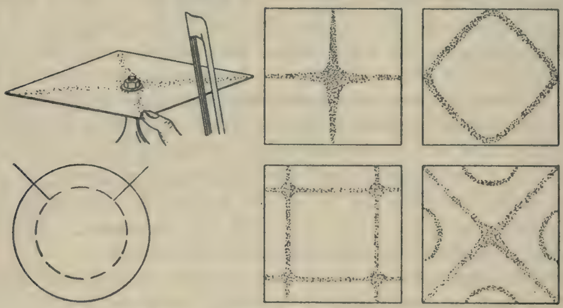


FIG. 139.

to induce it to remain a node. A few figures are given in Fig. 139 and many more will be found in Tyndall's *Sound*. Each has its own note. They depend on where the plate is bowed and touched and how it is supported. A very simple mode of vibration of a plate fixed at the centre produces a nodal cross, the alternate quadrants moving opposite ways. A uniform disc supported at points on a circle two-thirds its diameter and struck in the middle acts as a gong with this as a nodal circle.

The mica or metal diaphragms of telephones, gramophones, etc., are plates fixed round the edge ; they can vibrate in so many different ways that the 'resonance' to be expected at their own 'natural frequencies' hardly shows up at all, and they respond to all notes without exaggerating any.

§ 337 : **Gongs and bells.**

The ordinary dinner gong is a plate with a turned-up edge, the stiffness and extra weight of which brings a nodal circle out to the suspending string holes.

Bells can be looked upon either as deeply ' dished ' plates or as short cylinders. When struck in the ordinary way the circular mouth becomes elliptical and vibrates between this and an ellipse at right angles to it, Fig. 140. Four points 90° apart are moving radially, and since the outer arcs are longer than the flat inner arcs the points at 45° have to move tangentially. Hence the tangential drag of the wet finger on a tumbler rim evokes its note. These 45° positions are nodal ' meridians ' (diameters in plan) ; pellets hung in contact with the bell there are not driven off. Besides this motion characteristic of a cylinder the bell also has nodal circles like a plate, the whole rim heaving up and down and making the bell alternately shorter and taller.

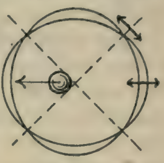


FIG. 140.

The five partials of the best modern English bells appear to run as near the frequency ratio ·25 : ·5 : ·6 : ·8 : 1 as the founder can get them. The highest is the loudest after the usual hard blow and gives the bell its name. The lowest two are heard in a muffled peal. The ·25 has 4 nodal meridians, the ·5 a nodal circle in addition, and the 1 has 8 meridians. The beating as the sound dies away originates from accidental irregularities in the bell.

A chime of steel tubes, 8 diameters long, slung on a rope through two holes about one-eighth below the top, and struck on the top edge, costs little more than a solitary monotonous church bell and sounds very well, unless the ringer attempts hymns.

§ 338 : **Singing sand, etc.** Some sands have acquired a reputation for emitting a musical note when disturbed. Such are to be found in patches on the beach at Studland and in Eigg. A smooth basinful gives a more or less definite note when prodded. The explanation is obscure, but these sand grains are very clean and uniform in size. I have noticed parts of the path up Ben Nevis, about 1000 ft. below the summit, where the footfall is a loud clear ' clink ' (about A 880 and its octave below) and where the stones certainly are particularly uniform in size, tight, and as clean as superabundant rain can wash them.

§ 339: **Membranes.**

Membranes are to plates what strings are to bars, their power of vibration is due to the tension put upon them and not to natural stiffness. Their vibration bears some general resemblance to that of plates and can be studied experimentally by scattering sand on them in the same way. The blow of a drum-stick on the drum-head can be likened to the fall of a drop into a teacup, circular ripples flow out and reflected at the *fixed circular edge* return and produce nodal circles as the disturbance continues. The soft stroke of a 'muffled' stick smothers out short waves and dulls the tone. In Sedley Taylor's Phoneidoscope a soap film is stretched over a cup sung into through a short speaking-tube, and shows the different and beautiful nodal patterns in brilliant colours for every note. The toy string telephone shows how well membranes can take up and reproduce notes and the string transmit them by longitudinal vibrations.

A membrane with *free edge* is the grass-blade between the thumb-knuckles of the hands closed as in supplication. Its squall when blown on is probably pitched by their resonating cavity.

These two types of membrane will be recognized in the following :—

§ 340. **The Voice.**

Stretched across the windpipe are two membranes, the 'vocal cords,' very roughly imitated by the strips of rubber in Fig. 135 E. When breathed through and tightened so that the 'glottis' between them becomes a narrow slit, they vibrate. For high notes they are very tight and only the thin edges quiver. The resonating cavity of the pharynx and mouth controls the pitch, and the tongue, teeth, and lips the articulation of the sound (see § 344).

Shouting with wide-open mouth means overblowing and straining the cords. A Megaphone becomes useful now, for in it a narrow conical mass of air first receives all the energy formerly spread out almost spherically, its vibrations have therefore much greater amplitude and at the nearly nodal reed end (§ 326) provide a greater back pressure. This supports the vocal cords and enables them to be blown very hard without injury. You work harder. The *directive* action of a megaphone is limited, for the 5-ft. waves of a man's voice diffract widely from its 1-ft. aperture, § 295.

§ 341. **The Ear.**

In the ear the air waves impinge on a stretched tympanum T (area 1·3 sq. cm.) whose motion is transmitted by a chain of bones (wt. 50 mg.) which reduces its amplitude to two-thirds, to the membrane covering the 'oval window' O (area ·028 sq. cm.) every square millimetre of which therefore gets about $\frac{3}{2}\times 1{\cdot}3 \div {\cdot}028=70$ times the force of the original motion. This intensified force is able to disturb the liquid filling the 'cochlea,' a 'snail-shell' cavity shown, unrolled, towards the right of Fig. 141. Along the cochlea, and dividing it into two compartments, stretches the long narrow tapering 'basilar membrane.'

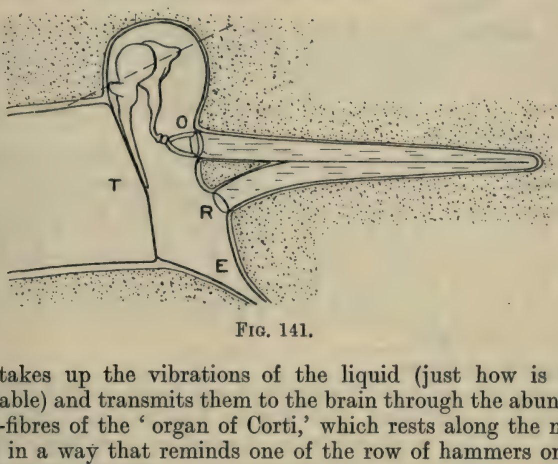


Fig. 141.

This takes up the vibrations of the liquid (just how is very debatable) and transmits them to the brain through the abundant nerve-fibres of the 'organ of Corti,' which rests along the membrane in a way that reminds one of the row of hammers on the strings of the piano.

The tympanum and mechanism of the middle ear can be destroyed without total deafness resulting, but they have protective and selective powers in addition to the transmissive and intensifying mentioned above.

The dotted line passes through the attachments of the bones to the skull. R is the pressure-relieving membranous 'round window' and E the Eustachian tube.

§ 342. **Audibility.**

The note of a whistle has been heard easily by ears which received only a twenty-thousandth of an erg per second from it,

and it is claimed that a millionth of an erg per second can just be heard. Ordinary loudness probably supplies tenths of ergs.

The *lowest audible pitch* probably lies between 15 and 30 per sec., depending on the instrument.

The *highest audible pitch* as tested by a Galton's whistle (a very small whistle with a graduated plug sliding in the end of the tube) lies between 30,000 and 40,000, but diminishes in age; elderly persons being often unable to hear the squeaks of bats, etc.

A seven-octave piano extends from 30 to 4000. Our perception of musical values gets very feeble at these frequencies, one note sounding very much like another.

§ 343. Quality.

The **quality, tone,** or **timbre** which enables one to distinguish what sort of instrument a note is being played on, is undoubtedly due to the co-existence with the principal note of overtones whose *numbers*, *relative pitches*, and *loudnesses* are very characteristic of the instrument.

Several considerations go to show that a simple harmonic disturbance, corresponding to a sine-curve wave, produces the impression of a simple or pure tone. One may be given:—As a note is dying away it loses its distinctive quality and reduces to the pure fundamental tone: now we know from Mechanics that always, as any motion dies out, the energy of the little rapid parts is dissipated soonest, and it is only the slowest swell that persists to the last.

Analysis of tones.

A note on any musical instrument can be analysed into fundamental and overtones. This may be done to some extent by a trained ear, but much better by resonators ready tuned to the expected overtones. The plain resonance tube can be used (for instance it often confounds the over-vigorous wielder of a tuning-fork by resounding to an unsuspected octave), but a battery of tuned resonators like those of § 323, each connected to its own manometric flame capsule, is better.

A new way is to photograph the vibrator's motion on a moving plate, or else to record the sound on a phonograph cylinder,* and then magnify the indented wax record by an arrangement of light levers, etc., on to a moving plate. In these ways one obtains wave curves which can be mathematically analysed into com-

* I think there is no need to attempt any description of phonographs and gramophones.

ponent S.H.M.'s, § 276. Results so obtained have agreed with the resonators, thus confirming the equivalence of the simple harmonic motion and the simple note.

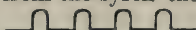
Synthesis of tones.

But the best proof of the theory is that the battery of resonators can rebuild the tones of different instruments. Each resonator is provided with its appropriate fork, kept going electrically, and when the proper ones are excited to the same extent as before, judged by the flames, *the original tone is reproduced*—flute, trumpet, violin, bell, and the vocal sounds, the Vowels.

§ 344. **Constitution of certain tones.** In these ways one finds that forks and wide organ-pipes give nearly pure tones, flutes their fundamental+weak octave, narrow pipes and strings give all at once the long series of harmonics already described, into which they so easily break. The 2, 3, 4, 5 frequency harmonics improve the musical tone (compare a fork with a piano, and then with the note and its octave together), but too great loudness of higher harmonics produces a brilliance apt to become strident. 6, 7, 8, 9 are choked by the soft blow of the piano hammer near the place they require nodes (about $\frac{1}{8}$ length), or in the violin by a good sounding board. The very high are weak in any instrument that can pretend to be musical, but sometimes afflict the user of the monochord. Harsh tones contain overtones not in harmonic ratios, squeaks mean exaggerated high ones. The peculiarity of bells in normally having the highest partial loudest has been noticed in § 337. Silver, beloved of popular fancy, contaminates bell metal, leading to flaws. By the way, some Brazilian frogs make perfect bell sounds.

The Vowels.

The human voice is very remarkable. Each vowel sound has one most prominent partial, of a perfectly definite pitch, without which the vowel is unrecognizable. The strengths of its higher and lower partials vary according to the singer's voice and the total effect gives more or less successfully the impression both of the vowel and of the key intended. But eee . . . for instance cannot be sung in a very low key, nor uuu . . . in a high one, because their essential partials are too far away from the desired note and the singer cannot bring them out without spoiling it. Even the best singers must often use substituted vowels and trust to articulation for intelligibility.

§ 345. The reader of § 277 will not fall into the very common state of wonderment that all these partials can co-exist, that e.g. an octave which wants a node in the middle is discoverable in the tone of a wire whose middle is visibly swinging to the fundamental. What really happens is that the sharp bend formed in the wire where plucked breaks into two peaks which run to and fro (there is photographic evidence of this) and the bellying shape is merely due to the persistence of retinal impressions. And from a pipe and from the syren there come separate sharp puffs, something like  and the marvel is that the trained ear can get from this the same impression of component notes as the mathematician gets of component S.H.M.'s.

§ 346: **Musical Scales.**

The **diatonic scale** consists of seven notes (and the octave) whose frequencies are in the ratios to the keynote

C	D	E	F	G	A	B	c
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
do	re	mi	fa	so	la	si	do

[*For instance* 264 * 297 * 330 352 * 396 * 440 * 495 528]

It makes no difference to these ratios what the actual frequency of the keynote is. Addition and subtraction have no place here.

Expressing each note in terms of the next below it we get

$\frac{D}{C}$	$\frac{E}{D}$	$\frac{F}{E}$	$\frac{G}{F}$	$\frac{A}{G}$	$\frac{B}{A}$	$\frac{c}{B}$
$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$

$\frac{9}{8}$ is called the interval of a Major tone, $\frac{10}{9}$ a Minor tone, and $\frac{16}{15}$ a Semitone.

Into the large gaps additional notes are inserted by multiplying the frequency of the note below by $\frac{16}{15}$ (sharpening it a semitone) and dividing the note above by $\frac{16}{15}$ (flattening it a semitone), giving a pair of notes in the position of each asterisk shown above.

Perfect concord demands that the very small difference between say C sharp and D flat ($\frac{16}{15} \sim \frac{9}{8} \times \frac{15}{16} =$ about $\frac{1}{80}$) should be respected, and probably this is done by good singers and violinists, but it crowds a keyboard. Further, musicians wish to use various keynotes, to select the present value of G, say, as the starting-point of a complete set of ratios. Then its next note should be $\frac{9}{8}$ of it and does not coincide with A, which is $\frac{10}{9}$ G, and a lot more slightly varied notes spring into being.

Fortunately the latitude nature allows in everything does not fail here; for resonance is never perfect. It is only a hypersensitive ear that is offended by squeezing the C sharp and D flat into one note, by making A/G the same ratio as C/D and so on, i.e. by ignoring the difference between major and minor tone and making the semitone half either of them. The modern keyboard contains twelve notes to the octave, each $2^{\frac{1}{12}}=1{\cdot}0595$ times the frequency of the one below it. Starting on any keynote the eleven above it form its **chromatic** scale, and skipping those in the asterisk position, its **diatonic** scale. The asterisks are all blacks in the key of C only. How nearly your piano is tuned to this **equal temperament** depends on the tuner.

The **actual pitch of A** lies between 440 and 455 in London nowadays, and seems to be a matter of enthusiastic disagreement among musicians.

§ 347: **Combination tones.**

It used to be taught that the simplest pitch ratios (2 : 1, 3 : 2, etc.) produced the best concords because numerical simplicity was charming. Helmholtz replaced this dictum by an explanation in which he used the **combination tones** described by Tartini, of 'Devil's Sonata' fame. To hear these tones the very best way is to get two penny tin whistles and paper over all the holes except say the third and fourth on each. Now put both in your mouth at once, leave only the third holes open, and blow. Whirring beats will be heard, for though nominally the notes are identical, exact tuning is not to be had at the price. But open the third hole on one and the fourth on the other, and you will hear a growl as if the beats, now too fast to hear as a tremolo, had blended into a note.

The frequency of this **difference tone** is, just like beats, the difference between those of the primary notes between which it is produced. One can prove this on a harmonium, or piano with forte pedal down, by sounding a high C and G, when the C below sounds out, one being twice and the other 3 times its frequency.

At first sight it seems obvious that the note *is* blended beats, but beats are only waxings and wanings of a definite note, and not at all the same thing as directed puffs of air. Mathematical theory, however, shows that there are several circumstances in which notes sounded together can and must give rise not only to this **difference tone** but also to a **summation tone** of frequency = the sum of the primaries, but a *very* weak tone, seldom actually audible.

Difference tones become feebler as the primaries get farther apart in the scale.

§ 348: **Concord and discord.**

Pure tones. Sound a fork, and its octave fork. The difference tone is the note itself, and introduces nothing fresh. But if the octave is a little sharp the difference tone is a few vibrations per second above the note, and beats rapidly with it, and rapid beats set anybody's teeth on edge. Sound, say, forks F, A, and c, frequencies 4, 5, 6. $F \sim A = A \sim c = 1$ and $F \sim c = 2$ and the effect is a string-like tone composed of 1, 2, 4, 5, 6. But frequencies 200 and 275, say, have a difference tone 75, a slow swell which seldom fits in with either, but keeps disturbing the ear at varying intervals before and after the primaries. Here is imperfect concord.

Compound tones. All ordinary instruments produce notes accompanied by series of overtones. This greatly accentuates concords and discords. For instance, in the first case above, ordinarily the first tone contains its octave as well (probably much louder than the difference tone) and *this* beats with the mistuned octave. And take again, with their harmonics :—

200	400	600	800	. . .
275	550	825	1100	. . .

There are here small differences of 50 and 25 giving rise to tones, weaker than the initial difference tone of 75 because between overtones only, but stronger because between closer frequencies. Is it unreasonable to suppose that their low grumbling distracts the ear and spoils its appreciation of the two notes, much in the same way as the hum of conversation spoils one's enjoyment of a concert ?

The above is such a partial account of concord and discord as one can give physically. It is assumed throughout that the first note is still sounding when the second is struck. Without this, odd notes are often not unpleasant. The Arab has evolved a musical scale different from our diatonic scale, and, on our theory, presenting more opportunities for discord. The desert music of the tent door and the bagpipes on the brae can charm us by those same imperfect harmonies that unfit them for the prolonging echoes of the aisle or the concert-room.

§ 349. **Doppler's principle applied to Sound.**

The following numerical example will show how this principle, fully explained in § 303, applies to sound :—

Example 1. The whistle of a locomotive travelling 56 ft. per sec. emits 525 waves per sec. Find frequencies of notes heard by a passenger, and by an observer beside the line as the train comes and goes.

The passenger hears the natural note. As the train approaches the stationary observer the frequency rises to $[1100/(1100-56)]\times 525 = 553$ and as it recedes falls to $[1100/(1100+56)]\times 525 = 500$.

This principle comes in with swinging bells, etc., together with the alteration in loudness due to their change of position: and the reader can supply many other instances for himself.

§ 350. **Wind.**

Sound 'carries' down wind mainly because the air moves faster higher up, where not impeded by friction with the ground, and makes the spreading waves overhang and beat downwards. Up wind the sound lifts off the ground and goes up: from a 70-ft. roof in a strong westerly breeze I have heard conversation on the ground-level 100 yards east.

To get some idea of the distortion of sound waves caused by wind blow on a hemispherical soap-bubble. In Fig. 142 the speeds of the wind low down and higher up are marked as frac-

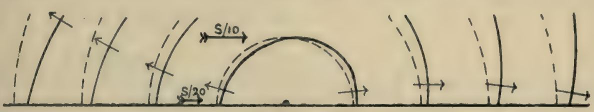


FIG. 142.

tions of the speed of sound. The little arrows are the directions of travel of the wave fronts to which they are perpendicular.

The good 'carrying' of sounds over water is mainly a question of the absence of obstacles and the wind eddies they cause. The exaggeration of sounds in the night may be ascribed to the surrounding silence.

EXAMPLES.—CHAPTER XXXVII

2. How would you determine whether any of the notes of a bell were harmonics ? [L.]

3. Knowing that 10 to 20 beats per second sound unpleasant, explain whether you would rather hear, on a piano, a note whose fundamental is 264 quickly followed by 440 or 470.

4. Explain why note of cycle-bell differs when approaching and receding. [L]m.

5. A person hears the whistle of an approaching train. Two notes of different pitch are, however, heard, one being due to reflection of sound from a bridge beyond the train. Explain why the notes heard are not of the same pitch, and show how to calculate what the alteration in pitch amounts to. [L]m.

6. At what speed of approach would a whistle in D be sharpened to E and at what speed of departure would it be lowered to C ?

7. It is said that in a tunnel sounds are heard best *up* wind. Give a possible explanation.

8. What is meant by the quality of a musical note ? How is the difference between sounds of fork and violin string accounted for ?

LIGHT

Chapter XXXVIII

RAYS AND SHADOWS. PHOTOMETRY

§ 351. Rays of light.

Sunbeams streaming into a room, or through gaps in the clouds when the sun is 'drawing water,' never fail to impress one with their rigid straightness, and it is natural to think of straight 'rays' along which Light travels, and of which many side by side build up a broader 'beam.' Our study of Optics will be developed mainly from this idea of rays, which involves no theory of the nature of light, but the Theory of Light as a Wave Motion will be kept in view.

§ 352. Shadows.

The terms transparent, translucent, and opaque are familiar. Opaque bodies obstruct light and cast shadows. If the source

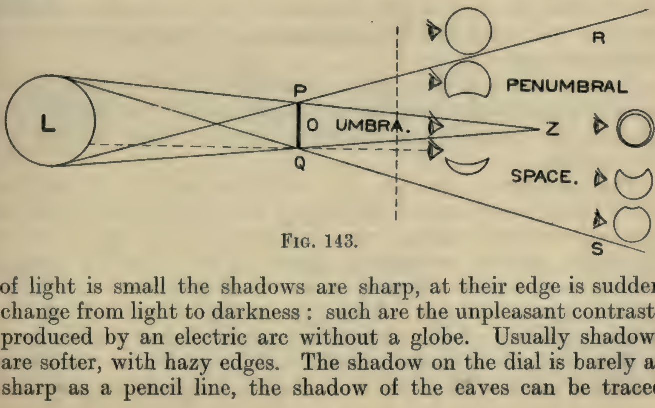


Fig. 143.

of light is small the shadows are sharp, at their edge is sudden change from light to darkness : such are the unpleasant contrasts produced by an electric arc without a globe. Usually shadows are softer, with hazy edges. The shadow on the dial is barely as sharp as a pencil line, the shadow of the eaves can be traced

quite definitely enough with a stick, and its edge spreads very hazily indeed when a thin cloud blurs the sun.

In Fig. 143, L is a broad source of light and O a circular obstacle. Into the space PQZ no light from L enters at all, here is the dark **umbra**, to an eye placed in it L is quite eclipsed. Outside the space RPQS there is no shadow at all, but to an eye in the intermediate spaces ZPR and ZQS, L is visible in part behind O, and the **penumbral** shadow here gradually deepens towards the umbra. On a screen in the dotted position there is a central uniformly dark umbra surrounded by a penumbra fading till it vanishes.

Beyond Z the shadow is all penumbra, O appears smaller than L and can never cover it entirely ; such is a little far-away cloud or tree in front of the setting sun.

The little patches behind the eyes show the fractions of L they can see. The total brightness in the shadow at each eye is proportional to the apparent area of the visible patch, e.g. the lowest eye on the left is in rather deep shadow, the lowest on the right hardly shaded at all, but is farther from L.

Note.—Of course all shadow is a question of contrast. When the sun peeps out it is not darker in the new ' shadows ' than it was before.

§ 353. In the **Pinhole Camera** straight rays from lighted objects pass through a small hole and form, on a plate beyond, an inverted picture of them. For the light that each part of the plate gets from the one small patch of the object's surface facing it through the hole is proportional to the brightness of that surface, and hence the light and shade and colour of the object are reproduced.

A small hole in a card, and a candle flame, enable one to show that the shape of the hole does not matter much : the sun shining through the irregular gaps in foliage throws rounded patches on the ground.

The roundness of the dots in a ' half-tone block ' is due to the same cause. Half an inch in front of the plate in the camera is a screen ruled with opaque cross lines, about 120 per inch, leaving of course *square* transparent spaces. But the light coming through each of these forms an image of the bright *round* window of the camera, the lens. For some purposes the shape of the dots is varied by putting a square or cross-shaped stop in the lens.

Too large a hole causes ' penumbral ' haze, but a *very* small hole also gives a hazy picture. This is inexplicable from the

'straight-ray' theory, but is to be expected from the train of waves of limited length which constitutes the ray on the wave theory of light, § 295.

Using a pinhole whose diameter is one three-hundredth its distance from the plate, one can obtain photographs which softly but clearly define everything beyond an inch from the pinhole, and give a more pleasing solidity in the stereoscope than do sharper lens photographs.

Photometry

§ 354. In these days of the 'rapid dry-plate' and nights of high-pressure gas and flame arcs, when tinder-box and snuffers are prized 'antiques' and a farthing rushlight is not to be had for a sovereign, we all take some interest in **Photometry**, the measurement of the brightness of lighting or the 'intensity of illumination.'

It is the useful **illumination of a surface** that is in question. When a surface squarely faces a 'standard candle' one foot away it is said to be lit with unit intensity—one 'candle-foot.'

A lamp that, put in place of the candle, lit the surface equally brightly would be said to be of one candle-power (1 c.p.). A lamp that at 1 ft. lights a surface with intensity 50 candle-feet and would require the (theoretically) concentrated light of 50 candles to replace it, is of 50 c.p.

The intrinsic brilliance of a lamp, i.e. its candle-power per square centimetre of flame, etc., does not concern us here. (*See Radiation.*)

§ 355. **The law of inverse squares.**

In the board 1 ft. from the candle (Fig. 144, top) cut a 3-in. square hole and hold up behind it another board twice as far from the candle. Light travelling in straight lines through the hole marks out a bright patch twice as broad and twice as high as the hole, or 4 times its area; at 3 ft. it is 'thinned out' over 9 times the area and so on. Hence it would take a 9-c.p. lamp to give a brightness of 1 candle-foot at 3 ft.

Thus *the brightness of illumination of a surface is inversely proportional to the square of its distance from the source of light,** cf. § 291.

Caution.—The source must not be broad compared with the distance, or the law fails; the diagram becoming confused like

* Hence the Continental unit 1 candle-metre is almost exactly one-tenth of the British 1 candle-foot.

Fig. 143. A printing frame is not quite 4 times as well lit at 3 in. as at 6 in. from a gas mantle. A sunlit whitewashed wall sends nearly as bright a light to your book at a yard away as at a foot.

§ 356: **The Cosine Law.**

Hold a card to face the light. Turn it obliquely, as in Fig. 144 (left), the shadow it throws gradually narrows—to nothing when the card is 'edge on.' Simultaneously the lighting of its

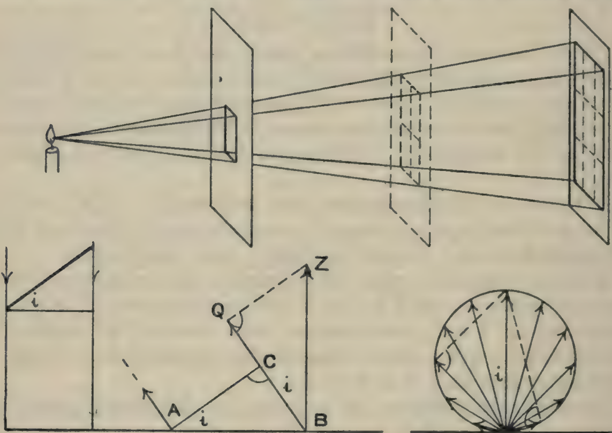


FIG. 144.

surface decreases—also to zero. The amount of light the card retains is proportional to the width of its shadow, i.e. to the cosine of the angle of incidence of the light, i. This amount has to be spread over the whole surface, and hence **Lambert's cosine law**—the *brightness of illumination of a surface is proportional to the cosine of the angle of incidence of the light.*

The converse is fairly true, the intensity of radiation in any direction from an extended luminous surface is proportional to the cosine of the angle between that direction and the normal; the vectors in Fig. 144 (right) show the intensity of radiation in their directions. But the apparent area of the surface seen obliquely diminishes in just the same proportion. Therefore it appears equally bright in all directions, foreshortening of area compensating reduced emission. For in Fig. 144 (middle) $BQ = BZ \cos i$ and breadth $AC = AB \cos i$. A sheet of paper

illustrates this, but glazed paper and high angles must be avoided. This also is the explanation of a lamp globe appearing merely a flat luminous disc. A flat flame is not a surface.

§ 357. Taken together these laws give

Brightness of illumination (in candle-feet) = candle-power of source × cosine of angle of incidence on surface ÷ square of its distance in feet

$$\mathbf{I}=\frac{\mathbf{c}}{\mathbf{d}^2}\cos i$$

and for perpendicular incidence ($i=0$) cos $i=1$.

The standard illuminant.

The standard of candle-power is the Vernon Harcourt Lamp, which burns a regulated supply of vapour of pentane (a very volatile petrol) in a well-ventilated room, and is legally equivalent to ten times the " Standard sperm-candle " now obsolete.

§ 358. **Photometers.**

Look at the moon and guess how many candles at a foot she appears equivalent to. Set an inexperienced amateur to photograph an ' interior ' without any exposure guide. Catch chequered sunshine on a paper and say how many times brighter is the sunlit part than the shaded. Then look at the end of this chapter : do we agree ?

But the eye can judge when two illuminations become equal (with a little practice, within 1 %) *and this is the foundation of all* **Photometers.**

These are the instruments used in comparing the candle-powers of sources of light. Some few are described in § 359.

For accuracy, in all photometers :—

(a) The two illuminated patches on the ' screen ' must touch each other along a line quite fine and sharp.

(b) No other illumination whatever should be in sight to distract the eye.

(c) All stray reflected or day light must be excluded from the screen.

Then when each patch is lit solely by its own lamp, and receives its light at the same angle (90° in most)

$$I_1 \text{ of 1st patch} = \frac{\text{c.p. of 1st lamp}}{(\text{distance})^2 \text{ of ditto}} = \frac{c_1}{d_1^2}$$

$$I_2 \text{ of 2nd patch} = \frac{\text{c.p. of 2nd lamp}}{(\text{distance})^2 \text{ of ditto}} = \frac{c_2}{d_2^2}$$

and when the lamps have been moved to and fro till the patches appear equally bright

$$I_1 = I_2 \quad \therefore \ \frac{c_1}{d_1{}^2} = \frac{c_2}{d_2{}^2} \quad \therefore \ \frac{c_1}{c_2} = \frac{d_1{}^2}{d_2{}^2}$$

The candle-powers are directly *as the squares of the distances of the lamps from the 'screen.'*

§ 359. **Photometers, a few patterns.**

Rumford; so-called 'Shadow,' Fig. 145 (R). The patches are produced as the shadows of a rod standing in front of a white wall, each shadow of course being lit by the other lamp only. It is a domestic contrivance; equally broad shadows mean equal

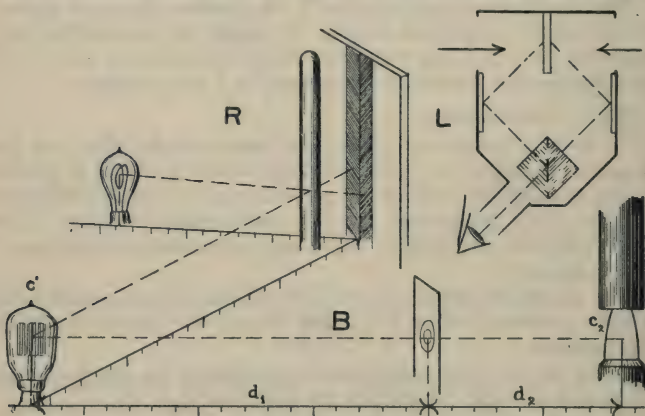


Fig. 145.

angles *i*, which is necessary; they should just *touch* each other; the brighter light on the rest of the wall is a hindrance. It is less affected by stray light than other patterns.

Ritchie wedge. A dead-white card is bent to a right angle and put in the straight line joining the two lamps with each face at 45°.

In the modern **flicker** photometers this screen is in various ingenious ways oscillated sideways about seven times a second so that the eye sees alternately its right and left faces and equality is obtained when 'flicker' ceases. These are easy to use, but often disagree with the usual patterns from some physiological cause.

Bunsen, Fig. 145, B. The screen, at right angles to the line joining the lamps, is made locally translucent and adjusted till the translucent part is neither brighter nor darker than the opaque, and 'disappears.' The translucency means that most of the light from the right passes through and is lost, and its place has to be taken by an equal amount of light coming through from the left.

Of translucent spots the worst is made by a greasy dirty finger, the best is printed on clean soft white printing paper by a clean hot metal disc rubbed with white wax, and is ring- or star-shaped.

The mean of readings taken on both sides of the screen must be used.

In the **Lummer-Brodhun** the screen is a white plaster plate in a box shown in plan at Fig. 145, L. The box is put in place of the Bunsen screen. Two right-angled prisms are squeezed in contact with each other except over cut-out patches of their surface. An eye looking as shown sees the right side of plate reflected in the right mirror *through* the prisms, while the cut-away patches totally reflect light from the left side of plate, via the left mirror. Brightnesses are equalized.

Abney's photometer does not employ the photometric laws, but depends on retinal persistence, a distinct physiological principle. The lamps are fixed equidistant from, say, a Ritchie wedge. In each distance is a rapidly rotating disc-shutter, having sectors cut out of it, which when wide open are quadrants, but can be closed gradually while the discs are spinning. When the screen halves are equal the discs are stopped, and the angular width of sectors open is read. If only 10° on one let through as much light as 80° on the other, its lamp was 8 times brighter.

With practice, it is not difficult to equate within 2 % or less lights of very different colours.

§ 360. One candle-foot is reckoned just adequate for reading fair print at night; less is bad. Much more is wanted for small work presenting less clear contrasts, e.g. sewing.

During the greater part of an average bright day the vertical illumination on a book lying flat on the table in the middle of a fairly light room will be something like 20 candle-feet, while outdoors it is 20,000 and often more.* No wonder one is blind on coming indoors suddenly after sunlight.

Sunlight varies, one hour at Aden has been found equivalent

to 48 hours' winter sunshine in Manchester. A white cloud may be several times brighter than blue sky.

The Moon is obliterated by the daylight sky and is therefore less than a hundred-thousandth of good daylight. At full it gives on a surface facing it, according to my measurements, about the illumination of a candle at 14 ft., say $\frac{1}{200}$ candle-foot.

* Evidently actual excess of illuminating power is not likely to be the cause of the occasional complaint that an artificial light is 'too bright.' But Contrast, difficult to avoid in artificial lighting, vexes the eye exceedingly. We have measured the iris and found it more contracted when looking at a bare glow-lamp than at the same lamp with a white card behind it, though in the latter case the eye is receiving nearly double the light.

EXAMPLES.—CHAPTER XXXVIII

1. Explain the production of luminous images by means of a screen having a small aperture and state the considerations which would influence you in choosing size of aperture. How would you determine the distance of an inaccessible luminous object of known size by means of the pinhole camera? [L.]

2. Define intensity of illumination. How can it be shown experimentally that the intensity of illumination due to a small source of light varies inversely as the square of the distance? [L.]

3. How would carry out experiments to test the laws of illumination? [L]m.

4. Describe a practical photometer and state the chief precautions for accuracy in its use. [L.]

5. Explain how to find the variations of the intensity of an electric light due to changes in the current. [L.]

6. How, by aid of a photometer, would you show that light is partly absorbed in passing through tracing-paper, and how determine ratio of light absorbed to light transmitted? [L]m.

7. Explain how the variation of the illumination due to a small source with the distance can be determined experimentally without assuming the inverse square law. [L.]

8. An 8-c.p. and a 16-c.p. lamp are 24 in. apart. Where, on the line joining them, do they give equal intensities of illumination?

9. At what angle must light from a 50-c.p. lamp 5 ft. away strike the wall to give an illumination of 1 candle-foot?

CHAPTER XXXIX

THE REFLECTION AND REFRACTION OF LIGHT

§ 361. **Laws of Reflection.** With a lamp, a cup of water, and a foot rule one can demonstrate the laws of reflection of light from a smooth surface (cf. § 298). Expressed in terms of rays these are :—

I. *The incident ray* (from lamp), *the normal* (perpendicular) *to the reflecting surface and the reflected ray lie in one plane*—i.e. the whole diagram lies flat on the paper.

A plumb-line held at arm's length will 'cut' the lamp and its reflection in the level water.

II. *The angles of incidence and reflection are equal.* These are the angles ii' between the two rays and the normal [see Fig. 152].

Putting the lamp and eye at equal heights on opposite sides of the room, the cup, whether on table or floor, will be half-way across the room when the reflection is seen in it.

Better 'proof' of the laws follows from the accuracy of the sextant, etc., in everyday practice.

§ 362. **Tilted mirror.**

It follows at once that if the mirror be tilted through an angle the reflected ray swings through double the angle. For when the

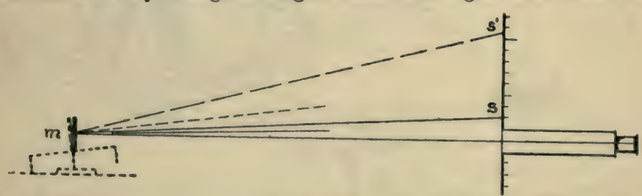


FIG. 146.

angle of incidence is increased, the angle of reflection increases equally, and therefore both together, which make up the angle between the direction of the light before and after reflection,

increase by twice as much. See dotted lines, Fig. 146. For instance, the reflections from the facets of a cut-glass tumbler move round on the tablecloth twice as fast as the glass is turned.

This gives an excellent way of measuring small angular motions such as occur in delicate galvanometers, etc. A small mirror, Fig. 146, reflects the image of a divided scale into a telescope, which is fixed and therefore defines the direction of the reflected ray. When the mirror moves the scale appears to move, the ray from s is reflected away and s' comes in sight, where sms' is twice angular motion of mirror. If all the angles are small, angular motion $= \frac{1}{2}$ diff. of scale readings $\div$ distance from mirror to scale.

If a lamp is used it stands behind a narrow slit in place of the telescope, and a reflected spot of light moves from s to s'. The mirror is concave (§ 395) with radius = distance of lamp and scale.

In the **optical lever** M is stuck on a little bar (dotted in fig.) which has a pin-point foot at each end and a pair in the middle, and stands on three of them on a fixed plate. The thin object of thickness t *to be found* is put under the middle feet and tilts the mirror through angle $t \div \frac{1}{2}$ length of bar $= \frac{1}{2}$ scale diff. $\div$ distance. Hence t.

§ 363: In the **Sextant** (Fig. 147) a small telescope T looks through the clear half (upper, nearest reader) of the 'horizon

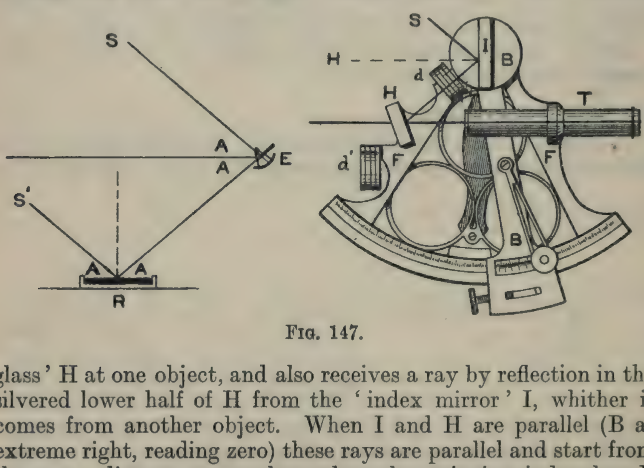


FIG. 147.

glass' H at one object, and also receives a ray by reflection in the silvered lower half of H from the 'index mirror' I, whither it comes from another object. When I and H are parallel (B at extreme right, reading zero) these rays are parallel and start from the same distant source, but when the swinging index bar B

carrying I is moved round the graduated frame F to which H and T are fixed, the ray SI turns through double the angle. The angle SIH′, e.g. the altitude of a star, is therefore obtained by moving I round till the reflected star appears on the horizon, and in general, the angular distance between two objects by making one apparently overlap the other.

On land, where the horizon is obstructed, the reflection of the star in the level surface of oil or mercury (' artificial horizon ') is sighted instead, and now the angle measured when star and its reflection lie side by side is double its altitude. For ES and RS′ are parallel, RE makes the same angle with the horizontal as RS′, $\therefore\ S\hat{E}R = 2A$.

For convenience the sextant's graduations are figured double. The diagram shows the fine adjustment screw and magnifying glass for the vernier which reads to 10 sec. of arc. The sun is being observed through the dark glasses *d* which can be turned out of the way, as are *d′*, when not required.

§ 364. **The reflected image in a plane mirror.**

Somehow one always thinks of Echo as a sprite dwelling at the very margin of the mocking woods, but in a looking-glass one sees the image *some distance behind* the surface. What distance ?

In Fig. 148, E sees the object O reflected along ME where angles at M are equal.

E′ (the left eye say) sees it along E′M′ where angles at M′ are equal.

Consequently it must appear to be at I where these directions cross, and the reader easily proves that I is on the perpendicular OP produced as far behind the mirror as the object is in front.

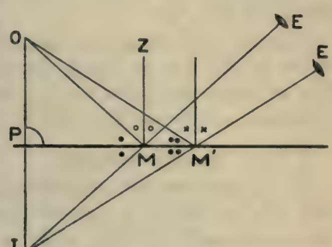


Fig. 148.

Having no real existence it is described as a *virtual image* and its apparent distance is actually judged by the means described in § 444, for a *single* line OME gives *no* information as to the distance of I. These means soon fail. You never thought of the moon's reflection as 100,000 miles or more below the pond.

Treating similarly other points O′ on a solid object the image is found to be equal in size but ' laterally reversed ' i.e. upside down only *or* left for right only, not both together.

§ 365 : **Multiple reflections in parallel mirrors.**

Sitting in a tea-room between parallel mirrors on opposite walls one sees vistas of rooms fading off each way into green distance, and the waiter standing facing opposite ways in successive rooms. His back is reflected in X; his face, the visible frame of mirror X, and its contained reflection of his back, are all mirrored in Y; then mirror Y and all these contents are reflected in X, and so on.

Draw (Fig. 149) parallels at equal distances XY and a perpendicular to them through the object B. Then mark off the images in two series, thus :—

X *series :* BX in X; then its reflection BXY in Y, as far behind Y as BX is in front; then *its* reflection BXYX in X and so on.

Y *series :* BY in Y; its reflection BYX in X; BYXY in Y and so on. [Suffix a letter for each reflection.]

Then *a* ray which has been reflected twice on its way to the eye is obtained by joining BXY and eye; but BXY is BX reflected in Y, ∴ from where the line meets Y draw to BX, and similarly from where this meets X draw to B. This is for the twice-reflected image seen by looking into Y. For the twice-reflected image seen in X draw from eye to BYX, BY, B.

One of the two thrice-reflected rays from BYXY is shown, via BYX, BY, B.

[NOTE.—The image is seen by looking in the mirror whose letter stands last in the string of suffixes; this string drops the last letter as each reflection is 'undone'; angles of incidence and reflection are equal throughout the ray.]

§ 366 : **Inclined mirrors.**

If the mirrors are slightly inclined to each other the images are seen to lie on a circular arc whose centre is where all the now radiating 'mirrors' run in to meet. In either mirror the images now go out of sight round the corner. You draw back to the outer end and go close to the glass to see as far round as possible, but you now find a limit to their number.

If you wish to know anything at all about this subject of inclined mirrors you must actually get two good-sized pieces of looking-glass (unhitch a couple of bedroom mirrors from their supports), set them parallel, put an object between them, gradually turn them so that their straight edges close together, and continuing, open to any angle. Compare what you see with your own figures, which you draw with angles gradually increased

from parallelism. Draw first the succession of equally inclined 'mirrors,' then plant out the two series of images, always in the same fashion, except that now the straight line has curved round into a circle centred at the point whence all the mirrors radiate.

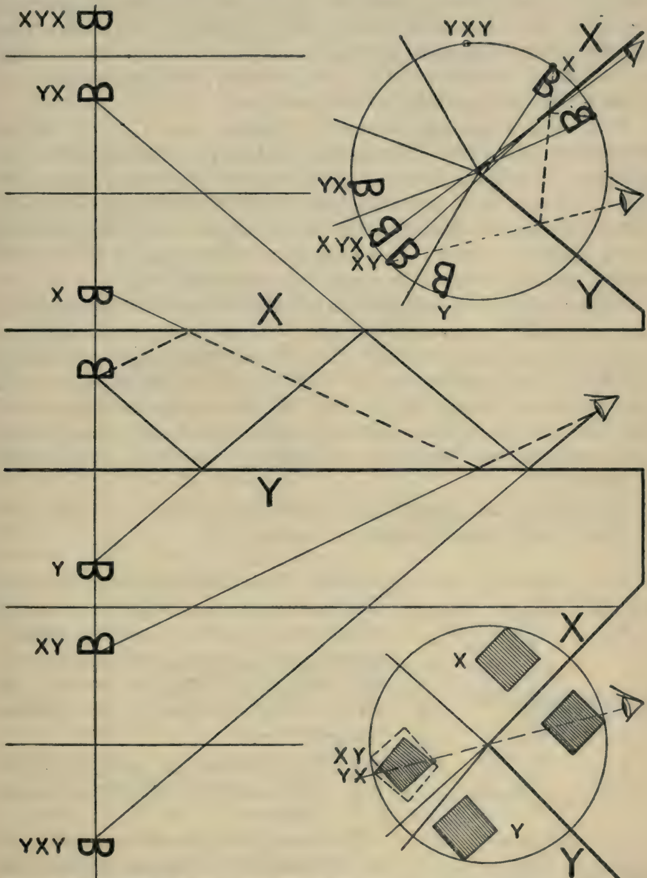


Fig. 149.

Fig. 150.
Fig. 151.

You should also dissect a toy kaleidoscope.

A diagram for inclination 80° is drawn in Fig. 150. Images **X**, **Y**, and **XY** are visible to an eye anywhere, and are all that are visible to the lower eye. An eye above the line joining **YX** to the corner, produced, sees also **YX** in **X**. **XYX** is so far round that to see it at all the eye must be very close to mirror **X** and then it is seen very close to the angle which probably cuts off the view of part of it. The actual path has been drawn in—from upper eye to **XYX**, from where this cuts **X** to **XY**, from where this cuts **Y** to **X**, from where this cuts **X** to object.

YXY cannot possibly be seen by looking into **Y**, i.e. it cannot be seen at all. Any diagrammatic attempt to construct a line of sight to it will end in unequal angles of incidence and refraction; try it. Hence both series stop short of the first image which has worked round far enough to lie out in front of the mirror whose letter ends its name; the line joining this image to the eye cannot possibly appear to come from that mirror.

As the inclination increases to 90° the failing **XYX** passes out of sight behind the angle and **YX** and **XY** join, to appear as only one image lying across the angle. The part of the image seen in **X** is **YX** and the remainder seen in **Y** is **XY**. If the 90° is exact these join without a break, but in Fig. 151 the mirrors are opened to 92° and an eye in the position shown sees the combined image being crushed out, for it can see nothing of **XY** in **X** and nothing of **YX** in **Y**.

Note that the thick line of mirrors in Figs. 150, 149, 151 divides 'real' from 'imaginary' space.

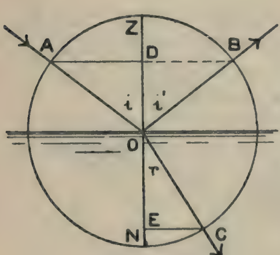


Fig. 152.

§ 367. **Laws of refraction.** Part of the light that falls on the surface of a transparent substance (or 'medium') passes into it, but in so doing becomes suddenly bent from its course. In terms of rays, the laws of this 'refraction' are:—

I. *The incident ray, normal to surface, and refracted ray, lie in one plane.*

[The angles made on opposite sides of the normal by the ray before and after refraction, called the angles of incidence and refraction, must be measured by their sines, then:—]

II. *Ratio of sine of angle of incidence to sine of angle of refraction is constant, and is called the Refractive Index of the second substance with respect to the first.*

Graphically (Fig. 152) describe a circle about the point where ray meets surface. Draw normal, and draw AD, CE parallel to surface. Then AD/AO=sine i, CE/CO=sine r, CO=AO.

$$\therefore \frac{\text{sine } i}{\text{sine } r}=\frac{AD}{CE}=\mu, \text{ the refractive index.}$$

(Bending is towards normal on entering more refractive medium.)

These laws have already been derived from the **wave theory** in §298, where μ=ratio of speed in first medium to speed in second.

On the older theory which supposed the **emission** of elastic particles from the source of light, reflection was a bounce off the surface, while refraction was explained as due to an attraction exerted on the particles by the second medium, as they approached it, which increased their velocity downward without affecting it horizontally, i.e. on the whole the speed increased. Foucault's direct experimental proof that light travelled slower in water (μ=4/3) than in air upset this and agreed with the wave or 'undulatory' theory. §481.

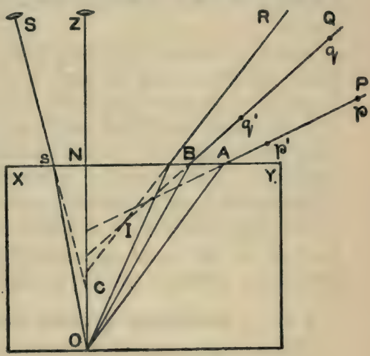


FIG. 153.

Some **Refractive Indices** relative to air are :—

Realgar	2·45	Flint glass (dense)	1·72	Carbon disulphide	1·65
Diamond	2·42	,, ,,	1·62	Ethyl benzoate	1·51
Phosphorus	2·16	Crown glass (common)	1·52	Benzene	1·50
Mono-brom-naphthalene	1·66	Rock salt	1·54	Alcohol	1·36
Canada balsam	1·53	Fluorite	1·43	Water	1·33
Glycerine	1·47	Ice	1·307		

Media with high indices are spoken of as 'optically dense.'

The Sine Law can be tested as in Fig. 153. The rectangle is a block of glass lying on a drawing-board (or trough of water standing beside a board). O is a scratch. Mark on board the

surface XY and ONZ perpendicular. Sight O and stick in pins pp', qq', etc., along various lines of sight. Draw $p'p$, PA, etc., AO, etc., and where PA produced meets NO put letter M.

AMN is an angle of incidence, sine i=AN/AM.

AON is corresponding angle of refraction, sine r=AN/AO.

$\therefore$ μ=AO/AM, and BO/BM′, etc., should give same result.

[This is an alternative way to Fig. 152 of performing the sine construction.]

§ 368. **Apparent reduction of depth in refractive media.**

In Fig. 153 two close lines Q, R will cut at I, light from O appears to reach the eyes from I, which is therefore its ‘ virtual image.’

Looking nearly vertically down, with both eyes to judge distance (§ 364), along ZN and S*s*, C is the position of the image of O and the apparent depth SC is only $1/\mu$ the real depth SO or NO.

A glass block appears only two-thirds its true depth and water only three-quarters ; stand shoulder-deep in it and look down at yourself. A pole slanting into water will appear bent *upwards* for all parts of it under water appear lifted *up*.

Looking obliquely I is much nearer the surface. The shallow bottom of the pond appears impassable, but sinks down under your boat to reappear just as shallow a few yards astern.

§ 369 : **Successive parallel layers of different indices.**

When a ray passes from medium μ_1 to medium μ_2 its refraction takes place according to *sine angle* in (1)/*sine angle* in $(2)=\mu_2/\mu_1$, which is the *Index of* (2) *relative to* (1) (e.g. water 4/3, glass 3/2, water-glass 9/8). Further refraction into a third medium is governed by *sine new angle in* (2)/*sine angle in* $(3)=\mu_3/\mu_2$ and so on.

If successive surfaces are parallel both angles in each layer are equal (alternate angles) and the whole refraction becomes

$$\frac{\textit{sine angle in } (1)}{\textit{sine angle in } (2)} \times \frac{\textit{sine } (2)}{\textit{sine } (3)} \times \text{etc.} = \frac{\mu_2}{\mu_1} \times \frac{\mu_3}{\mu_2} \times \text{etc.}$$

$$\text{or} \quad \textit{sine first angle/sine last angle} = \underline{\mu \textit{ last}/\mu \textit{ first}}$$

the final direction depends only on the first and last media and is the same however many different **parallel layers** the light has traversed. [This will probably appear plainer if one recollects that μ is the ratio of wave-velocities.]

Light therefore resumes its original direction after passing through a parallel-faced sheet of glass, and this must always be

the case when first and last media are the same. But any particular ray, though parallel to itself, is slightly shifted sideways, this **lateral displacement** varying from O when normal to nearly the thickness of the plate when very oblique (rise of I, § 368).

§ 370: **Atmospheric refraction.** The atmosphere can be regarded as usually a succession of parallel strata gradually increasing in density and refractive power from above downwards,

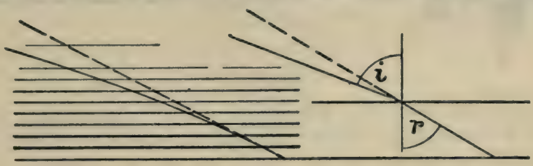


FIG. 154.

Fig. 154. Starlight entering it obliquely therefore gradually changes direction along a curved path and strikes the earth more steeply, i.e. the star is seen slightly raised in the sky as if in the dotted direction, Fig. 154. Taking advantage of the last paragraph one need consider only the index of the air next the earth, thus (cf. Fig. 154, right) :—

Example 1. Calculate rise for star 10° above horizon, assuming earth flat, μ air $=1{\cdot}00029$. [L.]

$$\text{sine } i = \text{sine } 80° = {\cdot}98480 = 1{\cdot}00029 \times \text{sine } r.$$
$$\therefore r = 79° \ 54'. \quad \therefore i \sim r = \text{rise} = 6'.$$

The refractive lift increases from 0 at the zenith to about $\frac{1}{2}°$ near the horizon (where curvature of earth modifies calculation). The sun or moon just touching the horizon is really just below it; there is several minutes' real gain in length of day, made possible because the sphere which catches the sunlight is the earth enlarged by its shell of atmosphere.

Ex. 2. Show that if the earth were flat and the air no clearer than usual there would be a belt $1\frac{1}{2}°$ high all round the horizon in which stars, etc., could never be seen.

§ 371. **Mirage** is by no means confined to the tropical desert. It can be seen by anyone who on a calm sunny summer day will lie down on the warm shingle and look close over it out to sea. Distant funnels appear strangely drawn out, and the ship may even seem floating high up on its own inverted reflection in a shimmering silvery sea. The hot surface warms the air just above it and makes it less refracting. Any ray from a slight

elevation which is too low to escape this hot air therefore bends 'away from the normal' and reaches the eye as if it came from a lower point, such as A′ instead of A, Fig. 155, while rays

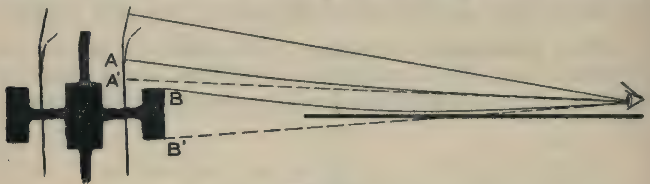


FIG. 155.

venturing still lower and getting into hotter strata may become so oblique that at last they suffer total reflection (§ 373) at the lowest and hottest layers and reach the eye as if from B′, etc.

§ 372. **The Prism.** Looking through a triangular prism (e.g. the 'lustre' from a Victorian vase) objects appear lifted up towards the narrow end, the *refracting angle*, of the wedge. That is, light passing through a plate (of higher refractive index) whose sides are not parallel has been permanently *deviated* 'towards the thick end.' The amount can be found by applying

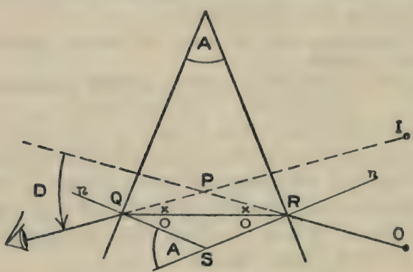


FIG. 156.

Fig. 152 to both faces in turn, but when the light passes symmetrically the deviation proves to be least—a **minimum deviation, D**—and from Fig. 156

from triangle PQR, angle $x=\frac{1}{2}$D

,, ,, QRS ,, $o=\frac{1}{2}A=\angle r$ in glass

angles $x+o$=angle PQS=external $\angle i$ in air.

$$\therefore \mu=\frac{\text{sine } i}{\text{sine } r}=\frac{\text{sine }\frac{1}{2}(A+D)}{\text{sine }\frac{1}{2}A}$$

NOTE.—For a very **thin prism** this reduces to $D=(\mu-1)A$ since small angles approximately=their sines (cf. § 300). And it now makes no significant difference whether the light passes symmetrically or not.

§ 373. **Total Reflection.**

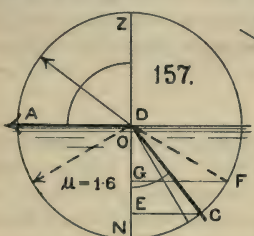
In § 299 we saw that waves travelling practically parallel to the surface of a slower medium send into it waves at a sharp angle. Or in rays, when Fig. 152 has become Fig. 157 (thick line) the ray AO at ' grazing incidence ' is refracted along OC where

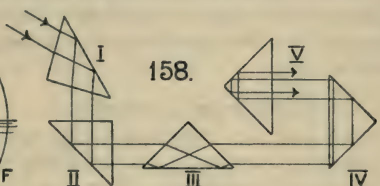
$$AD=AO=\mu\times CE$$

$$\text{or sine } 90^\circ=1=\mu \text{ sine } r \text{ or sine } r=1/\mu.$$

A ray FO cannot pass back through the surface, for $\mu\times FG$ exceeds the radius AO or μ sine r exceeds 1, which it is impossible for sine i to do.

CON the **critical angle** has therefore its sine$=1\div\mu$ of medium (relative to lighter). Any ray which attempts to pass out of the





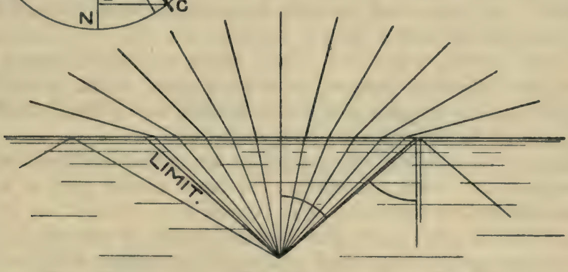


FIG. 157. FIG. 159. FIG. 158.

heavier medium more obliquely than the critical angle cannot do so, but is **totally reflected** back from the surface according to the ordinary laws of reflection.

This accounts for the brilliance of the under side of the water

surface in a tumbler, the silvery look of air bubbles in liquids, or cracks in glass or ice; light in the dense substance happens to strike the crack or bubble too obliquely and is reflected from it.

The reflection is not absolutely total (§ 299), but often exceeds 99 % even for surfaces which fall short of perfect smoothness and cleanliness. Mercury poured into a test-tube dipping under water appears less brilliant than the air-filled part. A little water on top of the mercury apparently cuts the tube in halves.

The lustre of a cut diamond is due to the abundant total reflections from the facets at the back, resulting from the small critical angle. The insides of the backs of glass prisms are total reflectors for rays striking them at incidence greater than $41\frac{1}{2}°$. Different ways of using a right-angled prism are drawn in Fig. 158; (i) is reflecting skylight vertically down, (ii) is used by photo-engravers* to laterally reverse process negatives, (iii) is put in front of a magic lantern to erect the picture on the screen, (iv) is used in prismatic binoculars, and (v) answers question 21.

Fig. 159, in which the angles are accurate, shows that a fish might see in the calm surface the whole sky and landscape (badly flattened) in a circular picture 97° in diameter, framed in reflected pond bottom.

§ 374. **Visibility and invisibility.**

A self-luminous object is visible, but most objects depend on obstructing, bending, and reflecting light from without, and so producing **contrasts**. Everyone knows the protective invisibility of birds and animals in their natural surroundings, but we mean more than this.

'Clear' water, a sheet of 'clear' glass, or a good mirror may show the reflected images of objects (and often leads from experience to a suspicion of its presence), but is itself invisible. We have all blundered into such surfaces.

Smash the glass, and the fragments are visible by their varying refractions and total reflections, the most visible part of a chip depending on the direction it is viewed in. 'Grind' its surface or powder its fragments and the multitude of reflections from scratches or grains flings light practically equally in all directions. A cloud is a swarm of droplets; froth, of bubbles; snow, of crystals; paper and fabrics, of fibres. Each individual, under the microscope, is perfectly pellucid, but light incident on the

* With whom it is the fashion to silver the prism face. This merely protects it from the atmosphere and keeps it clean.

innumerable and irregularly placed swarm suffers so many local and differently aimed reflections, etc., that it is scattered (or 'irregularly reflected') equally in all directions, i.e. the object is equally visible in all directions.

Similarly light gets through them, but irregularly, they are *translucent*.

The face of pressed paper has been so far flattened that it shows much nearly regular reflection or gloss, especially very obliquely.

Polishing glass and metals is a process of making finer and finer scratches until all are much smaller than the wave length of light.

Reduce refraction, and reflection is reduced also. Ice has nearly the same refractive index as water, and in water its outline nearly disappears while its contained air bubbles remain extremely visible. A glass rod is more refractive and not so invisible as the ice in water, but in ethyl benzoate or oil of cedar it utterly disappears. Oiled silk and oiled ground glass are nearly transparent.

It is difference in refractive index at irregular interfaces that produces the well-known visible streaming of hot air in cold, of petrol vapour in air, of whisky in water, etc.

Opacity helps visibility; directly, as in threading a needle against the light, or in obscuring the reflection from white paper (Indian ink v. watery ink); or indirectly by letting less light leak through and so maintaining reflecting power (contrast clearness of printing on heavy opaque white paper and on tracing paper); and also by casting shadows.

Colour is a selective opacity (§ 420), its utility in producing contrast needs no comment here.

Uniform illumination in all directions destroys all contrast and causes invisibility. Take one instance, striking though very imperfectly conditioned; floating dust enough to make an indoor sunbeam look solid is invisible in the broad daylight outdoors.

EXAMPLES.—CHAPTER XXXIX

3. State the laws of reflection and refraction of light rays and explain how they can be accurately verified experimentally. [L.]

4. State the laws of reflection and prove that the image is as far behind the plane mirror as the object is in front. [L.]

5. Prove that the angular convergence or divergence of a pencil of light is unaltered by reflection in a plane mirror. [L]m.

6. How would you arrange mirrors to illuminate the larynx from a lamp behind the head ? [L]m.

7. Investigate the Rule that the number of images visible in inclined mirrors is 360° ÷ angle of inclination.

8. Define the refractive index of a substance, and explain how that of a liquid can be measured. [L]m.

9. Show how the bending of a ray of light passing from air into a liquid is accounted for on the wave theory. What meaning has the refractive index on this theory ? [St. A]m.

10. Show that the depth of a liquid is always greater than it appears to be and prove that it is at least the refractive index times greater. [L]m.

11. Why on looking vertically into a pond does depth appear only three-quarters ? If eye were in water, how would apparent distance of object in air be affected by the water ? [M.]

12. Show that a thick mirror of glass (μ 1·5) silvered on back reflects normally incident light like a metal mirror one-third thickness of glass in front of silver. [L.]

13. A candle in front of a thick glass mirror is viewed from the right, $\mu = 1{\cdot}5$. Draw path of rays to eye ; why is more than one image seen ? [L]m.

14. Explain carefully how the deviation of a parallel beam of light by a glass prism varies with the angle of incidence of the beam on the first surface of the prism. How does it depend for a particular angle of incidence on the angle of the prism and on the refractive index of the glass ? [L.]

15. Explain how a right-angled glass prism with its largest face horizontal inverts without colouring objects viewed through it in a horizontal direction. [L.]

16. What is the 'critical angle' ? How may a real erect image of a luminous object be obtained by means of a convex lens and a glass prism whose angles are 45° and 90° ? [L.]

17. Prove that the largest refracting angle of a prism which will transmit a beam of light is twice the critical angle. [L.]

18. $i = 60°$, $r = 30°$, calculate μ and critical angle ; define latter. [M.]

19. Two bodies of indices 1·8 and 1·3 are in contact, find the angle at which total internal reflection begins. [M.]

20. Show that a man in a diving-bell covered with a flat horizontal plate of glass would not see the distorted view in Fig. 159.

21. Looking into the largest face of a right-angled prism an eye is seen in the corner. Whichever eye is shut, that reflection remains open. Explain this.

22. Show that through hot air overlying a cold sea it may become possible to see very distant objects usually hidden below the horizon, and that instead of being drawn out vertically as in Fig. 155, ships, etc., will appear very flattened and low in the water.

Chapter XL

LENSES

§ 375. **A Lens.** Suppose a small prism of narrow angle A at height AL above an axis LF, Fig. 160. At L on the axis is a prism of angle zero, i.e. a flat piece. Of the plane waves of a broad 'parallel' beam of light arriving from the left, the portion

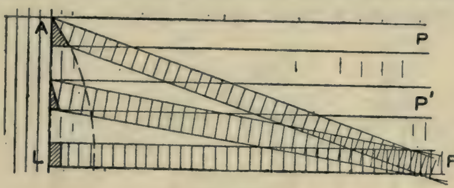


Fig. 160.

falling on A will be bent down (§§ 300, 372) and overlap the portion from L at F, where the illumination will be increased at the expense of the stretch of shadow AP.

The slope of the beam AF is the *small* deviation produced by A and can be expressed, as always on railways, as a Gradient of AL in AF or AL in LF, since AL is hardly distinguishable from the arc of a circle of radius AF or LF [recollect that angles get exaggerated in making plain diagrams].

Half-way between L and A put a prism of angle $\frac{1}{2}$A, this inclines its light $\frac{1}{2}$AL in LF and again increases the light at F, leaving P′ in shadow. And if all LA is filled with prisms whose angles are proportional to their distances from L, all the light will be concentrated near F.

With advantage, a curved piece of glass replaces separate prisms (mere thickness matters little) and the curve must be such that the angle increases regularly in proportion to the distance from the axis LF. Now to walk in a circle one must change one's direction equally at every step and the whole change is proportional to the distance walked. That is, **a circular arc** will suit

our purpose, provided that it is so slightly curved that it does not signify whether we measure along the arc or along the chord LA. We have arrived at a piece of a ‘ plano-convex ’ **lens** which will concentrate all the sunlight falling on it to a small focus (hearth) F in the midst of a cold shadow PP′.

Lenses are pieces of refracting substance bounded by surfaces which are portions of spheres (plane=infinite sphere). Half a dozen varieties are distinguished in Fig. 161, double or bi-convex 1 and -concave 2, plano-convex 3 and -concave 4, and meniscus or periscopic convex 5 and concave 6 [or concavo-convex]. Of course they merge one into another, one seldom finds a perfectly flat face. The spectacle-maker will sell you the half-dozen for a shilling or so.

Convex lenses are thickest in the centre and concave thinnest. Do not be surprised if these ‘ optical middles ’ of your chipped-

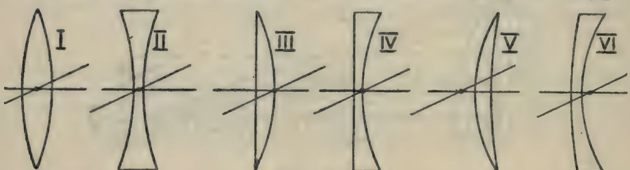


Fig. 161.

edge lenses are not just in the middle, the size and shape of the edge—round, oval, oblong, etc.—is more or less an accident. Break a lens and examine the bits.

We shall treat of **‘ thin ’ lenses** whose thickness is small compared with the other distances concerned, and our angles (exaggerated in diagrams) must never exceed a few degrees.

The reader will think that the spherical surface which demands this limitation is a poor makeshift. Curves better suited to particular purposes are known, but are deficient in all-round utility. Only plane, cylindrical, and spherical surfaces can be accurately ground and polished ; a great telescope lens can be touched up by trial in the final stages, but all others are finished spherical and *combined* to correct one another, § 433. In a modern camera lens there is not much to grumble at but the price.

§ 376. Consider a **Convex Lens** then, with plane ripples of sunlight falling on it (burning glass). They leave it not as separate streams as in the disjointed diagram, but curved to circular ripples which all close in on the burning focus F and then spread

out beyond it, Fig. 162 (A). Translated into *rays* (i.e. lines of travel of waves, always perpendicular to them) parallel rays become convergent radii which all pass through F and then diverge indefinitely. Working in a dusty or smoky room the parallel sunbeam is seen to become a cone, brightening as it approaches the vertex F and then spreading till it becomes too diffuse to

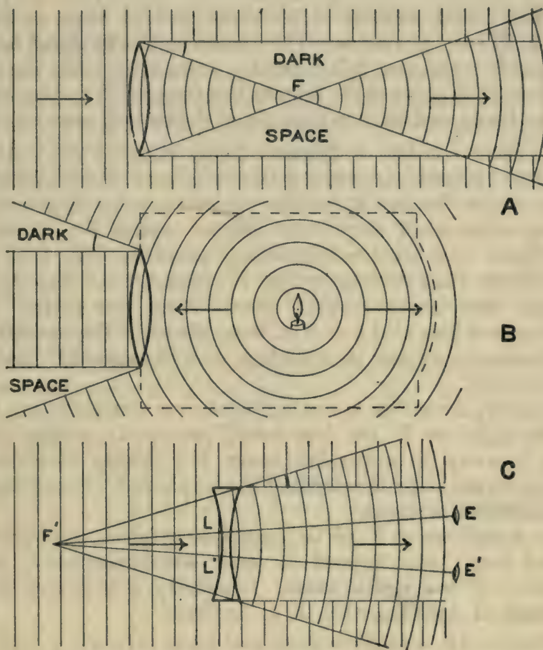


FIG. 162.

follow. All round the cone is the dark shadow-space robbed of its sunlight.

Using moonlight, which is not blinding, put the eye at F and look at the lens, i.e. get someone to hold and move it till he sees the bright focus on the pupil of your eye. Every part is sending light to your eye and the whole lens appears ablaze. Move your eye into the dark space and the lens becomes a black disc hiding

the moon. Put your eye in the cones and the size of the bright patch seen becomes less and less the farther you go from F, i.e. the less light you are getting.

Conversely, let F be a little lamp emitting light on its own account. Bulging spherical waves spread along all rays or radii. Those that fall on the lens are retarded in the middle by the greater thickness of slow-speed substance there, issue as plane waves, and travel straight in the same parallel beam as before (Fig. 162, **B**) but backwards. This occurs in railway signal lamps, brilliant only when seen full in front; a feeble light fills the dark space because all parts of the lantern box (suggested by the dotted lines) are lit up and scatter light through the bull's-eye window.

§ 377. Now consider a **Concave Lens,** Fig. 162 (C). All its constituent 'prisms' are turned the other way about and plane incident waves become spreading circles—parallel rays become divergent—just as if they came from a centre F′. Standing behind, your eyes receive light along directions LE, L′E′ and are convinced that the source is at F′, whence both lines appear to come. For instance, the 'direct-vision view-finder' is a strong concave lens held 8 or 9 in. from the eye; the sun and the whole landscape appear in miniature at a distance LF′ (inch or two) beyond it.

Conversely, if another (a convex) lens were concentrating light from the right on F′ the lens would prevent it getting there, sending it away in a parallel beam, the thicker slow-motion substance at the outside retarding the ripple-ends just enough to make the ripples straight.

There is nothing at F′ to be caught on a screen, no hearth of light and heat; only *through the glass there appears to be something there;* F′ is a **virtual focus.** In practice it is located as the intersection of sight-lines EL, E′L′ *produced.*

[Caution.—In sunlight a weak real focus may occur near F′, due entirely to light *reflected from front* of bi-concave.]

§ 378. **Optical centre of a thin lens.**

If the lens is slanted a little where will F be? Experiment, and you find it stops where it is. Near the middle of the lens a point L can be found such that straight rays drawn through it *meet both faces of the lens at places where they are parallel.* These rays therefore pass *without bending,* suffering only a trifling lateral displacement (§ 369) which in a 'thin' lens is ignored. L is the **optical centre** of the lens in Fig. 161, it has been found as the

intersection of two rays (shown), each of them satisfying the above condition. Lens diagrams are started by drawing straight rays through it. One of them happens to be perpendicular to the lens, but this is hard to find in practice, *single-lens diagrams have no fixed 'centre-line.'*

On any of these central rays are points, F for convex, F′ for concave, *on both sides at the principal focal distance f of the lens from L. This is the same on both sides*, the illusory difference with a meniscus 'landscape' lens explains itself in Fig. 161 (V).

§ 379. Rays from miles away are parallel enough, yet why is the focus of the sun, with a good lens, a sharp round patch and not a point? Bundles of parallel rays come from different parts of the sun, but the bundles are not parallel to one another. Each has its own point focus, all these lying side by side build up the patch. Some bundles start from less brilliant parts, their foci look dark—sunspots. An **Image** of the distant object has been formed in the principal focal 'plane' of the lens.

I trust it is clear from the foregoing why a lens should not be drawn with a *solitary* dot on each side invidiously exalted as 'principal focus.'

§ 380. Now take light spreading in circular ripples along rays from a point not far away, Fig. 163. These, hindered so much

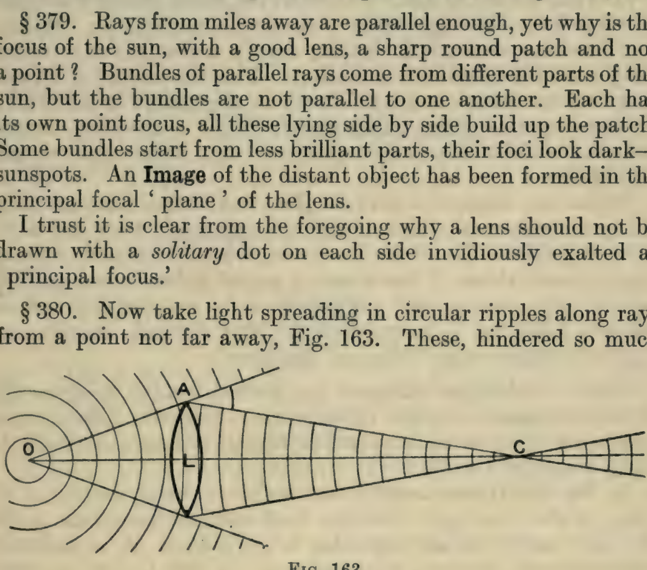


FIG. 163.

in their middles by the thicker slow-speed region, become concave and travel down radii to a centre, which is found thus :—

(1) It lies on the undeviated central ray OC . . .

(2) Ray OA is bent just the same amount as before, since the particular direction of incidence hardly affects the deviation by a thin prism (§§ 300, 372).

When some of the radiation from one point concentrates at another, the second point is the **image** of the first **object** point, and they are at **conjugate focal distances** from the lens, or mirror.

O can be a pinhole in a dark lantern, the eye placed at C will see the whole lens flashing full of light, as in § 376. O and C are interchangeable as far as the lens is concerned.

A Concave Lens lets the middles of the waves through faster, and they bulge more, as if they came from the virtual image of O, on the same side of the lens as O. It is *not* interchangeable with O.

§ 381. Thus lenses produce images by distorting the waves of light, and a full description of their action can be obtained only by the mathematical study of these waves, which is vastly difficult. In this book the bare outlines of the process will be obtained by forsaking waves and plotting only a few of the rays which map out their directions of motion. This GEOMETRICAL OPTICS is an artifice always employed by those who design or use lenses. It demonstrates many things clearly; that it does not explain all that minute observation detects is only to be expected (see §§ 474–8).

Notice the distinction between Real and Virtual Images. Real Images are formed where rays come and meet, they are to be seen actually in the air by an eye anywhere within the cone of rays beyond them. I have seen a parrot industriously pecking at one and getting very perplexed at its unsatisfying lack of flavour and its indestructibility. But Virtual images are apparitions seen only ' through ' the glass.

§ 382. Relations will now be worked out to connect the refractive index μ of the material of a lens, the radii r_1 r_2 of curvature of its faces, its principal focal distance f and conjugate focal distances a and b.

In Fig. 164 (I) the angle A between the faces of the lens at its edge is also the angle between their radii of curvature there, for each radius is perpendicular to its sphere. A is therefore the difference between the angles at C_1 and C_2, or speaking railway fashion, between the gradients AL in LC_1 and AL in LC_2, where L is the optical centre and C_1 C_2 are the centres of the spheres of which the left and right lens faces form parts. Putting $AL=1$, $AC_1=LC_1$ [$\because$ angles all supposed small*]$=r_1$, $LC_2=r_2$.

$$A=\frac{1}{r_1}-\frac{1}{r_2}$$

* Actual values in diagram A 16°, C_1 28°, C_2 12°, D 8°, μ 1·5.
In II A 16°, each C 8°, D 8°.
In III $C_2=16°$, $C_1=0°$.
Distances equal to AL are marked along from the optical centre.

Now taking a ray through A at the chosen unit distance from, and parallel to, the central ray LC, it is bent down through angle $D=(\mu-1)A$, § 300, and meets the central ray at the principal focal distance LF. D therefore = gradient AL in LF = 1 in f.

$$\text{Hence } \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

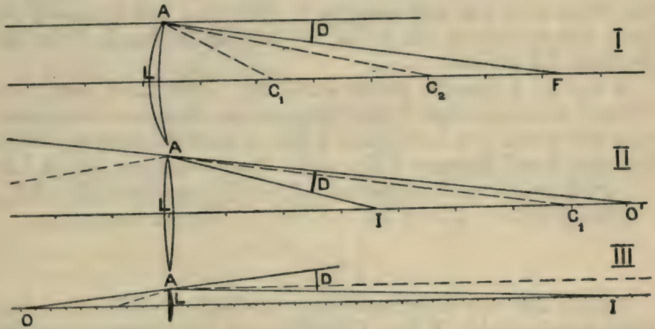


FIG. 164.

§ 383. A word as to this important $1/f$, the **Focal Power** of the lens. Speaking in focal *lengths* the most imposing figures mean least, a lens of huge focal length is nearly flat glass ; the less the focal length the more the lens does. Hence the optician prefers to describe lenses by $1/f$, their 'focal power' or 'strength,' and has invented a special unit for the purpose, the **Diopter.**

The focal power of a lens in diopters is the reciprocal of its focal length in metres.—

Thus a 1-m. focus lens has a power of 1 diopter ; a 5-diopter lens, written +5D, is a convex of 20 cm. focal length ; −8D is a concave of 12·5 cm. or 5 in. focus.

$$\text{Strength in diopters}=\frac{1}{\text{real } f \text{ in metres}}=\frac{100}{f\text{ cm.}}=\frac{40}{f\text{ in.}}$$

$1/r$ is a 'curvature' (§ 113), hence the relation we have found may be put :—

To calculate the focal power of a lens, multiply the difference of the curvatures of its first and second faces by its [refractive index minus one].

For common glass this $\mu-1$ is *approximately* $\frac{1}{2}$. If the faces

bulge opposite ways the difference of curvatures becomes their sum; the spectacle-maker reckons nearly enough thus:—Strength $=\frac{1}{2}$ sum of curvatures, calling convex $+$, concave $-$.

§ 384. Suppose the lens placed in light already converging on a point O′, Fig. 164 (II). Of many rays, one passes straight through its optical centre L, another cuts its prismatic edge A, is bent through angle D, and meets the axial ray in I. By § 375 all the other rays will also meet in I. The lens has increased the ray's down-slope to AL in LI, by adding the angle D to the gradient AL in LO′.

A ray at A parallel to the axial ray would have been bent through the same angle D (for the particular direction of incidence on a thin prism does not matter, § 372) to meet the axial ray at the principal focal distance LF; angle D = down-slope AL in LF.

∴ gradient AL in LI = gradient AL in LO′ + gradient AL in LF.

Putting AL = 1, LI = a, LO′ = b, LF = f, this can be written

$$\frac{1}{a}=\frac{1}{b}+\frac{1}{f}$$

Taking now the more usual case in which rays are spreading from a real object O, Fig. 164 (III), the lens again causes the definite change of gradient D downhill to the right, and this alters the actual gradients from AL in OL *uphill* to AL in LI downhill. One can think of the change of effort experienced in walking or riding over the hill-top A; or looking at the triangle AOI, its exterior angle D is equal to the sum of the interior and opposite angles at O and I.

Hence measuring from lens and putting

a distance of im*a*ge; referring to light *a*fter refraction
b „ „ o*b*ject; „ „ *b*efore „

$$\frac{1}{b}\text{ uphill}+\frac{1}{f}\text{ downhill}=\frac{1}{a}\text{ downhill.}$$

In algebra we must use the algebraic way of representing opposites, by opposite signs, hence $1/a=-1/b+1/f$, or what comes to the same thing

$$\frac{1}{a}=\frac{1}{-b}+\frac{1}{f}$$

So that if due regard is paid to b having now to be measured on the opposite side of the lens from f and a, by giving it a $-$ sign, there is no need to alter the main signs of the formula already found.

§ 385. To recapitulate. A convex lens causes a definite change of gradient, $1/f$ downhill, in the gradient of any ray meeting it at the unit distance above the centre. A straight central ray is the base-line of the gradients.

Per contra, a concave lens causes an uphill change $1/f$.

Putting these into algebra, where uphill and downhill must have opposite signs, gives

$$(\pm)\frac{1}{b}+(\pm)\frac{1}{f}=(\pm)\frac{1}{a} \qquad \text{I.}$$

Or if we choose to write this

$$\frac{1}{(\pm)b}+\frac{1}{(\pm)f}=\frac{1}{(\pm)a} \qquad \text{II.}$$

We are replacing the 'uphill $=-$downhill' convention by another, that distances measured opposite ways from the lens have opposite signs.

These alternative conventions—

I, that *uphill is minus downhill*,

II, that *distances measured opposite ways from lens or mirror have opposite signs*, are of course only two different ways of putting the same thing. Recollect that they are *alternatives*, choose one and stick to it, don't be misled into using both at once or you will be contradicting yourself.

I. Call *the downhill action*, characteristic of the convex lens, *positive*.

II. In the alternative plan, of measuring distances, this means that *the direction the light goes off after encountering the lens or mirror is positive.*

As a matter of convenience I have drawn all lens and mirror diagrams so that *this is towards the right*, the natural way one draws a line.

§ 386. Why have I chosen the signs this way ? The lens in the diagram is of the kind that bends rays together and makes them pass through a real focus, an actual hearth of light and heat, (situate of course on the side light goes off) : it is a burning glass, a magnifying glass ; the sort that everyone thinks of as soon as the word 'lens' is mentioned ; the *convex lens* that every maker and user of lenses always calls *positive*. The lens that comes as an afterthought (except perhaps to the short-sighted), that enfeebles light and diminishes things, the *concave lens* that

of itself can produce only virtual images, is *negative*, $-1/f$, $-$Diopters.

Later we shall see that concave mirrors and their real foci also come positive, as in practice, while convex mirrors and their virtual images are negative.

There has hitherto been taught to elementary students a convention which has made the position of the object all-important, always $+$. It contrarily makes magnifying mirrors $+$ and magnifying lenses $-$, it condemns the whole study of lenses to be carried on under the shade of the minus sign; it is an academic convention that one gets tired of warning men is right against universal lens-practice.

In rejecting this we shall have this one trouble to meet: for lenses, and for lenses only, the real object will be affected with a minus sign. But we shall have *real images always* $+$ and our eye will always be on the $+$ side. And after all it is the Image that is the *raison d'être* of the lens or mirror.

§ 387. Learn the statement of § 385. Learn also the formula if you like, but learn *all* about it. For it looks so simple, but it can trip you up five times out of six, and it probably will. Far better stick to the statement, and always make a rough sketch. The lazy man who will not make diagrams, and just memorizes 'lenses' as '1 upon etc., etc.' might more profitably teach it to the parrot.

NOTE.—The down-sloping and the axial rays are two of a convergent pencil of rays, and down-slope measures 'convergence.' The increase in convergence is the focal power of the lens. 'Divergence' is *minus* convergence, best stick to $\pm$ 'convergence' only.

Here is the whole argument in a typical case. Light goes off to right. Given a Lens, of course it has passed *through*, $\therefore$ came from left.

[Real] object actually emitting light $\therefore$ on [left], b from lens. Ray OA uphill, its gradient $-1/b$.

Lens [thickest] in middle bends it down through angle $D=[+]$ $1/f$, therefore gradient now $= -1/b+1/f=1/a$.

Does $1/a$ come out $+$, the ray AI is downhill towards the central base-line ray and actually meets it in a real image I (distant a on right).

Is $1/a$ $-$, the ray still goes uphill, but at a different slope, as if it came from a displaced source, i.e. a virtual image.

§ 388. There is a safer way still. There is no + and − in a picture. It is only when one has to translate the visible up and down, or left and right, of a diagram, into algebra, that those troubles come.

The following **Standard Geometrical Construction** is absolutely indispensable, and you must make yourself thoroughly familiar with it. Carefully drawn to scale it saves all calculation. A little practical acquaintance with simple lenses will show that the superior accuracy of calculation is largely fictitious. And rest assured that good graphical construction is at least as acceptable everywhere as calculation from ready-made formulæ.

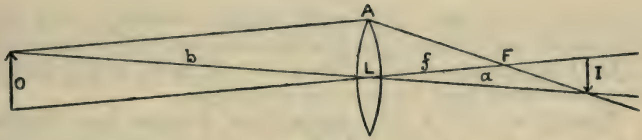


FIG. 165.

In Fig. 165, from both ends of the object O (placed parallel to the lens) draw axial rays straight through the optical centre of the lens. I call these the ‘ Scissors Rays ’ from the way they open and shut as the object is moved to or from the lens. Necessarily the image also, wherever it is, has its ends on these two rays.

From one end of the object (I have used the top) draw a third ray parallel to that from the other end. This ray is bent down and crosses the latter at F, at the Principal Focal Distance from the lens, and then continues and meets its brother ray in the image of the one point of the object from which both sprang. Draw in the rest of the image parallel to lens and object (here it is evidently inverted).

Any number of rays can now be drawn from points on the object to the lens, in cones. They will converge after refraction to corresponding points on the image found by running axial rays straight through. Pictures made in this way are pretty but rather meaningless, and it would be better to draw in the extreme rays only and fill in with ripples, as in Figs. 162, 163.

§ 389. The Standard Construction readily shows what happens to the image when the object moves to or from the lens.

The two parallel rays can be regarded as 'Rails.'* The object runs on the rails, always keeping its ends on them.

They, and their 'crossover' at F, remain fixed, and only the line OLI alters its inclination, scissors fashion, see Fig. 166.

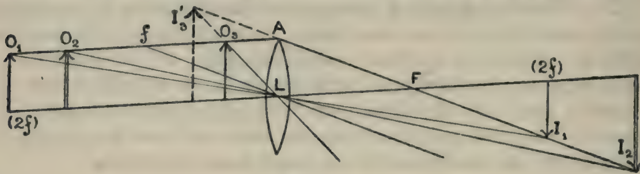


FIG. 166.

When O is far away, OLI is only slightly inclined to OLF, and I is near F ; as we expect, for F is the real image of O at 'infinity.'

As O travels nearer OLI tilts more and more, and I recedes along AFI away from the lens and gets bigger.

When O is at twice the principal focal distance from the lens I is also at $2f$; for then OLI is at half the inclination of AF, therefore the length of the hill LI, always nearly $=$LF$+$FI, is just twice FI, i.e. is $2f$. Hitherto O has approached faster than I receded, now *object and real image have come to their closest,* 4 *times the principal focal distance*—shown by measuring on a diagram, or by calculating in the appropriate $1/a-(-1/b)=1/f$ what a makes $(a+b)$ a minimum, or by experimental 'copying full size.'

Henceforth I recedes faster, till when O reaches distance f, I has gone to infinity, OL having become parallel to AF.

O moving nearer still, **within the principal focal distance,** O_3L and AF spread apart, never to form a real image, but appearing to eyes on the right of the lens as if they came from a point I'_3, a point on an enlarged erect **virtual** image. This is the important case of a magnifying glass.

As O moves close up the construction fails in exactness because the angles become too large, but it shows that ultimately object and virtual image *nearly* coincide on the lens surface : a reading glass laid right on the page has practically no effect.

§ 390. **Concave lens.** The action of a concave lens, with its virtual images, has already been explained in § 377. In calculation its focal power must be written $-1/f$. The standard construction

* Of course one 'rail' is also one 'scissors ray,' but in practice you will not find this causing any confusion.

is applied to it in Fig. 167. The same two axial rays are drawn and the third laid down parallel. But this now bends *up* in the direction found by joining A and F′, which is at the principal focal distance along the parallel axial ray on the *same* side as the

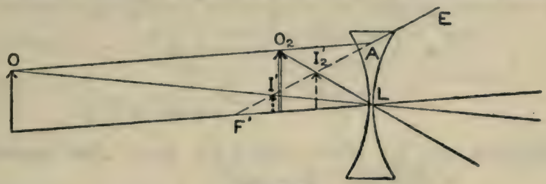


FIG. 167.

object. Virtual I′ is at the point where this prolongation AF′ cuts the fellow-ray from O, OL; for it is seen along LO and along the deviated direction EA. The whole image lies between the 'scissors.'

As O runs along the 'rails' from infinity up to the lens, I′ runs from F′ up to the lens. Image is always virtual and smaller than object, m increases from 0, up to 1 when lens touches object.

§ 391. **Magnification.**

The Magnification is the ratio of the length of the image to that of the object.

Since both lie between the 'scissors' axial rays their lengths are evidently proportional to their distances from the lens.

$$\text{Magnification, } m = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

and if they lie on opposite sides of the lens the image is inverted.

Taking heed as to conventional signs, for a real image $m = -a/b$ where $-$ is interpreted to mean 'inverted.'* For virtual images $m = a/b$.

Inspection of the diagram shows that m for a convex lens can have any value whatever for real images, but must exceed 1 for virtual.

* Hence multiplying the usual lens formula through by a;
m for real images convex lens $= 1 - a/f$
m „ virtual „ „ $= 1 + a/f$

§ 392: **Longitudinal magnification.** If the image of a small object is magnified m diameters, its thickness along the axis appears greatly out of proportion, being magnified m^2 times.

For let front of object be at b from lens and its back at $b+db$, where db is a small thickness, an increase in b (not $d\times b$, the letters are inseparable). The corresponding parts of the image are at a and $a+da$.

$$\text{Then } \frac{1}{a}=\frac{1}{b}+\frac{1}{f} \text{ and } \frac{1}{a+da}=\frac{1}{b+db}+\frac{1}{f}$$
$$=\frac{1}{b+db}+\frac{1}{a}-\frac{1}{b}$$

Multiplying out by $ab(a+da)(b+db)$ gives after cancelling
$$a^2db=b^2da+(b-a)dadb.$$

db is very small, or else back of object is hopelessly out of focus, $\therefore$ da is small, $\therefore$ $dadb$ negligible and

$$\frac{da}{db}=\frac{\text{thickness of image}}{\text{thickness of object}}=\frac{a^2}{b^2}=m^2$$

Thus the cheap-jack who advertises microscopes ($\times$ 100 diams.) made of a little globule of glass, as magnifying merely a million times cube, is sadly cheating himself.

Lens Problems

§ 393. We will now apply the foregoing to solve a variety of lens problems both graphically and by calculation. Graphically the draughtsmanship must be very exact, and all angles must be kept as small as is consistent with getting decisive intersections of fine lines: my sketches have angles too large, for clearness' sake. Algebraically, beware of signs. Make a sketch, just the axial ray OLI and the slopes OAI. The picture is right as it stands: *when* you are translating its visible ways into structureless symbols, *then* begin to use signs.

I. **An object is distant b from a lens of focal length f. Find position and magnification of image.**

This has been done in Figs. 165, 166, 167. Draw lens, and object at b to left, then rays drawn as described in § 388 give the image. Measure its distance a and its size compared with object.

By calculation:—

'slope before' $1/b+$focal power $1/f=$'slope after' $1/a$.

The 'slope before' is negative, being uphill to right. Cases are shown for $1/b$ — — —, $1/f$ + + —, respectively; a and a/b to be found.

II. Distance of image from object, and its magnification, are fixed. What lens is required?

Three cases in Fig. 168. Draw object 1 and image 2 the proper relative sizes, distance apart, and way up ($m-$, real, inverted; $m+$, virtual, same way). Draw rays in numbered order; where

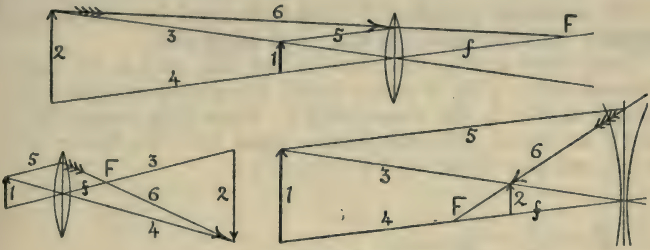


FIG. 168.

3 and 4 cross gives position of lens; draw 5 and 6, which gives focal point F, measure its distance f from lens. The figures show $m=3$ virtual and real, and $m=\frac{1}{3}$ virtual. If $m<1$ either the small diagram redrawn for real, or the third for virtual: notice how latter shows lens must be concave.

By calculation taking e.g. distance apart 10 in. (eighths of an inch in fig.), magnification $+\frac{1}{3}$.

Since $m=a/b=+\frac{1}{3}$ and for real object $1/b$ is an uphill, $\therefore$ $1/a$ is uphill. Distance apart of O and I is difference between length of long uphill b and short uphill a (lines 5 and 6, using 3 as base-line) to the summit A, $\therefore$ $10=b-a$. Since $b=3a$, $\therefore$ $10=2a$; $a=5$, $b=15$, these are the lengths of the uphill slopes from image and object respectively to lens [third fig., 168].*

The gradient $-1/15$ is converted by addition of $1/f$ into gradient $-1/5$.

$$-1/15+1/f=-3/15. \quad \therefore 1/f=-2/15$$

the lens is negative [with a focal power $-2/15\times 40$ (see § 383) $=-5\frac{1}{3}$D] a concave of $7\frac{1}{2}$ in. focal length.

* In figure the arrow 6 is more than 5 units long, it is at *too large an angle,* but distances to centre of lens will be found correct.

Example 1. If m were $-\frac{1}{3}$ (real inverted reduced image) and OI 10 in., show lens is convex, $f=1\frac{7}{8}''$, $a=2\frac{1}{2}''$ [second fig.].

Ex. 2. For $m=+3$, OI$=10$ in., calculate f [first fig.].

Ex. 3. A lantern objective is to project an image 8 ft. square on a screen 20 ft. away from a slide 3 in. square. Find f and distance from slide.

Ex. 4. In a photographic studio the sitter cannot be farther than 21 ft. from the plate on which an image $\frac{1}{20}$ life size is required. What is greatest permissible focal length of lens ?

III. **An image magnified m times is produced by a lens of focal length f: how far apart will image and object be ?**

Three cases in Fig. 169. Draw 1 the lens, lines 2, 3; then 4 through end of f till it cuts a dotted line placed m times as far from 2 as 3 is, in the image point I. Draw 5 the image, line 6, and 7 the object. Measure distance apart IO. The figures show $m=3$, real and virtual, and $m=\frac{1}{3}$ virtual.

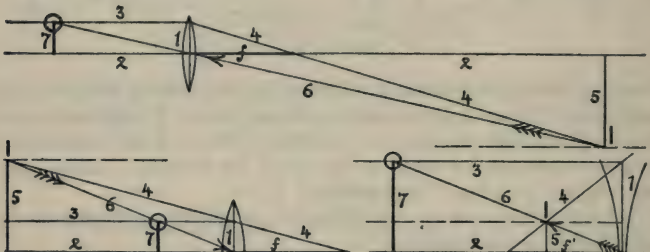


FIG. 169.

Calculation.

1. *Case of real object of which convex lens $f=9$ forms real image magnified thrice* [first fig.].

i.e. $a=3b$. Lens adds gradient $+1/9$ to gradient $-1/b$ and produces gradient $+1/a=+1/3b$.

$$-1/b+1/9=1/3b. \quad \therefore 1/9=4/3b.$$

$\therefore b=12$, $a=36$, and $a+b$, the distance between the feet of the opposite gradients, $=48$.

2. *Can a 10-cm. focus concave produce a real image magnified twice of real object ?*

By adding $1/f=-1/10$ to $-1/b$ from real object the necessary $+1/a$ cannot be produced.

3. *But try putting $1/b$ positive.*

$+\frac{1}{b}-\frac{1}{10}=+\frac{1}{a}=\frac{1}{2b}$ is quite possible and gives $b=5$, $a=10$.

Hence 'object' is 5 cm. and image, real, and same way up since a/b is $+$, is 10 cm., both to right of lens.

This is a new state of affairs. No real object can have passed through the glass to the wrong side; this is a **'virtual object.'** (Such was the point O′ in Fig. 164 (II).) It *was* a real image formed by rays converging (downhill) to the right, but now the concave lens has been put in their path and lessens their convergence; they form a larger image a little farther on; see *Telephoto Lens*, § 469, Fig. 220.

Try now putting $a/b=-2$ *or* $a=-2b$

$$+\frac{1}{b}-\frac{1}{10}=\frac{1}{a}=\frac{1}{-2b}$$

gives $b=+15$ cm. and $a=-30$ cm. There is a 'virtual object' on the right and a 'virtual image,' other way up, on the left. The lens farther from the original real image has actually bent the convergent rays uphill, as if they came from an image on the left. *See opera glass for near vision*, § 468.

4. *Let us again attempt the apparently impossible; require a convex lens to produce a diminished upright image.*

Put $a/b=+\frac{1}{3}, f=+10$

$$\frac{1}{b}+\frac{1}{10}=\frac{1}{a}=\frac{3}{b} \text{ gives } b=+20$$

i.e. a virtual object and $a=+6{\cdot}7$ a real image.

This is the case contemplated in the original all positive diagram Fig. 164 (I), and see *field lens of Huyghens eye-piece*, Fig. 210.

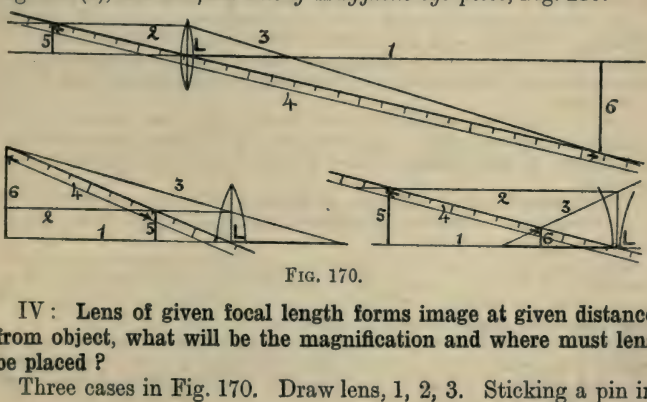


FIG. 170.

IV: Lens of given focal length forms image at given distance from object, what will be the magnification and where must lens be placed?

Three cases in Fig. 170. Draw lens, 1, 2, 3. Sticking a pin in

centre L, lay against it a straight edge 4 with given distance apart marked on it and move it about till it cuts 2 and 3 at the ends of this distance. Then 5 and 6 are object and image, compare their sizes; measure OL.

Calculation.—The cases shown have $1/b - - -$, $1/f + + -$, $1/a + - -$ respectively. $a \sim b$ is known, replace a by $b+$difference, solve and find a/b. But first make a sketch.

Ex. 5. Work out diagrams for a virtual object.

For problem V see § 448.

EXAMPLES.—CHAPTER XL

6. Give experiments which distinguish between shadow, real image, and virtual image. [L]m.

7. Show how to construct image in a thin biconvex lens. Object being at a distance exceeding f from lens, will an increase in f increase or diminish size of image? [Ab.]

8. Draw diagrams showing formation by convex lens of (a) inverted magnified, (b) inverted diminished, (c) erect magnified, images. [L]m.

9. Draw a curve showing for a convex lens the connection between distance of object from one principal focus and of image from the other. [L]m.

10. A lens intercepts light converging to a point 6 in. beyond and alters its point of convergence to 12 in. Find its focal length. [M.]

11. What focus lenses would produce an image distant 15 in. of an object distant 3 yd.? [L]m.

12. Where must a 10-in. lens be placed to project a magnified image on a screen 5 ft. from object? [L]m.

13. A convex 6·25 diopter lens projects an image on a screen 1 m. from object. In what two positions may the lens be placed? [L]m.

14. With a camera lens of 6 in. focal length a photograph is taken of a man 70 in. tall and 5 yd. away. Find height in picture.

15. Prove that a lens with a plane mirror behind it behaves like a spherical mirror whose radius of curvature is equal to the focal length of the lens.

16. A 10-cm. convex lens is held horizontally just above a liquid filling a tank 20 cm. deep. The image of a point 30 cm. above the lens is focussed on the bottom. Show paths of rays and calculate index of liquid.

17. A convex lens is projecting an image 9 in. away of an object 2 in. away from lens. Calculate distance of image of object 1·9 in. away; does this agree with § 392?

18. A horizontal telescope contains a pair of horizontal cross-wires one-tenth of an inch apart. It is focussed on a vertical staff 10 ft. away from the object-glass, which has a focal length of 10 in. Find length on staff apparently intercepted between the wires. [L.]

19. Calculate the curvature necessary for the faces of an equi-convex lens of 6 in. focal length made of glass of refractive index 1·55. Find the focal length of a lens which gives a 3 times magnified image of an object placed 2 in. from it. [L.]

CHAPTER XLI

SPHERICAL MIRRORS

§ 394. Returning to reflection, let us consider the image-producing properties of mirrors which instead of being plane are hollowed (concave) or bulged out (convex) into portions of a spherical surface.

An approximation to the continuously curved surface may be built up of many little flat facets. If hollow, all face inwards and reflect the light more or less exactly to one place; if convex, they face outwards and scatter it as if it originated at one place behind them. Here are concave mirror with real focus like convex lens, and convex mirror with virtual focus like concave lens.

The study of mirrors therefore resembles that of lenses, but is more simple, for there are no refractive indices coming into account. Mirrors of course reverse the direction of travel of the light, but in calculation the right-hand downhill remains positive whichever way the light is travelling on it. **$1/b$ for a real object** now becomes **positive**.

§ 395. **Reflection in a spherical mirror: Relation between radius of curvature, conjugate focal distances of object and image, and principal focal length.**

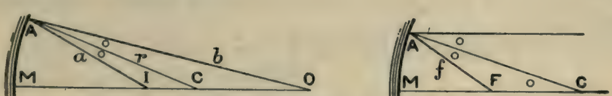


FIG. 171.

Take first the Concave Mirror, Fig. 171, described from centre of curvature C. Trace out two rays from an object point O.

One passes through C and strikes the mirror as a radius, i.e. perpendicularly, and returns straight back on itself. Another strikes at A making angle OAC with the radius AC, the 'normal.' It is therefore reflected at an equal angle CAI on the other side of

AC and meets its returning fellow-ray in I, which is therefore the image point of O. [All the rays of a whole cone starting from O and meeting the mirror will be concentrated on I.]

As with lenses, all angles must be small, § 375.

Gradient of AI is greater than that of AC by ∠e IAC.
,, AO ,, less ,, ,, ,, ∠e OAC.
Adding up, these equal angles cancel and
Gradient of AI+of AO=twice gradient of AC.
Putting AM=1, AO or MO=b, AI or MI=a, radius AC=r

$$\frac{1}{a}+\frac{1}{b}=\frac{2}{r}$$

Now put O very far away, OA becomes parallel to OM, gradient $1/b$=O, and MI becomes the distance f of the real principal focus.

$$\therefore \frac{1}{f}=\frac{2}{r}$$

§ 396. With a Convex Mirror, Fig. 172, the first ray, aimed at the centre, returns straight back on itself from M. The second

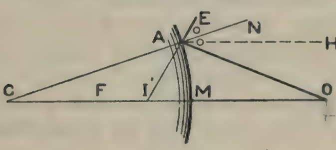


Fig. 172.

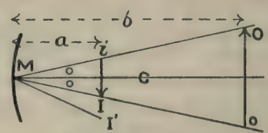


Fig. 173.

is reflected at A, so that the angles OAN, NAE on opposite sides of the radius (normal) CAN are equal. Reflected rays MO and AE both appear to have come from I′, which is therefore the virtual image of O.

Dot in AH parallel to CO, ∠e NAO= ∠e HAO+ ∠e HAN
∠e NAE= ∠e HAE− ∠e HAN
These are equal. ∴ HAO+HAN=HAE−HAN
or HAO−HAE=−2 HAN
or down gradient of AO−up of I′A=−twice up gradient of CA.

Now leaving the diagram and plunging into algebra, we must replace ‘down’ by + and ‘up’ by −.

$$\therefore \frac{1}{a}+\frac{1}{b}=\frac{2}{r}$$

With the usual convention as to sign then this holds in either case. The whole may be stated :—

§ 397. The sum of the right-hand downhill gradients of the going and coming rays = the r.h.d. gradient of the ray to the principal focus = twice r.h.d. gradient of radius to centre.

Or briefly, but less easy to translate into practice :—

The sum of the conjugate focal powers of a mirror = its principal focal power = double its curvature.

The focal power is evidently + **for concave** and − **for convex mirrors.** It is measured in Diopters as for lenses.

$1/f = 2/r$, reciprocally $f = \frac{1}{2}r$, **focal length = $\frac{1}{2}$ radius of curvature.**

§ 398. **Standard geometrical construction for mirrors, Fig. 174.**

Concave. From the ends of an object draw 'scissors' rays through centre of curvature C [which now replaces optical centre of a lens]. Both strike mirror radially (perpendicularly) and return back on themselves.

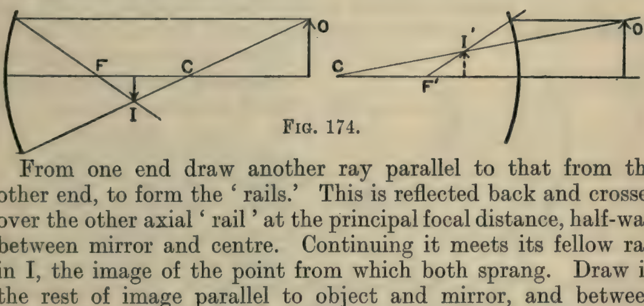


Fig. 174.

From one end draw another ray parallel to that from the other end, to form the 'rails.' This is reflected back and crosses over the other axial 'rail' at the principal focal distance, half-way between mirror and centre. Continuing it meets its fellow ray in I, the image of the point from which both sprang. Draw in the rest of image parallel to object and mirror, and between 'scissors' rays. Evidently it is real and inverted.

Convex. 'Scissors' rays return on themselves before reaching centre. The parallel ray is reflected directly away from F′, at the virtual principal focal distance, half-way to C, and I′ is where it crosses the direction of its fellow ray. Fill in image, evidently it is a small erect virtual image, the familiar little picture inside the reflecting globe, flask, teapot, etc.

§ 399. **Magnification.**

Since both lie between the 'scissors' rays, evidently the ratio of diameters of image and object = m = ratio of their distances from the centre of curvature.

A more practically convenient relation can, however, be deduced by calculation or as in Fig. 173. From mid-point of object draw

through C to M, join OM, oM. Ends of image lie on these rays; for if not let I′ be end. By symmetry ∠e OMC = ∠e CMI, by law of reflection ∠e OMC = ∠e CMI′. ∴ I and I′ coincide, and similarly i is on OM. Hence again

$$m=a/b$$

m is now + for real inverted image.

§ 400. **Motion of image.** As the object runs along the 'rails' of the standard construction all that happens is that the slanting 'scissors' ray OCI alters its inclination and cuts the fixed line AF (produced) at different conjugate distances, as in Fig. 175. There is no limit to its inclination, but those who wish to rely upon actual rays all the time can use instead of it the rays MO, Mo (produced if necessary) of the last paragraph.

The virtual image in the **convex mirror** starts at $\frac{1}{2}r$ beneath the surface for distant objects and slowly comes forward till image and object touch on the surface; m increases from 0 to 1.

With the **concave** mirror much more happens. The real image starts at F, $\frac{1}{2}r$ out in front, and comes forward to meet the object till they meet at the centre of curvature, image being inverted and same size as object ($m=-1$). The scissors ray now slants the other way and carries I rapidly out along AFI; the mirror is producing a large distant aerial image of a small object (m—large). When object reaches F, OC and AF are parallel, I has disappeared at infinity, § 451.

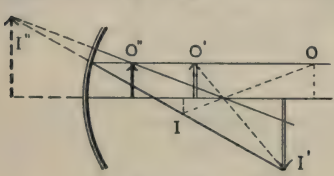


FIG. 175.

When the object is within its principal focal distance of the mirror, OC slants *less* than AF and can never meet it, but both appear to come from a point I″ behind the mirror on an enlarged upright virtual image, Fig. 175, which comes forward, diminishing till image and object touch on the surface. This is the use of a concave mirror as a magnifying shaving-glass, etc.

In **calculation** put $a=f+(a-f)$ and $b=f+(b-f)$, then $(a-f)$ and $(b-f)$ are distances from the principal focus. Then

$$\frac{1}{f+(a-f)}+\frac{1}{f+(b-f)}=\frac{1}{f}$$

or $f[f+(b-f)]+f[f+(a-f)]=[f+(a-f)][f+(b-f)]$

or $f^2=(a-f)(b-f)$

Plotting these distances of Image and Object *from the Principal Focus* as ordinates and abscissæ of a curve we get the rectangular hyperbola of Fig. 176, which sums up all there is to be said about distances and magnification in mirrors. Measuring from dotted axes gives a and b.

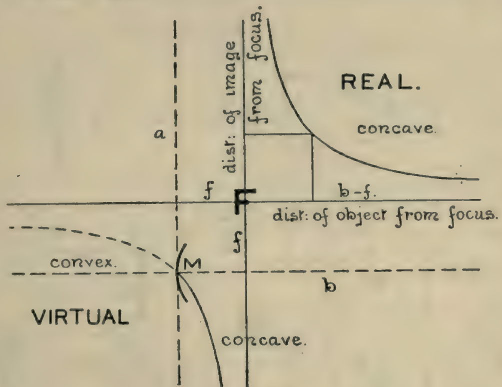


Fig. 176.

§ 401 : **Mirror problems.**

The standard construction solves mirror problems similar to those of § 393, taking similar precautions.

I. *Object b from mirror of focal length f ; find position and magnification of image.* Done in § 398.

II. *Distance of image from object and its magnification are fixed. Where and what is mirror ?*

Very simple construction ; join ends of object and image by the 'scissors' rays and by the two rays of § 399. The intersections of these pairs give C and M or M and C according as image is inverted or upright. Two cases in Fig. 177 (II).

III. *Image magnified m by mirror +f ; where are mirror and object ?*

Four cases in Fig. 177 (III). Draw 'rails' 1, 2 and dot in 3 parallel to them and m times as far from 1 as 2 is. On 1 mark M, F, C, describe M cutting 2 in A. Draw A F, it cuts 3 in I, draw IC, it cuts 2 in O. Draw in image and object to line 1, thus arriving at standard construction.

In the cases C, D, E the object is virtual ; light already con-

verging toward a real image is always more sharply converged toward a smaller one by a concave mirror (C, left as an exercise); may be formed into a larger real image by a convex mirror D as in the Cassegrain telescope, Fig. 212, or if not convergent enough appears as an inverted virtual image which may be enlarged (E).

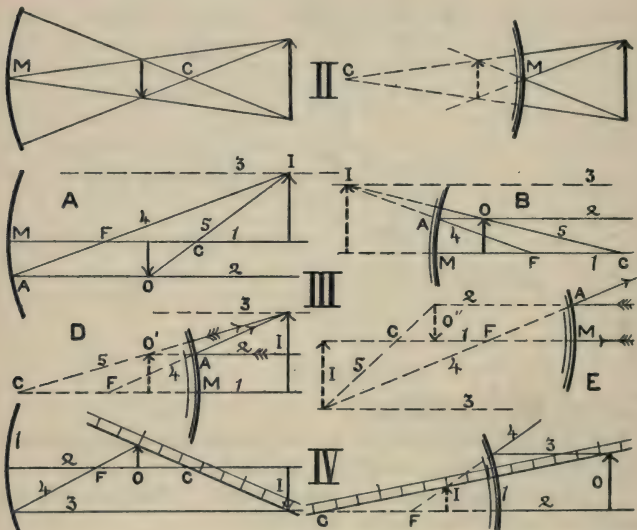


FIG. 177.

IV. *Mirror of given f forms image at given distance from object, what is m and where is mirror?*

Two cases, Fig. 177 (IV). Draw mirror 1, rails 2, 3; 4 to F, stick pin in C, and with marked length on straight edge rested against it find IO on 3 and 4.

V. *Given distances of image and object from mirror find f.*

Draw O, M; between rays 1, 2 from ends of O crossing at M mark I at given distance, the other two rays joining object and image cross at C; CM=$2f$. The diagrams come identical with those of Fig. 177 (II).

EXAMPLES.—CHAPTER XLI

1. A luminous point is inside a reflecting circle, half-way between centre and circumference; draw the reflected wave front after each portion of the disturbance has travelled a distance equal to diameter of circle.

2. Show that the focal length of a concave mirror is equal to half its radius of curvature. How would you find the focal length experimentally? [L.]

3. Show in a diagram the cone of rays by which an eye looking into a concave mirror sees one point of image of an object close in front. [L]m.

4. Why is the rear-reflecting mirror attached to a car made convex? Draw diagram showing positions of eye and object seen. [L]m.

5. Two reflections of the landscape are seen in a hollow glass sphere (e.g. a lamp bulb). Where are they inside the sphere, and what is the difference between them?

6. In 5 how do the images change as the object approaches? Can they coincide as regards distance?

7. Give diagrams of production of virtual images by convex and concave mirrors. Galvanometer mirror $f=2$ ft., lamp is at 3 ft., where must scale be? What if lamp were at 18 in.? [M.]

8. An object is 6 ft. in front of concave mirror 5 ft. radius, find image. Diagram. [Ab]m.

9. A pin 3 cm. long is 48 cm. in front of a concave mirror, the real image is formed at 16 cm. The pin is moved 24 cm. towards the mirror, draw a diagram and find the changes in the image. [L]m.

10. Convex lens produces real image of flame 50 cm. from itself. Concave mirror 100 cm. from lens reflects the light back through lens to form an image close to flame, what is f of mirror? [L]m.

11. Show that if a horizontal concave mirror is filled with a liquid its apparent radius of curvature is diminished in the ratio of μ of liquid. [L.]

12. The plane side of a plano-convex lens is silvered, and the lens then acts like a concave mirror 30 cm. focal length. $\mu=1{\cdot}5$, calculate radius of convex surface. [L.]

13. A plano-convex lens silvered on its plane side acts like a concave mirror of 20 cm. focal length. When the convex side is silvered it acts like a concave mirror of 7 cm. Calculate μ. [L.]

Chapter XLII

PRACTICAL METHODS FOR MIRRORS AND THIN LENSES

Out of a host of practical methods the following few are recommended as good and adequate to the elementary study of mirrors and thin lenses such as are commonly met with.

§ 402. For supporting things in position the '**optical bench**' of Fig. 178 is simple and most satisfactory. On a stout board is nailed a wooden metre scale; against this slide wooden uprights of the plain shape shown (pretty bits of cabinet-making or metal-work cause infinite trouble in use. Over the large holes in these are strapped by elastic bands the lenses, mirrors, post-card screens, or glass millimetre scales (for magnification measurements).

§ 403. *A broken skyline in sight through the* open *window at least* 100 *ft. away will serve for an indefinitely distant object.* (*Sun itself too dazzling.*)

Convex lens. Catch sharp image of the distant chimneys, trees, etc., on screen behind lens. Lens to screen $=f$, Fig. 178 (i).

Concave mirror. Ditto on screen half covering hole in upright in front of mirror. Mirror to screen $=f$, Fig. (ii).

§ 404. **Methods by return image.**

A luminous object is made by cutting a very small cross, or pricking a few pinholes, in a card on the first upright.* Somewhere behind is a lamp with a broad flame, or else with a diffusing ground glass.

* Instead of this a pin, well illuminated, may serve as object; its image is looked for in the air by an eye 2 ft. or so to right, adjusted till same size and always touching object, as tested by moving head sideways and best by getting both in focus together under strong pocket lens. More troublesome than lamp and screen, it is beloved of examiners.

Convex lens. Behind the lens is held a plane mirror (bit of good thick looking-glass) and lens is moved till image of the illuminated cross appears in sharp focus on screen close beside it, Fig. (iii). Lens to screen$=f$, for light that exactly retraces its path after reflection at a plane must necessarily be parallel.

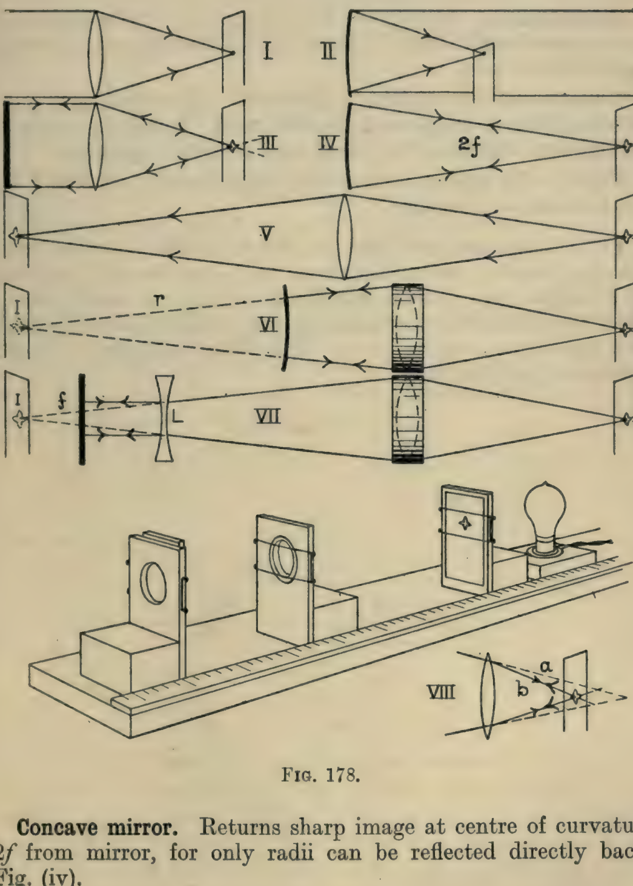


FIG. 178.

Concave mirror. Returns sharp image at centre of curvature $2f$ from mirror, for only radii can be reflected directly back, Fig. (iv).

Conjugate foci.

Convex lens, Fig. (v). Involves calculation by ready-for-use formula $1/a+1/b=1/f$. Pet examination method.

Conjugate focus and return image. With the best convex lens you can get (say a 6-in. lantern objective) form a real image I on a second screen at end of bench :—

Convex mirror. Insert as in Fig. (vi) and move till sharp return image seen beside cross. $IM=r=2f$, for the directly returned rays must be radii.

Concave lens. Set up the plane mirror. Insert lens and move till sharp return image seen beside cross. $IL=f$, Fig. (vii), for the light to and from the plane mirror is then parallel.

NOTE.—*The faces of lenses are of course mirrors, unsilvered, and can be measured as above.*

§ 405. A **convex** lens's faces can also be studied as in Fig. (viii). Having found f, as in (iii), move lens closer to the cross till at b sharp image returns from back surface, on which rays in glass must therefore be falling radially (most passing through) as if they came from its centre. Hence $a=r_2$ and $1/r_2-1/b=1/f$. Turn over for other face.

CHAPTER XLIII

COMBINATIONS OF LENSES

§ 406 : **Focal power of a pair of lenses in contact.**

In Fig. 179 (I) rays from O at the principal focal distance f_1 of the first lens are rendered parallel by it and the second then converges them to its principal focal distance f_2. The total bending is the sum of the focal powers [*or* f_1 and f_2 are conjugate foci on opposite sides of the combination]

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

The focal power of a close combination is the algebraic sum of the focal powers of the components.

If one is negative, as in the concave component of an achromatic lens, algebraic sum is of course numerical difference.

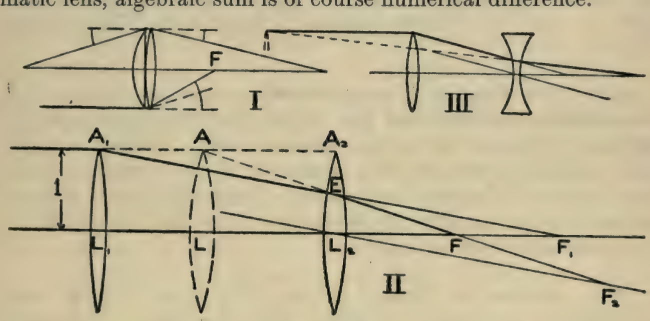


FIG. 179.

§ 407 : **Focal power of a pair of lenses separated by distance d.**

Taking two convex lenses, Fig. 179 (II), the axial ray $L_1L_2F_1$ passes straight through, the parallel ray is bent at A_1 and travels at gradient $1/f_1$ towards F_1, but meeting second lens is bent down to F, the principal focus of the combination. F_1 and F are conjugate foci of L_2; $L_2F_1=(f_1-d)$, hence $1/L_2F-1/(f_1-d)=1/f_2$.

This gives *position* of F.

Of more interest is the 'equivalent focal power' of the combination, i.e. the slope of BF. By similar triangles $EL_2/A_1L_1 = L_2F_1/L_1F_1 = (f_1-d)/f_1$ and putting $A_1L_1(=A_2L_2)=1$ as usual, $EL_2=(f_1-d)/f_1$. Now we saw in § 375 that the refraction caused by a lens is proportional to the distance from the axis: the full refraction of second lens at $A_2=1/f_2$, $\therefore$ at E refraction ($\angle$e F_1EF) $=1/f_2 \div 1\times(f_1-d)/f_1=(f_1-d)/f_1f_2$.

$$\therefore \text{ Final gradient of EF} = \text{gradient of } EF_1 + \angle\text{e } EF_1F$$
$$= 1/f_1 + (f_1-d)/f_1f_2$$

or **focal power of combination** $\underline{\dfrac{1}{f}=\dfrac{1}{f_1}+\dfrac{1}{f_2}-\dfrac{d}{f_1f_2}}$

This is less than when close, becomes 0 when $d=f_1+f_2$ (telescope), and is thereafter —.

The theoretical '**Equivalent Lens,**' which would have the same focus and focal power as both, is situate where FE cuts direction of ray it sprang from in A. To find it, position of F would be calculated as in first section, then f measured back from it. This 'equivalent lens' is in the 'second principal plane' of the combination. If light were passed through the other way we should find the same focal power but the 'equivalent lens' in a *different* position, the 'first principal plane.' See further Thick Lenses, § 410.

§ 408: The whole of this can be done by the exceedingly simple **Geometrical Construction** of Fig. 179 II (and III). Draw axial ray and L_1L_2. Parallel ray at A_1 becomes A_1EF_1. Parallel to A_1EF_1 draw L_2F_2, the refracted EF_2 meets this at F_2, crossing original axial ray in F, *Focus of combination.* Produce FE back to meet original ray in A, draw in equivalent lens AL; *Focal Length* of combination=AF or LF.

§ 409. **A Lens in Water.** If the thin prism of § 300 were immersed in a medium of higher refractive index μ' than air, the speed outside is now only $1/\mu'$ of what it was and the ratio of speeds outside and inside diminishes to $1/\mu' : 1/\mu = \mu/\mu'$ or $\mu/\mu' : 1$. That is, μ of prism relative to air is to be replaced by its index relative to the surrounding refractive medium.

This gives $D=(\mu/\mu'-1)A$ and for a lens $1/f=(\mu/\mu'-1)(1/r_1-1/r_2)$, so that the focal power is less than before, in the ratio $(\mu/\mu'-1):(\mu-1)$. e.g. if a lens of μ 1·5 is in water μ' 1·33, $1/f$

changes from proportional to $(1{\cdot}5-1)={\cdot}5$ to proportional to $(1{\cdot}5/1{\cdot}33-1)={\cdot}125$; the lens retains only a quarter of its strength. Such is an oil drop in water.* [And see § 442.]

If the surrounding medium is of identical index refraction ceases altogether (invisible, § 374). If of greater index, e.g. glass in carbon disulphide 1·671, $1/f$ becomes proportional to $(1{\cdot}5/1{\cdot}67-1)=-{\cdot}1$, i.e. changes sign, and begins to gain strength as a concave lens. Optically it is a cavity between two hollow refracting cheeks, like an air bubble in water,* or a water drop in oil.

** A drop of milk frothed and put under the microscope shows both in abundance.

Thick Lenses

§ 410: No actual lens is indefinitely thin. In § 368 we saw that a pond or thick flat glass looked into perpendicularly appears shallower than it really is, i.e. a virtual image of the bottom, parallel and quite unaltered in size, is seen in a new position, and the intervening distance is annihilated, so to speak.

The action of a lens may therefore be divided into—

(1) The action of a thin lens, due to its curvatures.

(2) The space-annihilating effect of its thickness.

It can in geometrical theory be replaced by a thin lens of equivalent focal power (§ 407) which takes in light in one position called the **first principal 'plane'** and then jumps the annihilated gap to a parallel **second principal 'plane'** and gives out the light. Graphically this means cutting the ordinary thin lens diagram in halves down the middle of the lens and putting the cut edges in the positions of the principal planes. For an ordinary thick bull's-eye these would be about one-third its thickness apart.

A combination of separate lenses (like a 'rectilinear' lens, or micro. o.g. or e.p.) also ranks theoretically as a 'thick lens,' but now *duplicates* space, the lens jumps back, the diagram halves overlap, sometimes very much (telephoto lens), and the planes may be far away from the actual lens. We have seen how to find them by geometrical construction in § 408. Evidently the ordinary formulæ apply when the distance of the object is measured from the first principal plane and the image from the second principal plane.

§ 411: Below is given an easy practical process which facilitates accurate measurements of lens combinations instead of the bungling guesswork that results from pretending them 'thin.'

A. **Focal length of 'thick' convex lens.**

In $\frac{1}{a}-\frac{1}{b}=\frac{1}{f}$ multiply by af and put $\frac{a}{b}=m$ the magnification.

$$\therefore (1-m)f=a$$

For a different magnification $(1-m')f=a'$

Subtract and divide out by $m-m'$. $\therefore f=\frac{a-a'}{m-m'}$,

$$\textit{focal length}=\frac{\textit{change of distance of image from lens}}{\textit{its change of magnification}}$$

Put up two cross scales with the lens between them, illuminate one and focus its image on the other. Observe the magnification m, i.e. the number of millimetres which 1 mm. division of the image occupies. Move the observing scale a definite distance farther from the lens, then move object scale only till image again clear, observe m'. Then f=the definite distance$\div(m-m')$.

NOTE.—m at the principal focus=0 (point image) and it follows that an axial ray can be divided into parts each=f and then m of image is simply the number of parts it is distant from F, Fig. 180. **This very useful device applies of course to all lenses.**

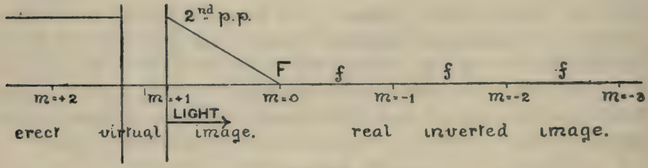


FIG. 180.

B. **Principal focal points.** Put lens both ways in sun (or method iii, § 404) and record distances of both foci from a scratch made on the lens mount. These distances are not equal, you do not expect them to be.

C. Measure inwards f from each principal focal point and you have the positions of the principal planes (or the 'nodes' of the lens in air). If either focus was inaccessible, mf and $(m+1)f$

measured inwards from an m times magnified image on that side give focus and principal plane.

Example. A certain $\frac{2}{3}$-in. micro. objective by Crouch.

Placing on stage and in e.p. (field-lens removed) scales of ·1 mm.

1 stage division = 8·6 e.p. divs., and pulling out draw-tube 2 in., = 11·5.

$\therefore f = 2 \text{ in.} \div 2{\cdot}9 = {\cdot}69$ in.

Focal pt. found ·16 in front of nozzle.

$\therefore$ 1st pp. is ·53 above nozzle.

A ·1 mm. scale laid on back of mount showed image when formed there was magnified 1·25 times, $\therefore$ back focus is $1{\cdot}25 \times {\cdot}69 = {\cdot}86$ in. from back of mount (in among the glasses), a further ·69 below this is the 2nd pp., ·05 below the nozzle.

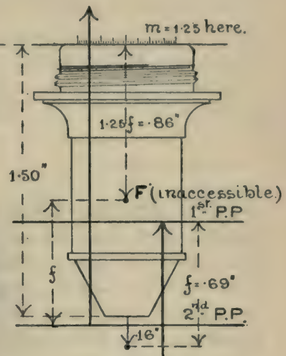


FIG. 181.

EXAMPLES.—CHAPTER XLIII

1. A lens of −1 diopter is placed in contact with one of +30 cm. focal length. What is focal length of combination ?

2. Prove that F of two lenses in contact is given by $1/F = 1/F_1 + 1/F_2$. [L.]

3. Two thin 6-in. focal length plano-convex lenses are gradually moved apart. Where will they focus parallel rays when separated (a) zero, (b) 1 in., (c) 6 in., (d) 12 in. ?

4. Show that if the convex and weaker concave lenses of an achromatic lens be gradually separated the equivalent focal power increases.

5. Draw the construction appropriate to a pair of lenses separated more than the focal length of the first.

6. Show that focal length of thin lens index 1·5 immersed in a flat cell of water is 3 times that of lens. [L.]

[Notice difference from § 409 ; the rays here escape into the air, suffering further refraction ; there they were entirely in water. This is a combination of a glass convex and a water lens of equal concavity.]

CHAPTER XLIV

COLOUR

§ 412. **Spectra.** Light a bunsen burner in front of a dark background, and shut its air-holes. From the far side of the room look at it through a glass prism standing on its triangular end. You have to look in quite a different direction and you see the luminous flame drawn out into a broad rainbow band, a **continuous spectrum,** the blue farther away from the real position of the flame than the red.

The breadth of colour depends on the material of the prism, a common 'lustre' gives only narrow colours. A prism made at home from three 3-in. squares of plate glass set in a triangle on a piece of wood, cemented bottom and edges with plaster, made tight with gum and filled with liquid carbon disulphide ($\frac{1}{2}$ lb., 6d., chemist's, malodorous and highly inflammable) will give magnificent spectra.

Now open the air-holes and support a little salt or soda on an iron wire in the bluish flame. Only the wire makes a streak of colour now, and you see only one distinct deep yellow flame. Put a little saltpetre on the wire, as it flares up you will see through the prism not one mauve flame but three distinct flames (overlapping partly with the 'lustre,' quite apart with the better prism), viz. a red flame on one side of the yellow and a fainter violet on the other. Open the air-holes wide to get the roaring green cone, and you will see in a row a citron-green, a green, a blue, and a violet cone. If bothered by overlapping, put before the flame a piece of tin with a narrow vertical slit cut in it, and you will see distinct narrow coloured slits.

Stand your good prism near one end of a 2-ft. board and stick a new pin upright at the other end. By the light of the flame you see several coloured pins. In the sunshine the pin becomes a coloured streak, down across which are faint dark lines, at least one each in the red, yellow, green, and blue. Crook the pin, and the dark lines go crooked likewise. You now realize that this

streak is built up of a succession of very close images of the pin of different colours, and the dark lines mean that some particular colours are missing from sunlight. The pin's head will not show any dark gaps at all, its various images are too broad and overlap them. Thus Newton, who let sunlight into his dark room by a round hole, never saw these gaps, and it was left to Fraunhofer, who used a narrow chink, to discover (ca. 1814) the absence of these images of it and to describe them as spectrum **'lines.'**

Now plant two pins before the prism in sighting-line with the dark 'line' in the sunshine yellow, go indoors and illuminate the pin with the salted flame, and you see its solitary yellow image just where the dark line was. And by lighting with magnesium wire you can see on a continuous spectrum a particularly bright green pin where the dark gap was in the green.

The separation of colours has been brought about in consequence of their different speeds in the glass, violet slowest, i.e. the refractive index has increased between red and violet, and according to §§ 300, 372 violet light is more bent round than red, and the colours are seen as in Fig. 182 (upper) in the directions which have been bent the appropriate amounts.

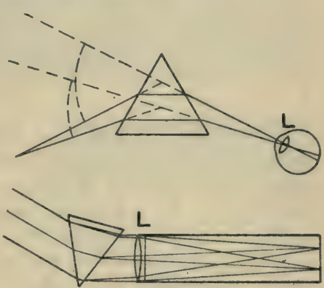


FIG. 182.

§ 413. For photographing the spectra of stars the lens and retina of the eye are replaced by lens and sensitive plate at its principal focal distance, the whole forming a long camera with a prism close in front, Fig. 182 (lower). The star's light coming in one parallel beam is refracted into different directions by the prism and concentrated into a string of images of the star on the plate, the latter slowly moving sideways to broaden out the beaded line into a band.

NOTE.—*Any attempt to produce a spectrum on a screen without a lens to form clear images of the narrow source can only result in a coloured blur.*

It is always best that the light should fall on the prism in one direction only. To secure this in **ordinary spectroscopes** another lens L′ first catches the light from the illuminated slit at its

principal focus and 'collimates' it (brings it into line). And since the spectrum is small, an eye-piece E is used to magnify it. Then for measuring purposes a scale of some sort is provided in the

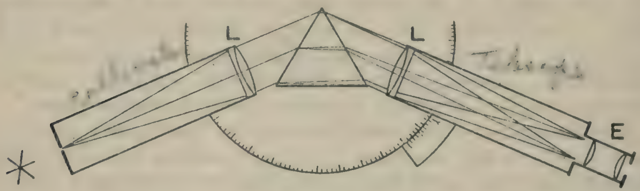


FIG. 183.

eye-piece, or else the telescope LE turns on a graduated scale; and the spectroscope becomes a spectrometer, Fig. 183.

The prism usually stands in its position of minimum deviation.

Large spectroscopes have several prisms in succession.

In pocket and micro-spectroscopes there is a slit, magnifying lens focussed on it, and 'direct-vision' prism, Fig. 184 and § 440.

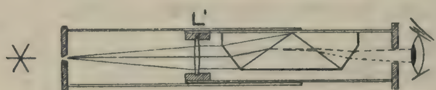


FIG. 184.

§ 414. The fact that a Diffraction Grating, § 296, will do instead of the Prism, shows that *different colours correspond to different wave lengths* of light (or reciprocally, different vibration frequencies), the red to the longer waves (slower vibrations). The grating enables spectrometer readings to be translated into wave lengths, § 296. A grating gives a very long but not very bright spectrum. The colours and lines follow in precisely the same order as with a prism but the red is enormously elongated, whereas a prism makes this short, and draws the violet out long.

§ 415. Putting the bunsen flame then before the slit of the spectroscope, there appears a spectrum of separate **bright lines,** meaning that certain definite frequencies of vibration can be detected in their source. **Bright-line Spectra characterize incandescent gases and vapours of metals.** See Fig. 185.

The *Flame* brings them out from the alkali metals and thallium, and the alkaline-earth metals: the latter contain also broader

lines or 'bands.' The volatile salts put in the flame soon oxidize and their non-metallic constituents make no difference after the first 2 or 3 sec. Mixtures give the complete spectra of every metal present. The pretty spectrum of the greenish-blue flame itself, with its four bands, sharp on one edge ('flutings') is that of carbon. 10^{-10} grm. of sodium suffices to give its yellow line.

The greater violence of the *Electric Spark* between points of the metal, or wet with solutions of metallic salts, or of the *Electric Arc* between poles charged with salts, volatilizes all metals and brings out spectra of many brilliant lines from their vapours.

Gases are made luminous by passing electric discharges, § 672, through them, rarefied to about 1 cm. pressure, in capillary tubes.

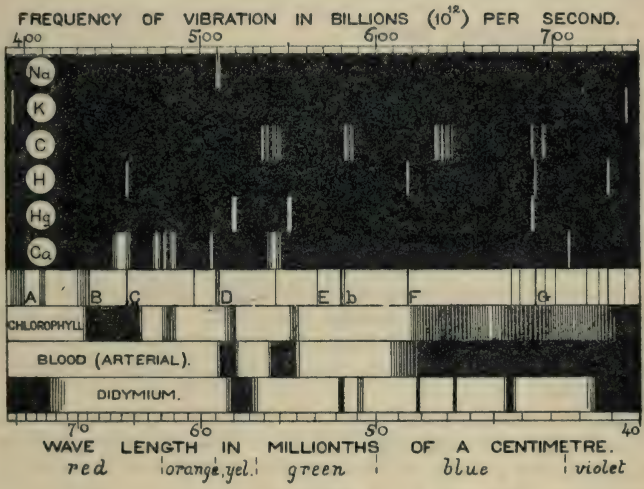


FIG. 185.

In addition to lines, nitrogen in a tube or in a leyden-jar spark gives many flutings in the violet. It gives the violet tint to lightning. Nebulæ show the bright lines of hydrogen, helium, etc. Comets show the fluted carbon spectrum.

§ 416. When hydrogen is compressed its few bright lines broaden out into indistinct bands. Matter being much closer packed as solid or liquid than as gas, it is not surprising to find

that **incandescent solids and liquids give a Continuous Spectrum,** all characteristic lines being blurred out. One may suppose that atoms close together interfere with one another's vibrations and produce a confused jumble of indefinite frequencies.

On this view the candle flame owes its luminosity to incandescent particles within it, easily deposited as soot. The continuous spectrum of the arc lamp is crossed by bright lines: by forming a real image and spectroscoping it bit by bit the continuous is seen to arise from the hot carbons and the lines from the faint arc itself. The latter has nowadays been made long and luminous by loading the carbons with calcium salts, etc. (see Fig. 185, Ca), which give brilliance even to a cool gas flame, and the highly efficient ' flame ' arc lamps are the result.

§ 417. Absorption Spectra.

But the continuous spectrum of the sun is crossed by the dark lines of which our prism revealed two or three, of which Fraunhofer listed some hundreds, naming the chief ones alphabetically, and of which modern spectroscopes show thousands. It was noticed that several of these tallied in appearance and position with the bright lines of the laboratory, D with the sodium yellow, *b* with the magnesium green, C, F, and G with the hydrogen red, blue, and violet, see Fig. 185. But why were they dark?

Sir George Stokes used the argument of § 301. Like the boats, the atoms of sodium in vapour should chiefly check and ' absorb ' vibrations of their own natural frequency. A few years later (ca. 1860) Kirchoff viewed limelight through a salted flame and saw the sodium line dark by contrast on the bright continuous background.

Had then the sun a cooler atmosphere of hydrogen and metallic vapours to select and absorb parts of the light from a dense central sphere? If so, at the moment when the eclipsing moon just blotted that sphere out, might not the glowing atmosphere round the edge show these same lines as bright lines, since it was only by contrast that they looked dark? At the next eclipse the red ring of ' chromosphere ' that flashed round the dark moon showed the spectra of hydrogen, calcium, and the element first discovered and named from its line there, helium. Since then a closer layer has been glimpsed showing many more of the Fraunhofer lines bright. Sometimes the outer chromosphere is so thick (' prominences ') that it actually produces a bright line down the middle of the broad black line—it absorbs broadly and re-

emits vibration concentrated accurately in one frequency—and by the light of these, hydrogen and calcium clouds are now daily photographed all over the surface of the sun.

Several hundred solar lines have been identified with Fe, Ti, Ca, Mn, Ni, Co, Cr, Ba, H, Na, etc.

Stars have been classified according to their dark lines; some have even more than the sun; Arcturus is very like him; Vega, Rigel, etc., bluer and probably hotter, show only hydrogen absorption.

The strongest bands in the red, A and B, get darker as the sun sinks and shines through a longer length of the earth's atmosphere. They are due to absorption by oxygen (which owes its blue tint as a liquid to their presence). A broad band in the yellow is due to aqueous vapour, and darkens before rain, which it is used by some in forecasting.

The Solar Spectrum thus represents the continuous emission of a dense incandescent mass, less the absorption of the gaseous envelopes of sun and earth.

§ 418: **Application of Doppler's principle,** § 303. If source and spectroscope are approaching each other the frequency of vibration appears increased, i.e. well-recognized spectrum lines are shifted a trifle towards the quicker violet, and the speed of approach is easily calculated in terms of the speed of light. In this way the speeds of approach or recession of several stars have been measured, Saturn's rings have been found to revolve faster inside than outside (meteor swarms), etc.

§ 419. **Extent of spectrum.**

The visible spectrum is only a small part of the whole known spectrum, which extends far beyond the red into much slower vibrations (longer waves) of 'dark heat' as the **infra-red,** and beyond the violet to higher frequencies—photographic—as the **ultra-violet.**

The flint-glass prism of an ordinary spectroscope practically blots out these extensions with their contained lines and bands. See also § 499.

Substances at lower temperatures emit mainly infra-red radiation; as the temperature is raised the emission contains higher and higher frequencies of vibration, a red glow becomes visible, brightening in tint as yellow and green frequencies appear (red-hot iron) and whitening as blue and violet begin to be emitted. 'Blue-hot,' a popular exaggeration, suggests the

way in which the increasing approximation to noonday whiteness of the very hottest metal contrasts with the tint ordinarily accepted as 'white heat.'

Colour

Colour of emitted light has been already dealt with.

§ 420. **Colour by transmitted light.**

All coloured substances produce *Absorption Spectra* when the light from a white source which gets *through* them is analysed in the spectroscope, and this discloses the cause of their colour. The absorption spectrum of the photographer's ruby glass, for instance, is a broad black shadow blotting out all except the red. A red signal shines through it with transparent brightness; to a green signal it is opaque, the received energy merely goes to warming it. A test-tube of weak pink permanganate solution held between lamp and spectroscope slit produces five dark bands in the green, a stronger solution blots out the green altogether. Hence its colour is what is left of white light after the green has been removed. Restoring this would complete the white again, and that is what is meant by the statement that crimson and green are **complementary colours.**

Cobalt glass lets through the extreme red without absorption, and the whole range of blue and violet with a slight absorption. There is much more of these last to start with and the light appears *blue*, but many thicknesses of glass increase this veiling of blue and violet and one sees through the bundle the unhindered *red*.

Chlorophyll (steep green leaves in alcohol) serves as a type of *sharp local absorption* in several places, see Fig. 185, hæmoglobin (few drops of blood in water) has a very characteristic spectrum changing with its state of oxidation, iodine vapour and NO_2 have very complex dark-line spectra, didymium salts absorb several scattered portions with the curious result that they appear almost colourless.

§ 421. **Colour by irregularly reflected light.**

Thus colour seen through is accounted for, but what of the colour of leaves, flowers, and earths, of feathers, fabrics, etc., looked *at* and seen by the light they scatter (§ 374)?

(1) A glossy leaf, or varnished picture, when regularly reflecting light to the eye, shows hardly a trace of its own colour; the smooth sea reflects noonday blue or sunset gold impartially.

(2) *Under the microscope*, by transmitted light, individual coloured grains, cells, and fibres are remarkably transparent, and

(3) by reflected light each shows a certain amount of internal reflection, like cut gems or rods of coloured glass.

That is, part of the light things scatter has dived through absorbing material, and therefore they show much the same colour as by transmitted light. The other part has come back uncoloured from the front surface. The proportion of the two parts varies greatly; silk dilutes its colour with surface light, velvet does not, satin looks either rich coloured or merely shiny according as its surface light misses or catches the eye. Wetting a sponge or varnishing wood means filling it with a medium of about its own refractive index, which does away with the more superficial reflections and permits the light to dive deeper and return more richly coloured.

Conversely, finely powdered bichromate, froth on beer, etc., show only a slight orange tint; so many little surfaces are flinging back the light before it can traverse any appreciable thickness of coloured substance.

§ 422. **Composition of the incident light.**

Thus the natural colour of all these things is the unabsorbed residue from full white light, and they will appear more or less altered if the light falling on them is not white. Hence the difficulty in telling colours by ordinary artificial light, which is deficient in blue and violet (e.g. try chrome alum solution). In monochromatic light, of course, everything reduces to one colour in varying brightness. Passing a geranium along a spectrum, it is dark in the blue, in the green a black flower with green leaves, in the yellow all dull yellow, in the red a red flower with black leaves. The face and lips are dull and grey in the yellow light of a salted bunsen flame. The pretty green mercury-vapour lamp has a spectrum of a yellow, a green, and a violet line, it shows up objects of these colours vividly, but is trying to the complexion, making it green and unspeakably dirty.

§ 423. **Metallic colours** arise rather differently. Gold leaf transmits bluish-green light about *complementary* to the colour it reflects. But it transmits very little, though only ·00001 cm. thick, and one may suppose that metals are so exceedingly opaque that any light which does enter them is quenched in

perhaps ·0001 cm., while the remaining colours cannot even get in and are flung back forthwith. (And see note in next section.)

Several *very intensely coloured substances* behave similarly. Indigo has a coppery sheen; crystals of permanganate, magenta, etc., opaque, but crimson when thinned out (as in solution), have the complementary green lustre; purest methylene-blue is golden.

§ 424: **'Anomalous dispersion.'** Very thin hollow prisms can be filled with strong solutions of these dyes. They throw spectra unexpectedly long, broken in pieces, and coloured in quite the wrong order.

By the analogy of boats (§ 301) it appears that a medium, containing particles which can vibrate with a definite frequency of their own, will absorb waves of that frequency and prevent them passing through (absorption band). It can be shown that waves a little shorter travel through very fast, while waves a little longer struggle through only very slowly. That is, close to the blue side of an absorption band the speed in the medium is great; $v/V=\mu$ is small, sometimes less than 1. Crossing to the red side where the waves are longer the speed is very small, μ is very large, but sinks quickly to a normal value farther on.

NOTE.—This sudden slowing of speed, almost equivalent to an abrupt stop, of course produces a reflected wave, i.e. a surface colour, a little redder than the absorption band.

Thus magenta, with its metallic absorption and lustre in the blue and green, when formed into a prism refracts violet feebly and yellow very strongly, and the colours run Violet—gap—Red, Orange, Yellow (most refrangible) a score times as far apart as glass would throw them. This was called **anomalous dispersion,** but it is now recognized that glass, etc., act in the same way. For glass absorbs strongly just beyond both ends of the visible spectrum (§ 419), so that the red is the short-wave side of the infra-red band and μ is low; the violet is the long-wave side of the ultra-violet band and μ is increasing fast.

The **vapours** of iodine and the metals retain all the characters referred to in these paragraphs. Sodium vapour has well repaid study.

NOTICE the distinction between these anomalous spectra which the substance, in prism form, constructs, and its absorption spectrum where it merely obstructs.

§ 425. **Calorescence.** Tyndall filtered sunshine through a black solution of iodine in carbon disulphide, which was quite opaque to light but transmitted 'dark heat,' i.e. 'infra-red.' Focussed on a strip of metal it was raised to a bright red heat. In this **calorescence,** radiation has been excited containing wave lengths from perhaps 100 microns to ·5 micron (green), and it continues a little while after the supply of energy is cut off. Of course it is all a perfectly normal thermal effect.

§ 426. **Fluorescence and Phosphorescence.**

Sun shining on the 'canary glass' (containing uranium), now common in ornaments, makes it glow with a green light quite different from its ordinary pale yellow tint. Even if filtered through blue glass (short waves only) it still excites the green (longer wave) **Fluorescence.**

Further, by using a rotating perforated disc arrangement (phosphoroscope), the glow can be seen persisting for a small fraction of a second after the sunlight is cut off. This is brief **Phosphorescence.**

Comparing these effects with calorescence, one is led to the view that both alike involve the storage of supplied energy for at least many million vibration periods (one green vibration occupies about 2×10^{-15} sec.) and its re-emission in a fresh form.

But the blackened metal will absorb all wave lengths and in re-emitting starts at the bottom of the infra-red and gives nearly the full radiation, mostly invisible, of a continuous spectrum (§§ 419, 497), whereas the fluorescent substance emits its own *selection* of short bits of visible (or ultra-violet) spectrum and shines brightly with but little expenditure of energy. Substances fluorescing give spectra not unlike the bright line and band spectra of gases.

For any particular substance the exciting light vibrations must be within certain limits of frequency. This can be shown by using any fluorescent solution, e.g. very weak quinine bisulphate (blue), decoction of horse-chestnut bark (blue), fluorescin and eosin [red ink in water] (green), paraffin oil (violet). Holding these in the sunshine the fluorescence does not spread far into the liquid, evidently the first layers have used up the effective rays. Lamp-light is less effective, yellow glass stops the glow, blue does not. This points to the blue, violet, and ultra-violet as usually the exciting radiation. (Early investigations of the ultra-violet spectrum were made by forming it on a quinined screen.) Yellow

and red rays, however, best call forth the fluorescence of various dyes, and the red glow of chlorophyll (green leaves in alcohol, filtered). 'Dark heat' (infra-red) never causes fluorescence or phosphorescence although, as heat, it may modify them. Hot luminous paint shines brighter but briefer ; many things will not glow at $-180°$, but eggshells then acquire the power.

The chemical luminescence of oxidizing phosphorus, the flash between lumps of sugar rubbed together in the dark, the fluorescence excited by cathode rays, X-rays, or radium, and the vital luminosity of fungi, glow-worms, *noctiluca* (of the sea), etc., can be only mentioned.

§ 427 : **Interference colours.**

In a soap bubble, in the floating shreds that mock early efforts at glass-blowing, in the thin film of oil on water, of oxide on hot polished metal, of tarnish on Roman glass, of air or water squeezed between clean plates of glass, of air in cracks in glass, mica, ice, and opal, there appears a play of **'Newton's colours'** which are due to **interference** (§ 292). The thin transparent film has two surfaces, each of which reflects back a small fraction of the incident light. That reflected from the back surface has had farther to go than that which came to the eye at once from the front surface. Suppose it happens to be just half the wave length of some particular spectrum colour behind ; interference smoothes out its waves and so destroys that colour, and the light that reaches the eye is of the complementary tint. Examined by the spectroscope there is a black gap in its spectrum.

As the film thickens the wave length destroyed must increase. The thinnest coloured film removes the violet and appears straw-yellow ; a thicker appears orange as the blue goes ; then purple as destruction reaches the green while violet has reappeared in the spectrum ; then blue as the long waves of red interfere. [The engineer will recognize the tempering tints of steel.]

Thickening still, more than one colour can be removed at once by the odd half-wave-length lag, e.g. $2\frac{1}{2}$ waves of red = in length $3\frac{1}{2}$ green = $4\frac{1}{2}$ blue. The complements amount to pale tints of pink and green, fading away altogether in thick films to a white, which, however, yields a spectrum showing many equidistant dark gaps. The monochromatic light of a soda flame continues to show yellow and black bands even in thick films, there being no other colours to overlap them.

The presence of coloured streaks in a film evidently means that it is wedge-shaped, the brightest tints near the thin end.

The tints change when looked at obliquely in the same way as by thickening the film, for rays penetrating across the film and back have farther to go when oblique.

§ 428 : **Diffraction colours.**

In § 296 it was pointed out how a surface, divided into equidistant patches, of which alternate ones reflect incident disturbance, will throw off trains of waves of different lengths in different directions. Such a surface therefore, with several thousand striæ or dots to the inch, will break up white light and throw off colours. This *diffraction-grating* structure exists in mother-of-pearl, in labradorite (of black Norway granite), on the finely striated microscopic scales producing the metallic glory of the butterfly or the diamond beetle (*Entimus imperialis*), etc., and possibly accounts in part for the lustre of the drake's head and the peacock's tail.

A swarm of minute particles, all the same size, scattered in the path of light, will break it up in a similar way, the angle at which a particular colour is thrown off now depending on the diameters of the particles. In this way thin cloud produces coloured '*coronæ*' round the moon, particles not quite in the line of sight diffract off waves which reach the eye, the farther out of line the longer the waves (Fig. 186), i.e. the red corona is outside the blue. Similar rings round all bright lights indicate a misty deposit in the humours of the eye and an indifferent condition of health. With particles of various sizes the colours blend into a colourless haze.

FIG. 186.

What happens if the spaces in Fig. 119, or the diffracting particles, are no larger than the smallest waves of visible light ? They can then throw off blue only, and that in a direction at right angles to the incident light. It can be calculated that still smaller particles would not be entirely without effect, but would scatter a light preponderatingly blue. Dilute HCl poured into hypo solution sets free sulphur in a blue opalescence turning white as the particles grow. Seen through the liquid a candle

appears orange-red, for its blue light has been thrown out sideways.

A more beautiful blue largely due to this cause is seen in water-softening tanks when a fine chalky precipitate is settling out, but the best example is the **blue sky.** On Ben Nevis 100 particles per c.c. is the lowest record of dust in air, so there is no lack of reflecting motes. Nearer the horizon, larger particles turn the blue into white, as happens above the dusty town, and completely in mist and cloud. The residuum from white light, after many miles of blue-scattering air, appears in the hues of sunset.

The action of very minute particles in scattering a trifle of the light falling on them is utilized in the **Ultra-microscope,** an instrument employed for examining the number and motions (§ 265) of colloid particles in liquids. The drop of liquid is lit from one side with a horizontal beam of intense light, and is examined from above through a powerful vertical microscope. With arc-light particles ·015 micron diameter, and with sunlight particles ·005 micron diameter, have been made, not visible indeed, even in a microscopic sense, but manifest as specks of light on a dark background. [1 micron = ·001 mm.]

§ 429 : **Rainbows** are caused by refraction and internal reflection in myriads of spherical water drops on which the sun (or the moon) is shining. The bright primary bow is returned after one reflection (*not* total) inside the drop. In Fig. 187 the paths of several equi-spaced parallel rays of sunlight meeting the upper half of the drop have been exactly traced. It will be seen that they emerge in very scattered directions, except three which are practically parallel, i.e. the drop throws back a much more concentrated reflection in this direction than in any other. This is a direction of minimum deviation, the obtuse angle turned back (between the dotted lines) being here less than for any of the other rays. Hence raindrops lower down in the sky will each reflect a little light to the eye along paths such as PQ, but drops near a certain greatest height reflect a lot and appear very bright.

As the light has suffered two refractions the minimum deviations for red and violet are of course different (180°—42·1, 180—40·2), i.e. the brightest red and violet come from drops nearly 2° apart, and a spectrum is drawn out in the sky.

Referring to Fig. 188, the line from the observer's head to its shadow is 180° away from the sun, and therefore all reflecting drops lying at 42·1° off this line appear red and all at 40·2° blue;

i.e. the rainbow forms part of a circle with its centre in the direction of the shadow of one's head and with outer angular radius 42·1° red, and inner 40·2° blue. Inside the bow is a light haze, outside a dark space. At the top, inside, are 'supernumerary bows' caused by diffraction, since the bow suddenly limits the broad reflected waves (cf. § 295). Each observer sees

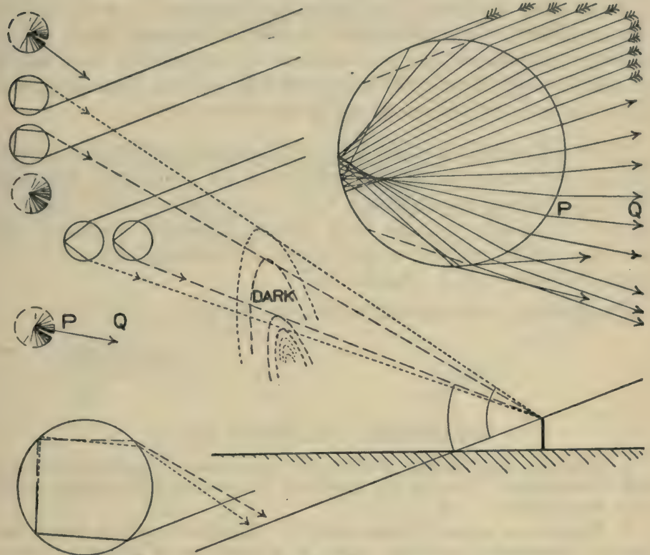


FIG. 189. FIG. 188. FIG. 187.

his own bow, built up from all the drops lying in a cone from his own eye to the distant margin of the rain. The lower the sun the more bow can be seen in the sky; the rest of the circle has a background of earth, and to get enough drops to show it one must stand in the midst of drenching spray from a fall or a hose. The diagram for a drop at the bottom of the circle would be Fig. 187 inverted.

Other light shining on the lower part of the drop is twice reflected inside and emerges to give the larger 'secondary bow,' weaker on account of the double loss on reflection, and with its colours inverted, red of angular radius 50·8° and blue 54·5°,

having been separated as shown in Fig. 189. It bounds the dark space, and outside it there is hazy reflection again.

The little figures interspersed with the chief drops are marked to show the directions over which diffuse reflection of the first and second varieties takes place. There is a gap of 9° in which no reflection occurs, for drops between the two bows the observer looks into this gap and sees only the dark cloud.

Bows from three and four internal reflections lie on the side of the sun, where the brightness of the sky obliterates them. That from five reflections lies just outside the secondary bow but is of course very faint and wide and has seldom been glimpsed. Many more have been detected in the laboratory.

§ 430 : The **Halo**, a white ring of 22° angular radius surrounding the sun or moon, is due to refraction at minimum deviation through floating ice-crystals. Colours are sometimes visible in it, the red inside and the blue outside (contrast coronæ), and one occasionally sees a solitary speck of cloud, 22° from the sun, brightly iridescent, a ' mock sun.'

For **Coronæ**, often miscalled haloes, see § 428.

EXAMPLES.—CHAPTER XLIV

1. Describe experiment illustrating connection between radiation and absorption. What is the evidence for iron and hydrogen in the solar atmosphere ? [M.]

2. What is an absorption spectrum ? Describe how you would test for the presence of carbonic oxide in a specimen of blood. Draw a figure to indicate the arrangement of the apparatus. [L]m.

3. Describe an optical test for blood in a red liquid. [L.]

4. Explain the red colour of (a) a coke fire, (b) a dark-room lamp, (c) a poppy petal, (d) a strontium flame, (e) copper, (f) sunset, (g) noble opal, (h) chlorophyll solution.

Chapter XLV

ABERRATIONS OF MIRRORS AND LENSES

Aberration dependent on Shape of Surfaces of Mirror or Lens

§ 431 : **Spherical Aberration.** Set a cup of tea in a direct light. On the surface appears the familiar bright cusped curve of light, called a **Caustic,** reflected from the semicircular margin of the

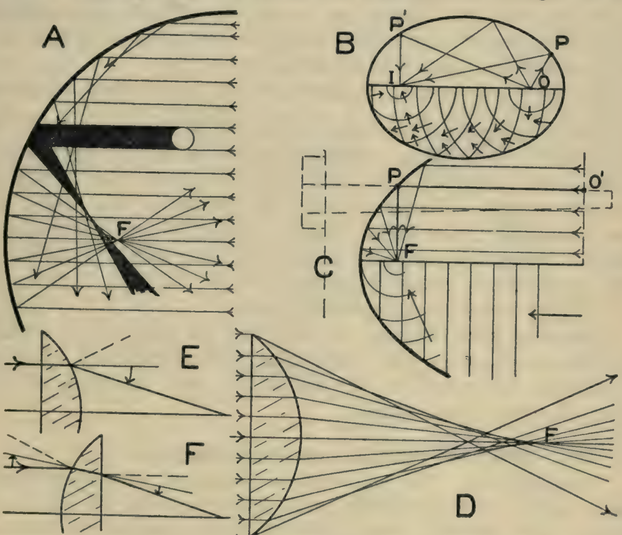


Fig. 190.

cup. Pass a vertical penholder across the lamplight ; its pointed shadow (Fig. 190, A) sweeps round, the tip 'rolling' on the caustic and in every position blotting out a little bit of it.

This little bit was evidently the focus of all the rays that fell on the now darkened patch of mirror. The rays are not all reflected to one hearth, the complete semicircular mirror has a complex succession of foci instead of the single point, though the brilliance of the cusp still tells us that a large proportion of the light is condensed thereabouts. This imperfection in focussing of circular and spherical surfaces is referred to as **Spherical Aberration.**

§ 432 : Mirrors can be made that are free from this. Knowing that the image is the point at which light arrives in the same time from the object, by whatever path, stick in pins at O and I, Fig. 190, B, loop a thread round them and pencil-point P and carry P round. OP+PI is constant, hence an **ellipse** reflects all rays emanating from one of its foci strictly to the other.

For O at infinity, fasten one end of the thread O′ on the edge of a T-square on which the pencil runs, hence the **parabola** O′P+PF=constant reflects to its geometrical focus (§§ 451, 453) all rays arriving parallel to its axis, Fig. C.

The lower halves of these figures show the reflection of waves.

§ 433 : Spherical aberration occurs with Lenses as well. A thick bull's-eye held in a bright light in smoky air produces a 'pulled-out' cone, Fig. D, quite like the middle part of the reflection caustic. The outer rays are refracted too much, the focal distance of the outer 'zones' of the lens is unduly short. Instead of a sharp cone, and image, there is a sort of bottle-neck, with a moderate image anywhere within half an inch or so, and always round it an unpleasant haze.

A reading glass forming on the wall an image of a distant lamp shows this quite well. Or looking through the lens, spherical aberration accounts for the distortion and smearing of the print all round the outside.

§ 434 : **Means of reducing spherical aberration.**

'Stopping down' the lens to an inch diameter with a perforated card, and so cutting off the outer rays, removes the haze and gives a more definite focus. But the objection to this way of reducing spherical aberration is at once apparent; it cuts off light. It is all that can be done, however, with spherical mirrors.

Fortunately, with lenses, the fact that the larger the angle, by far the greater the aberration, gives another means. Reduce the

amount of bending that occurs at any one refraction and share it equally among several refracting surfaces; n may be needed instead of one to produce the required total, but each involves perhaps only $1/n^2$ as much aberration, so that altogether there is only about $1/n$th, e.g. in Fig. F the same bending as in Fig. E is shared between two surfaces about equally; the haziness of the image is halved. And see how the same idea is carried farther in Figs. 204, 206, 208.

In complex lens combinations Focal Power is mainly a question of the extra thickness of glass on the axis; Chromatic Correction, (see further) of how this is allotted among different sorts of glass; and Spherical Correction of 'dishing' the lenses, without alteration of strength, from biconvex to meniscus, so as to alter the angles at which rays strike them.

§ 435: **Astigmatic beams.**

The caustic of Fig. 190 is in one plane, a thin sheet of light reflected at a semicircle. Rotate the whole diagram about its axis MF, Fig. 191,* the semicircle becomes a hemisphere on which is a darkened zone, the dark point P traces out a dark ring, but by symmetry, every ray still passes through the axis somewhere. Now think of a little beam of light which would fill a quarter-inch patch of the dark zone. Reflected it first all passes through a little length PP′ on the ring, making a minute bright line (standing out perpendicularly to the paper) which might be caught on a screen. Continuing it then all passes through the axis between Q and Q′, the screen held hereabouts would show a second bright line at right angles to the former (and in the plane of the paper).

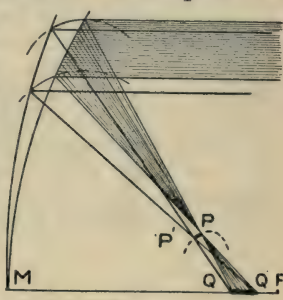


FIG. 191.

These are the primary and secondary **focal lines** of an **astigmatic beam,** they form 'adze edge' and oblique 'axe edge' of the volume of light between them, which no-

* The shaded part is an enlargement of the black of Fig. 190, A and lies in the plane of the paper. The rest of the figure is supposed to be produced by lifting up from the plane of the paper about the fixed hinge MF.

where passes through a focal point: hence the name *astigmatic—pointless.*

Images built up of little lines like these instead of tiny circles, looking as if 'smudged while wet,' irritatingly impossible to see distinctly, are characteristic of *Oblique Reflection or Refraction.* Turn your stopped-down reading glass askew and it draws out the image either horizontally or vertically according to its distance.

One might say that the focus of a large lens is built up of the little focal lines, pointing in all directions, of the oblique beams from all parts of it; a sort of asterisk, a spot with hazy margin, an image spoiled by 'spherical aberration.' Uncover the reading glass while askew, the total aberration is now vastly worse on one side and receives the apt name of Coma.

Chromatic Aberration, dependent on Nature of Refracting Substance

§ 436. The spreading apart or **Dispersion** of the spectral colours which accompanies the deviation of white light when refracted, and is of course the whole aim of the spectroscope prism, becomes a nuisance among lenses. For these, bending the blue more than the red, bring it to a shorter focus, and a good image becomes impossible. In Fig. 192 (vastly exaggerated) at B there would be a sharp blue image of the star with a red fringe round it and at R a red image with a blue fringe. This is called **Chromatic Aberration.** Fortunately Dollond discovered in 1757 how to correct this and make images and lenses **achromatic**, Fig. 193.

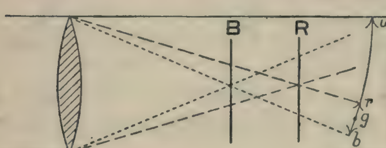


Fig. 192.

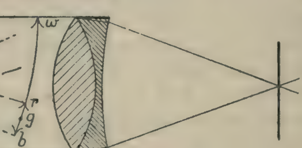


Fig. 193.

§ 437. *The difference of the deviations, or the* **Dispersion,** *blue—red, produced by a 'thin' prism, is* P *times the average deviation* where P is a small fraction characteristic of the material the prism is made of, and called its **dispersive power.** In Fig. 192 angle *br* is P times angle *wg* (not large).

The dispersive power is related to the observed refractive indices thus :—

In a prism of small angle A, deviation $D=(\mu-1)A$ [§ 300]
Hence $D_{blue}-D_{red}=(\mu_b-1)A-(\mu_r-1)A=(\mu_b-\mu_r)A$
$=P\times$ mean deviation $=P(\mu_{mean}-1)A$

$\therefore\ P=\dfrac{\mu_b-\mu_r}{\mu_{mean}-1}$ [accurate, independent of 'small angles']

In the following table are given the difference of index for the brilliant blue and red hydrogen lines (Fraunhofer F and C), the mean index for yellow (Na), and P, for some common substances. These colours are largely used in calculating lenses for visual purposes.

	$\mu_F-\mu_C$	μ_{Na}	P
Hard crown glass ..	·0086 ..	1·518 ..	·0166
Diamond	·0251 ..	2·418 ..	·0177
Water	·006 ..	1·334 ..	·0180
Dense flint glass ...	·0171 ..	1·620 ..	·0276
Denser ,, (Jena)	·0243 ..	1·717 ..	·0339
Carbon disulphide .	·034 ..	1·627 ..	·054

Fig. 194 shows the actual deviation and dispersion by a 30° prism of some of these substances, and Fig. 195 an Achromatic Prism of English crown (30°) and flint ($\frac{1}{2}\times30°$) deviating a ray $15°-\frac{1}{2}\times18°=6°$ and dispersing it $·25°-\frac{1}{2}\times·5°=0$.

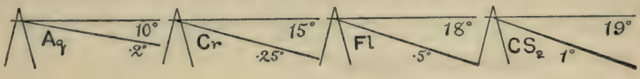


FIG. 194.

§ 438. **Achromatic Lens.**

A prism or lens which shall deviate all colours equally must be made of two parts, the second of which just neutralizes the colour-spreading effect of the first—produces equal and opposite *dispersion*—without destroying all the *deviation*.

Hence $(D_{blue}-D_{red})=P_1\times$ deviation of first lens, must be equal and opposite to $P_2\times$ deviation of second lens.

In lenses close together the deviations are produced at the same distance from the axis and measure their mean focal powers.

Therefore *for an achromatic lens the product* [*dispersive power* $\times$ *mean focal power*] *is equal and opposite in the two component lenses*.

$$P_1 \times \frac{1}{f_1} = -P_2 \times \frac{1}{f_2}$$

Hence one lens is concave, and whichever lens is focally weaker must be of the more dispersive glass.

The joint focal power is got by adding, recollecting one is negative, $1/F = 1/f_1 - 1/f_2$ (§ 406).

Example 1. What lens of glass $P' = \cdot 03$ will achromatize a 6-diopter convex of fluorite $P = \cdot 01$ and what is the combined power ?
$\cdot 01 \times 6 = -\cdot 03 \times x$. $\therefore x = -2$-diopter (concave) and together they form a lens $6-2=$ convex of 4 diopters.

NOTE.—Three glasses, and sometimes fluor-spar, are used in the more perfectly corrected, so-called 'apochromatic,' lenses.

§ 439. Separated pairs of lenses of the same glass can sometimes be made achromatic, as in an eye-piece, Fig. 209, lower part. The more bent ray strikes the eye-lens just so much nearer the centre that its greater refrangibility is neutralized by the smaller angle between the lens faces there. It leaves parallel to the red ray (longer dots) and therefore inseparable from it by an eye focussed for parallel light.

One can write down from § 406 expressions for the focal powers of the pair for red and for blue light. These must be the same; in their difference equated to zero substitute P and one finds that *the distance apart of the lenses must be half the sum of their focal lengths.*

§ 440 : **Direct-vision prisms.**

Conversely, prisms can be combined to give dispersion of

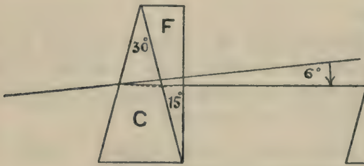


FIG. 195.

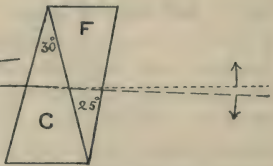


FIG. 196.

colours without deviation of the mean ray. Making the *mean deviations equal and opposite* :—

$$D=(\mu_1-1)A_1 \qquad \therefore \frac{A_1}{A_2}=-\frac{\mu_2-1}{\mu_1-1}$$
$$-D=(\mu_2-1)A_2$$

the thin prism angles are inversely as the (mean ref. index-1) and one is inverted.

In Fig. 196 the deviation is $(1{\cdot}5-1)\times 30°-(1{\cdot}6-1)\times 25°=0$
and the dispersion $0{\cdot}25°-({\cdot}5°\times 25/30)=-{\cdot}21°$
(exaggerated in the diagram). Since dispersion$=$P$\times$deviation
$\therefore$ net dispersion$=(P_1-P_2)\times$original deviation.

Thick prisms of this sort are very useful in pocket spectroscopes, Fig. 184.

EXAMPLES.—CHAPTER XLV

2. A white stone lies on the bottom of a pond. Its edges are generally observed to be fringed with colour, blue and orange. Explain this, and state which is the blue edge. [L.]

3. What is observed near the boundary of 'total reflection' of white light ?

4. Draw a section of a 30° prism and paths of red and blue rays through it, originally coinciding (a) for normal incidence, (b) for any other incidence. [L]m.

5. Explain 'dispersion' and describe some experiments to illustrate it. How would you compare the dispersive powers of two substances ? [L.]

6. Describe how to measure dispersive power for different colours. [D]m.

7. How may a double prism be constructed (a) to give deviation without dispersion, and (b) to give dispersion without deviation ? [L.]

Chapter XLVI

THE EYE

§ 441. An earthworm seems sensitive to light anywhere near its anterior end. In several animalculæ this sensitiveness is supposed to be concentrated in an ' eye-spot.' In the ' compound eyes ' of insects better provision is made for localizing light and shade; the central nervous tissue sends a fibre into each of surrounding hundreds of long narrow tubes, like so many gun-barrels, radiating in most directions of the sphere. Along each comes the light gathered solely from the direction in which it is aimed, to help build a patchwork or mosaic picture of the world without.

A mosaic could be obtained by packing nerve-endings like a velvet pile on the back of a hollow chamber in the front of which was a small hole—a pinhole camera. To gain more illumination the pinhole is enlarged and covered with a lens, and there results the eye of vertebrates. The nervous ' pile ' of the retina is so fine that the ' mosaic grain ' vanishes.

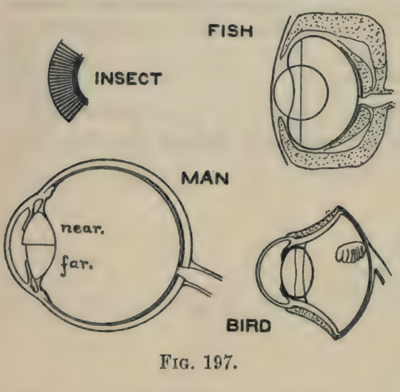


Fig. 197.

In a fish's eye, Fig. 197, a dense spherical lens has to do all the refraction. In land animals the clear hard spherically bulged front of the ' Cornea ' does most. The Lens separates the anterior ' aqueous ' and posterior ' vitreous ' ' humours,' both of them jellies which are, optically speaking, water. It is less curved, and is variable in curvature and position to ensure the clear focussing

on the retina of light from different distances and so to 'accommodate' vision.

§ 442. Hence a first approximation to the action of the human eye is obtainable by regarding it as a case of **Refraction at a single spherical surface,** Fig. 198.

A number of rays are already travelling towards a focus O′. In their path is placed the spherical surface AS, radius SC, of a medium of refractive index, μ. The rays now focus at I, real image in the medium of now 'virtual object' O′ in air. Selecting two rays, one passes straight through C, the other EA is refracted at A (radius-normal CAN) so that sine NAE$=\mu$ sine CAI. The angles being small are practically equal to their sines; dot in HA parallel to SC; let SO′$=b$, SI$=a$; then

$$\angle\text{e NAE}=\text{NAH}-\text{EAH}=\mu\text{CAI}=\mu(\text{ACS}-\text{AIS})$$

$$\text{or slope of NC}-\text{of EO}=\mu\text{ (slope of NC}-\text{of AI)}$$

$$\frac{1}{r}-\frac{1}{b}=\mu\left(\frac{1}{r}-\frac{1}{a}\right)$$

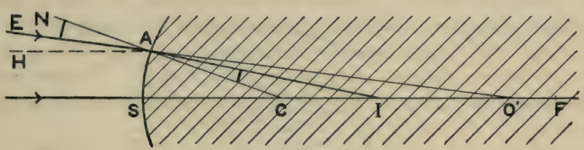


FIG. 198.

If the rays were parallel in air $1/b=0$ and a becomes principal f in medium$=\dfrac{\mu}{\mu-1}r$ while if light were parallel in the medium $1/a=0$ and b becomes principal f in air $=-\dfrac{1}{\mu-1}r$. *Thus the focal distances in the two media are different, and are proportional to their refractive indices* (cf. § 409). This is perfectly general, and the relation found above holds, with the usual sign convention, in all cases.

To this approximation the Eye may be considered as a bulk of water, $\mu=4/3$, with a refracting cornea of radius ·5 cm. In the case calculated such an eye is receiving light that has come through a convex spectacle lens.

Such an eye would of course lose all refracting power in water.

The imperfect sight that the human eye retains under water is due almost entirely to the denser 'crystalline lens.'

§ 443. **Accommodation of vision.**

At rest the normal eye is adapted for parallel light from a distance. For near objects the 'ciliary muscle' encircling the lens pulls on it all round, causing its rather flat front to bulge and thus making it stronger. See Fig. 197, which is drawn to natural size and correct curvatures.

In Birds the lens is forced forward by the hydrostatic pressure of the vitreous humour when encircling muscles squeeze inwards the overlapping bony plates which surround the eye. There is also a highly vascular organ, the *pecten*, into which blood can be forced so as to increase the total contents of the eye and again to drive the lens forward.

Distinct vision is possible only in the interval between limits of distance D and D′, called the **near and far points of the eye.** Both can be found with the **optometer**, which is just a convex spectacle lens with an object sliding beyond it on a graduated bar. The nearest and farthest distances d and d' are noted at which the object can be clearly seen by the eye close behind the lens, from their reciprocals is subtracted the focal power of the lens, and the remainders, re-inverted, are D and D′. The **accommodating power** of the eye is $1/D - 1/D'$, it is the angle marked *a.p.* in Fig. 199.

As an example of normal sight (at age 19) G.L.S. had D ·08 metre, D′—7·7 m. (for meaning of — see § 385) and therefore accommodating power 12 diopters.

§ 444. **Judgment of distance. Stereoscopic vision.**

Shutting one eye, the effort of focussing the other on near objects enables some estimate of their comparative distances, and practice, such as the one-eyed man gets, develops this faculty. But the possession of two eyes gives us solid-seeing—stereoscopic —vision and a far better means of judging distances.

Always the distance is that at which two lines cross. For a single eye these are the extreme right and left rays that enter the pupil, only $\frac{1}{8}$ in. across. Rays to two eyes separate twenty times as far; the turning inwards of the eyes increases very perceptibly as the object on which both are bent comes closer, and there is no difficulty in detecting difference of distance.

§ 445. Although the whole field of vision is large only a *very* small portion is perfectly sharp. This is to be expected from the known optical imperfections of the lenses (see § 433); the one well-formed spot of image falls on the minute *fovea centralis*, where alone the retinal filaments all terminate in the thicker 'cones' and are quite unobscured. The rest of the retina consists mainly of thin-ended 'rods' and is actually overspread by a layer of bloodvessels (seen against a dim surface when sunlight enters the eye very obliquely; the dancing spots seen on the sky when lying on one's back on the grass are reputed to be blood corpuscles).

This is a great advantage, for it compels attention to one thing at a time. And binocular vision enhances the advantage. Looking at a jumble of things with one eye you will find its attention wanders from one attraction to another much more than does that of both together. Two eyes looking at the world from different points of view form slightly different pictures, you make these coincide in the point looked at, but they fit together nowhere else, everything else is blurred and in fact doubled—hold up two fingers in line, look at either and the other appears on both sides of it—but vision off the axis is so imperfect that this doubling usually passes unnoticed.

§ 446. **Chromatic aberration** of the eye accounts for the 'standing out' of colours in front of a pattern showing violent contrasts. The distant purple light of a shunting engine appears to an eye slightly out of focus as a blue dot with a red ring round it. Window bars, seen only through the edge of the pupil when a book is held close so as to obstruct most of the eye, are margined with blue and orange.

§ 447. **Spectacles.** The normal eye can accommodate itself to focus clearly light divergent from a point 8 to 10 in. away, or of any less divergence down to 0 (parallel light), or even very slightly convergent, and its accommodating power is measured by the extreme angle *a.p.*, Fig. 199.

But if its refraction is always abnormally strong, then unless the rays are divergent they will be bent in to an image before reaching the retina, i.e. only near objects can be clearly seen. This is **short sight** or **myopia.** The trouble of the short-sighted is that he cannot see distant objects: provided with a *lens which makes parallel light diverge* so as to be just within his outer

visual limit he will be content, i.e. a **concave lens** is added to his eye to give a normal combined refraction.

If the refraction is abnormally weak, rays divergent from near points cannot be brought to an image by the time they reach the retina. A **convex lens** must be added to render such rays more nearly parallel and enable the **long-sighted** or **hypermetropic** eye to see clearly at the near distance. For the trouble now is that nothing is clear till 2 or 3 ft. from the eye, a distance at which most print is too small to read.

A spectacle lens is an optical instrument: through it *one sees, not the object but the virtual image of it*, and the lens must be such as to form this virtual image at a distance within the wearer's range of accommodation. You cannot 'make the eye see things,' you must make images where the eye *can* see them. Glasses are so familiar that one is apt to forget or even disbelieve this statement; but borrow a pair and try to walk downstairs, looking at your feet.

§ 448. In **Spectacle Calculations** the problem is

V. Given distances of image and object from lens, find f.

a is where the patient can see; given b, find f. Both a and b are $-$, the light leaves $+$ to enter the eye. Distances are reckoned from the usual position of a spectacle lens, an inch from the eye.

Example 1. *A short-sighted person can see only between* 4 *in. and* 2 *in.*

Give him glasses to make his 'far point' very distant, so that he can safely walk about. b very great, $1/b=0$, while $1/a=-1/4$

$$0+1/f=-1/4$$

i.e. *a concave lens of* 4 *in. focus; or of* $-40/4=-10$ *Diopters.*

This is a foregone conclusion; *parallel* light has been made to diverge from $a=f$ by definition.

His near point through glasses will be b when $a=-2$

$$1/b-1/4=-1/2$$

i.e. 4 in. from the eye. [That this is still near does not matter unless perhaps in treating squint, when f might be computed for $b=-8$ when $a=-2$.]

Ex. 2. *A long-sighted person cannot see nearer than* 1 *metre.*

He wishes of course to see objects at the normal 25 cm. Putting $a=-1$, $b=-{\cdot}25$

$$-1/{\cdot}25+\frac{1}{f}=-1/1$$

$\therefore$ $1/f=+3$ *Diopters; a convex lens* $\frac{1}{3}$ *m. focus.*

By construction, Fig. 199.

Ex. 1. Easy.

Ex. 2. Draw lens and axial rays 1, 2, mark in object 3 and image at visible distance 4. Draw 5 parallel to 2, then 6 through A cuts 2 at f.

Elderly people often lose nearly all power of accommodation—**presbyopia**—and frequently cannot even converge parallel light: rays must be already coming to a focus at some point behind the eye. They require two pairs of spectacles.

Ex. 3. *An elderly person whose single distance is* 2 *metres* $(=+a)$ *behind eye requires glasses for vision at* 4 *m.* (*walking*) *and also for reading at* $\frac{1}{4}$ *m.*

$+1/2=1/f+(-1/4)$. $\therefore 1/f=\cdot 75$ Diopters for walking.
$+1/2=1/f+(-1/\frac{1}{4})$. ,, 5 Diopters for reading.

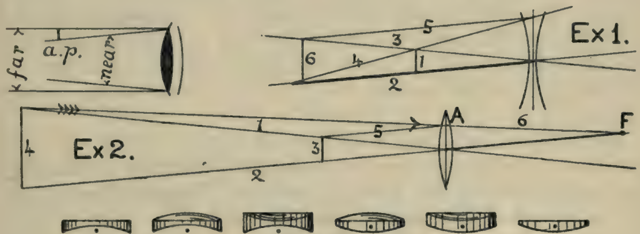


FIG. 199.

§ 449. An **astigmatic eye** contains a refracting surface which is elliptical, curved differently in the vertical and horizontal planes. Any image it produces is distorted, like your face in a teaspoon. A pattern of radiating lines cannot be seen clearly all at once, when some are distinct those at right angles are blurred, and require refocussing. Print becomes illegible from the blurring of the horizontal strokes. Astigmatism is unhappily common among students; the ellipticity if slight is practically corrected by appropriate stress in healthy eye-muscles, but if more serious, eye-strain and headache drive one to the oculist, who prescribes for each eye separately a compensating lens. This is plane or spherical on one surface and cylindrical on the other; along the straight axis (dots in the row of lenses in Fig. 199) of the cylinder there is only the sphere's curvature, at right angles there is sphere's $\pm$cylinder's. Thus one gets the effect of an ellipse (an ungrindable surface) and distorts the light ready for the afflicted eye.

§ 450. **How the eye sees colour.**

It has been tacitly assumed throughout that the eye blends all the spectrum colours into a tint. If all are present in normal proportion it is unconscious of tint—white light; when some are weak or absent the tint perceived is **complementary** to that which the abstracted parts would add up to.

An overdose of one colour forced on the eye fatigues and almost blinds it awhile to that colour: feeble white light will for several seconds afterwards appear defective in that colour and therefore tinged with its complementary. Hence the old advertising device of a red disc at which one stared for half a minute in a strong light and then looked up and saw a blue-green disc on the ceiling. Hence also a reason for colour harmony and discord; the eye, fatigued by one colour, makes its perpetual little excursions over the other; if it receives the complementary only, the fatigue recovers; if the new tint still contains some of the old, plus a new strong colour, there is double fatigue and discord.

Now this red and blue-green which together make up white light (i.e. the whole spectrum) can be nearly matched by two small selected bits of it, say Lithium red and Thallium green lines. The yellowish tint of these combined needs only a little Indium blue line to make it practically colourless. In fact there are three spectrum colours, roughly recognizable as vermilion, emerald green, and royal blue, by combining which in proper brightness any colour whatever can be closely imitated. Maxwell did this by putting adjustable sectors of paper of these colours on a disc, and spinning it; also in better ways with pure spectrum colours.

From this arose the **three-colour theory of vision,** that the eye has three colour-sensations only, red, green, and blue, and that its notion of any colour depends on the relative proportion in which these three are excited. This seems at first sight to suggest further analysis of a pure spectrum colour of perfectly definite frequency, but the case is really analogous with the setting in motion of two strings on a piano by a fork intermediate in pitch, § 324. The theory is, however, objected to by physiologists, and need not be insisted on; sight and colour blindness are in the province of the sister science. Sufficient that it has brought forth colour photography and three-colour-process printing.

Note.—Complementary coloured lights added together produce

white: as the first green flush of the sprouting wheat spreads over the red fields of Devon both colours are blended and lost in a grey paleness. But complementary paints mixed together look black. Compare under a lens the white of a Lumière 'autochrome' transparency and the black of a three-colour process print. For evidently adding lights together is the very reverse of subtracting red, green, etc., in succession from white light by putting absorbing films in its path.

Common blues and yellows transmit a little green, blue and yellow lamps together give a *white* illumination faintly tinted green, blue and yellow paints give a green mixed with *black*, i.e. pure, but not nearly so brilliant as the single pigment: try it.

The Spectrum Top is a 4-in. card disc, divided into a black and a white semicircle. From the black half projects a curved arc, concentric with the disc, of black, an inch long and $\frac{1}{12}$ inch thick. Spun steadily to the right in a strong light the arc appears as a red ring; spun to the left it appears blue. It seems that the light of the white card suddenly appearing on both sides of the dark line, as the black semicircle goes away, tends to spread by 'irradiation' (dazzling) over the narrow dark space, but the red sensation spreads fastest, and the dark line appears red. And conversely the red sensation disappears fastest and the dark line invading a white space appears blue.

Similarly, when walking past a high open-paled fence, the skylight flickering obliquely into the eye from between the palings often appears tinged with red.

EXAMPLES.—CHAPTER XLVI

4. Describe the eye as an optical instrument. What forms of lenses are required for long and short sight and what disadvantages are experienced in their use? [St. A]m.

5. What is meant by long sight, short sight, and astigmatism? A short-sighted eye cannot see clearly anything at a distance greater than 6 in.; what lens should be used to enable the eye to see distant objects? [L.]

6. A short-sighted person cannot see anything distinctly at a greater distance than 25 cm. What lens will enable him to see things distinctly at a great distance? [L]m.

7. What lens would enable an eye of D_f 8 in. to see objects at 48 in.? What lens would enable an eye of D_n 3 ft. to read at 1 ft.? [L]m.

8. A short-sighted person can see distinctly objects at distances ranging from 10 to 20 cm. from the eye. Give the focal power or dioptric strength of suitable spectacles, and calculate the new near and far points. [L.]

9. Walking along a road paved with granite setts and looking steadfastly at a point about 6 ft. before your feet, a curious rippling movement appears both before and behind the point looked at. How do you account for this?

10. How would you test an eye for astigmatism and how decide what kind of spectacle lenses to recommend? [L]m.

11. Using a lens of focal length 7·3 cm. the limits of distinct vision were found at 7·6 and 4 cm. Calculate the limits without the lens and the accommodating power.

12. Prove that an air bubble in a glass ball or a goldfish in a globe will appear nearer than it really is, at its true distance, or farther off, according as it is nearer the surface than the centre, at the centre, or beyond it.

CHAPTER XLVII

OPTICAL INSTRUMENTS

§ 451. **Apparatus for projecting an intense beam of light.** The **Catoptric** (cata=down, back) **lanterns** of a lightship are lamps in the foci of **parabolic** mirrors. This shape reflects all rays from its geometrical focus accurately in parallel lines. The flame being, however, more than a mathematical point, the whole beam gradually widens out as described in the next section.

In **searchlights** the brilliant 'crater' of the large positive carbon faces the parabolic reflector, Fig. 200. Direct light is now prevented from scattering out by a broad collar round the carbon.

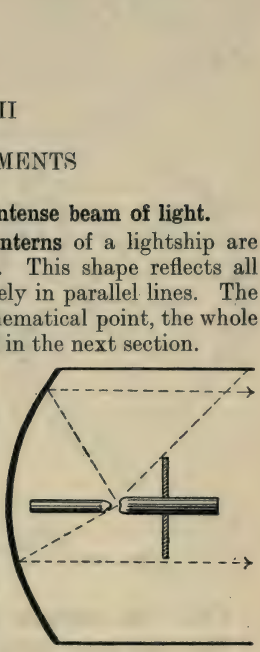


FIG. 200.

In **Dioptric** (dia, through) **lanterns** (simplest type the common bull's eye) the lamp is at the principal focal distance from a lens, which makes the direct rays from any particular point of it into an approximately parallel beam. If there is a reflector it must now be differently arranged, for if it sent parallel light as before, the lens would focus and then scatter it.

Fig. 201 is a section through a very perfect 'catadioptric' lighthouse lantern designed to utilize all the light possible. A plano-convex lens of diameter *ab* and short focus FL would be very thick. Thickness does nothing useful and merely absorbs some of the light. The lens is therefore built up of concentric rings of glass, each of which has had its superfluous thickness removed. [These 'echelon' lenses, cast in one piece, are common on ships' lights: sometimes the flat face is stepped down instead of the convex]. The angle of the outer rings is modified from the 'spherical' to reduce aberration.

Encircling the lens are rings of prismatic section (6 triangular

and 7 trapezoidal), most carefully angled and placed so as to catch and *totally reflect* the light into one direction, without getting in one another's way.

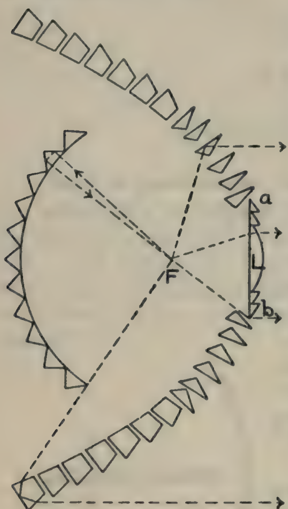


FIG. 201.

Revolving 'flashing' lighthouse lanterns have 2, 4, or more panels each built up in this way, and emit this number of brilliant radii. Our lantern is to send all its light through one panel, accordingly the back is filled in with a portion of a spherical mirror reflecting all the radiation from that side of the flame *back into it*. Mostly, this passes through it and travels out with the direct rays; any part which is absorbed in the flame makes it hotter and therefore brighter; either way it is utilized.

Since a silvered surface is apt to tarnish the mirror is built up of glass rings of right-angled prism section which totally reflect the light. [On a small scale the 'Holophane' reflectors for glow lamps act in the same way.]

§ 452. **The character of the beam produced.**

Since each point of the flame is approximately a principal focal point of the lens, or mirror, light from these devices passes out in *many* parallel beams slightly inclined to one another. Or really, in a solid cone of angle FLF′, Fig. 202, but truncated at the broad face of the lens or mirror. This spreading causes it to approximately follow the inverse square law (§ 355) when examined at considerable distances away.

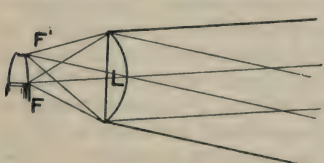


FIG. 202.

The ideal parallel stream of light that retains its brightness quite undimmed by distance cannot therefore be artificially produced. The best one can do, in the absence of the mathematical point source, is to use as source a small pinhole in a plate

(cf. spectroscope collimator), intensely lit from behind, say by focussing an image of the arc-carbon on it. Or on the large scale at sea, to use a flame as concentrated and brilliant, and lenses and mirrors as far from it, and hence as large, as practicable, and so to reduce the angle FLF′ of the truncated cone.

Suppose a 1000 c.p. lamp was illuminating the whole inner surface, $12\frac{1}{2}$ million square metres, of a kilometre radius sphere. Concentrating this outflow of light into a cone—a ' jet '—of angle 3° means concentrating it into a 50 m. diameter patch on this sphere, or 12,500,000 : 2000 sq. m. = 6250 times = 6,250,000 effective c.p., and a smaller angle would evidently produce a yet more brilliant and penetrating flash.

§ 453. **Illumination of objects under the microscope.** The problem here is the converse of the foregoing, viz. to convert almost parallel light into a strongly convergent cone, at the apex of which is placed the small object.

In the **parabolic illuminator** the light is totally reflected inside a glass paraboloid and concentrated towards its hemispherical hollow middle, whence it falls obliquely all round on the object, Fig. 203. A black patch stops axial rays and the object appears luminous on a dark ground.

FIG. 203.

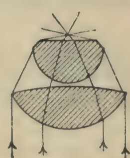


FIG. 204.

The substage condenser, Fig. 204, consists of two or more fat lenses. Their curves are calculated to equalize the refractions at successive faces and so diminish spherical aberration, which would grievously dull the point of the cone of light.

The illuminating advantage of this wide cone over the ' parallel ' lamplight can be compared to that of daylight over moonlight. It pours in light on many sides instead of in one narrow direction only. The lenses produce a real image of the lamp flame in focus on the microscopic object. Its intrinsic brilliance cannot be greater than that of the flame. A quite small lens could have produced an image the same size and just as bright. But the widely lit image is visible in so many more directions, anywhere perhaps within 60° or 70° of the axis, and the more of this spreading light the microscope can take up, the more like a full daylight view and the less like a moonlight silhouette will the field of view appear (see also § 476).

§ 454. **The camera lens.**

A convex lens projects a real inverted image on a plate. In Fig. 165, etc., the image of a straight line has been drawn as a straight line parallel to it, but if the diagram be constructed carefully for 3 or 4 distinct points of the object it will be found that the image is actually curved ; the image of a flat sheet would be in focus on the inside of a saucer.

This difficulty was overcome in the **Landscape Lens**, a meniscus, hollow towards the view and with a limiting circular hole or 'stop' about $\cdot 1f$ in front of it, Fig. 205. This gives a flat image, in focus all over a good-sized plate, but distorted so that a square has bulging sides. Turned the other way round it makes a square 'cushion shaped.' Hence the symmetrical **'Rapid Rectilinear'** in which a pair of meniscus lenses face each other (front lens dotted, Fig. 205), with a stop midway between, and give a flat undistorted image.

The **Stop** is a very essential part of the complete lens, it removes the haze in which 'spherical aberration' would otherwise envelop the picture. Diminishing its size also reduces the obvious outstanding difficulty of focussing on a flat plate images of objects at all sorts of distances away. For suppose the cone of light from the lens is not coming to a focus till F, Fig. 205. Evidently a smaller stop makes the cone narrower, and the circular patch in which it strikes the plate (the cross line) smaller and more like a true focus.

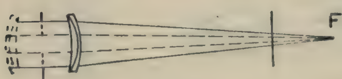


FIG. 205.

Unfortunately, cutting down the size of the window in this way necessitates a lengthened exposure. The diameter of the aperture is stated as a fraction of the focal distance (which is usually nearly enough distance of plate), e.g. $f/8$, $f/11$, etc. The light it transmits from a given outside brightness is of course proportional to its area, or to $(f/11)^2$, etc. Hence the illumination on the plate, c.p.$\div d^2$, § 355, is $(f/11)^2 \div f^2$, etc., and the exposure to catch a given quantity of light is inversely proportional to this, i.e. directly proportional to $(11)^2$, etc.

Modern 'anastigmat' lenses are elaborations of the types described and can work at wider apertures and wider angles because their astigmatic and spherical errors have been greatly reduced.

The **'portrait lens'** type, shown in Fig. 206 (right), consists of a

convex and about $f/3$ behind it a weaker correcting lens, and at wide aperture gives exquisite definition over a rather limited central area.

All photographic lenses are of course carefully achromatized.

That ancient seaside joy, the Camera Obscura, in which a 45° mirror or reflecting prism behind the lens throws the image down on to a horizontal screen, has now, like the *Cetacea*, taken to the sea entirely, and forms the head of the periscope of the submarine.

§ 455. **The Magic Lantern,** Fig. 206.

The photographer is often in a difficulty because the dark parts of the object do not send enough light to affect his plate. Conversely, when he comes to project the finished picture on the screen, it is invisible wherever the lantern slide is too feebly lit. The condition to be fulfilled in lighting the slide can be put thus: Regarding the projection lens or 'objective' as an eye, which is to form a bright image (enlarged) on its retina, the white sheet, it must see all parts of the slide brilliantly lit from behind. In 'daylight enlarging' a broad white painted board reflects the skylight, but at night a flame three inches square is impracticable, a diffusing piece of opal glass in front of the lamp is wasteful of light, and what is actually employed is a 'condensing' lens system arranged so as to everywhere bend and send the light received from the lamp into the 'eye' of the projection lens. The face of this **condenser** will then appear to it as a uniform blaze of light; its image of this constitutes the well-known circle of light on the screen.

The strong lens-system of the Condenser most commonly consists of a pair of large cheap plano-convex lenses back to back,

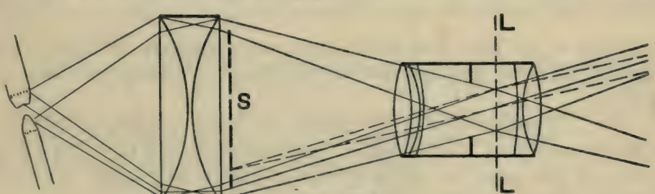


FIG. 206.

this arrangement reducing the spherical aberration. The breadth of the source of light smoothes out their imperfections. They focus the lamplight into a rough image of the lamp somewhere

inside the Projection Lens. This must be a good achromatic lens of wide aperture, that shown is a 'portrait' lens. For if too small, the image of the lamp will more than suffice to fill it, and any overlapping light is of course merely stopped. Such lenses of large aperture being costly, the source of light should be small, a condition admirably fulfilled by limelight or electric arc.

In Fig. 206 the rays are drawn as they would be refracted by a thin lens at LL, about equivalent to the portrait combination. The solid lines are converging towards points on the screen. The dotted lines and the middle solid line form the standard construction for a small portion of the slide S.

In practice one lights up, inserts a slide, and moves the lens till the scrap of picture visible is in focus ; then one removes the slide and moves the lamp to and fro and sideways till the whole circle is bright. A little smoke or dust in the air will then show the path of the light as described.

§ 456. **Magnifying lens or 'simple microscope,' and its Magnifying Power.**

The use of a convex lens as a simple magnifier is figured in Fig. 207 (see also Fig. 168). The object is within the principal focal

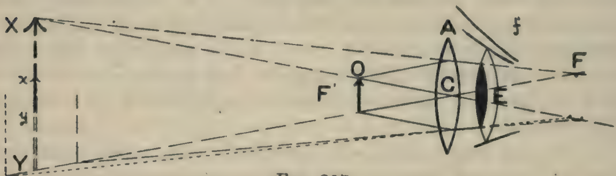


FIG. 207.

distance of the lens, and the virtual image appears the right way up and at some distance within the observer's range of distinct vision.

The 'Magnification,' § 391 [diam. of image÷diam. of object] may be anything from 1 when object touches lens (reading glass lying on the page) to infinity when object is at f from lens and image is seen by parallel beams.

But the image is **virtual,** therefore the closest point from which it can be inspected is the other side of the lens. One cannot always go close up to it as to the real image on a lantern sheet. To an eye placed close behind the lens it appears to vary very

little in size wherever it is. For it is not clearly visible until several inches away, its ends always lie on the 'scissors' rays CX, CY so that its angular diameter XEY never differs much from XCY.

Now one naturally brings an object to the *nearest* distance of distinct vision, D_n of § 443, before calling for a magnifying glass, so the **Magnifying Power (m.p.)** *is defined as the number of times the apparent (angular) diameter* [XEY] *of the image contains the angle* [xEy] *that the object would subtend when placed at the nearest distance of distinct vision.*

[Perhaps the simplest way of stating the action of any magnifier is this: it enables the user to bring an object nearer, so as to subtend a greater angle at the eye, and yet see it clearly. A pinhole in a card held to the eye can do this, for it is a 'small stop,' § 454, and reduces the size of the 'circles of confusion' of an out-of-focus image. A well-known advertising firm occasionally provides readers of Bradshaw with this magnifier, but it demands a good light.]

This is nearly enough the same as (*linear size of image at* D_n $\div$*size of object*), but this as a definition is objectionable, for it arbitrarily and quite unnecessarily ties the image down to a fixed distance, a distance moreover which the observer mostly alters by slightly drawing back the lens, for it involves maximum strain on the eye.

In a diagram of course one has to dot in virtual images possessing real size, but actually they are merely apparitions possessing only angular diameter and quasi-distance. *This should be borne in mind whenever they are dealt with.*

To Find the Magnifying Power then, look with one eye through the lens at a scale, and with the other eye at *another* similar scale 10 in. away.*

With a little adjustment the large virtual image—the magnified scale—can be seen overlying the second scale. Both being in clear focus they must be at the same distance and therefore the number of divisions covered by one magnified division = m.p.

Ten inches (or 25 cm.) is the standard D_n for which all m.p.'s are specified, but a short-sighted eye gains relatively less advantage than this, not on account of a difference of distance of the image, but because it can, *unaided by lenses*, see the object nearer, under a larger angle, than a normal eye.

* Or as under microscope, § 462.

The **relation of magnifying power to focal length** can be deduced thus :—In Fig. 207, taking XC as base-line,

Right-hand down-slope of AF=slope of OA$+1/f$

$$\text{i.e. } (-1/a)=(-1/b)+1/f$$

changing sides and multiplying by a throughout,

$$a/b=a/a+a/f$$

$$\text{i.e. (by § 391) } m=1+a/f$$

and Magnification m becomes Magnifying Power when $a=D_n$

$$\therefore \underline{\text{M.P.}=1+D_n/f}$$

e.g. a 1-in. focus lens has m.p. 11 ; but to a person with $D_n=3$ in. it is only 4 times as effective as his shorter sight.

If the lens is held away from the eye the least value of a is (D_n—distance lens to eye) and the m.p. is reduced. But by slightly shifting the lens and refocussing the eye a can be increased till vastly greater than the separation of lens and eye, XE′Y again=XCY and the normal m.p. is regained : try this with a pocket lens.

§ 457. The large angle at which rays from an object O strike the lens, Fig. 208 (i) results in much spherical aberration, apparent as the blurring all round the very limited field of view of a strong lens. The back of the lens has refracted the ray AE much less. Fig. 208 (ii) shows that an equally strong plano-convex lens,

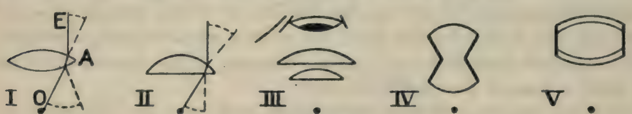


FIG. 208.

flat face to object, shares the refraction more equally between the two faces, giving much less aberration and therefore a wider clear field.

Magnifiers of successive plano lenses like Fig. 208 (iii) up to m.p. 200 were much in use before compound microscopes had been successfully achromatized. A powerful pocket lens is the ‘ Coddington,’ a little sphere of glass cut away to a dice-box shape as in Fig. iv, to stop off marginal rays. Its field is flat, but its working distance is small. It is a development of the spherical drop of melted glass with which the earliest studies of ‘ animalculæ ’ were made. The best pocket lenses of the present

day are the ' aplanatic ' cemented triplets, such as Fig. v, in which four surfaces share the refraction.

It is remarkable and fortunate that every simple magnifier, used correctly, is achromatic. The greater bending of the blue rays (short dots) causes the blue image to appear farther off, as in Fig. 207, lower half, but since all the images lie between the ' scissors ' rays they appear to an eye near C to cover one another very exactly, their sum total being a colourless image. Contrast Fig. 192.

§ 458. **Eye-pieces** for telescopes and microscopes are interesting varieties of magnifying glass, consisting of two lenses spaced apart. The ' object ' they are used to magnify is a ' real image ' formed by the object-glass or -mirror of the instrument. They replace the single convex eye-lens of elementary theory, which gives only a small field of view very badly blurred and coloured all round. According to § 439 two lenses of the same sort of glass separated by half the sum of their focal lengths form an *achromatic* combination. And by sharing the deviation equally among their four surfaces *spherical aberration is minimized*, § 434. These ideas are put into practice, as far as other considerations permit, in the following :—

The Ramsden or ' positive ' eye-piece, Fig. 209, has two plano-convex lenses flat sides outwards, of equal f, and d rather less than $\frac{1}{2}(f+f)$, actually $\frac{2}{3}f$ apart. The principal focal planes lie $\frac{1}{4}d$

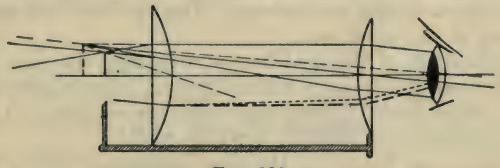


FIG. 209.

outside either end: in one lie the image and the cross-wires, micrometer, etc., for which this eye-piece is usually employed. Parallel light then leaves for the eye at the other end ; for abnormal vision the whole eye-piece is pulled out or pushed in a little.

[In the figure the dotted virtual image ' formed by ' the ' field-lens ' was found by the standard construction of which the dotted lines form part. This image lying at f from the ' eye-lens,' the long line was drawn through centre of latter, all rays will emerge

parallel to this. Then working *backwards* such rays were drawn from margins of pupil, eye-lens converged them to dotted image, then field-lens to real image.]

The Huyghens or so-called 'negative' eye-piece, Fig. 210, has two plano-convex lenses, convex sides towards incident light. The first or 'field-lens' has from 2 to 3 times (for minimum spherical aberration) the focal length of the little 'eye-lens' and the distance between them $=\frac{1}{2}$ sum of focal lengths (for achromatism).

The field-lens receives rays from the object-glass *before* they have come to a focus and brings them more quickly to an image

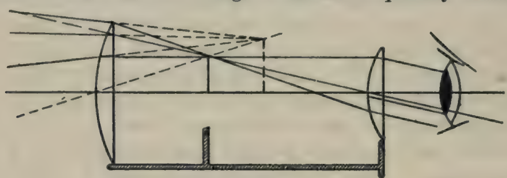


FIG. 210.

in the plane of the field-of-view diaphragm. This is inside the eye-piece and lies in the focal plane of the eye-lens, it is responsible for the familiar black circle. On it can be placed pointers, micrometer scales, etc., but since the field-lens has distorted the image somewhat at the edges this eye-piece has to give place to the Ramsden for accurate micrometry. The eye-lens then magnifies everything in the usual way.

The advantage of the field-lens in enlarging the field of view is evident from the figure ; without it rays such as shown would continue their dotted paths and miss the eye-lens altogether, only a small middle of the field would be visible.

[The figure was drawn by the same scheme as before, but one extreme ray at present missing the eye has been shown. Below is a half-section of the brass casing, showing the diaphragm.]

§ 459. **Telescopes.**

Telescopes are contrivances to improve our view of distant objects by increasing their apparent angular diameter, i.e. by forming an image of them which subtends a greater angle at the eye than the objects did.

We seldom use a telescope to get a better view of small objects than we could get at arm's length, if accessible. We are content

with a picture of the distant scene on a moderate scale, showing the detail that might be seen in a very sharp photograph of it under a magnifying glass.

To secure the desired sharpness the photographer would make use of a 'focussing magnifier,' a pocket lens which he fixes up in focus on the grain of the ground-glass screen, then adjusting the camera till the picture also comes into sharpest focus. The whole constitutes a telescope, for in most forms of telescope one first forms a small real image of the distant object, and then—but without troubling to catch it on a screen or to develop and fix it*—examines it in the air with a magnifying glass.

The image can be formed either by a convex lens, the **object-glass**; or by a concave mirror. Taking the former, as the more usual, Fig. 211 shows how an image of a very distant object (e.g. the moon) is formed *in the principal focal plane of the object-glass* G; bundles of parallel rays (extreme pair only shown), originating from different parts of the object, focussing into the points which build up the image.

This is also the principal focal plane of the **eye-lens** E and the image will appear to an eye, focussed for parallel light, looking

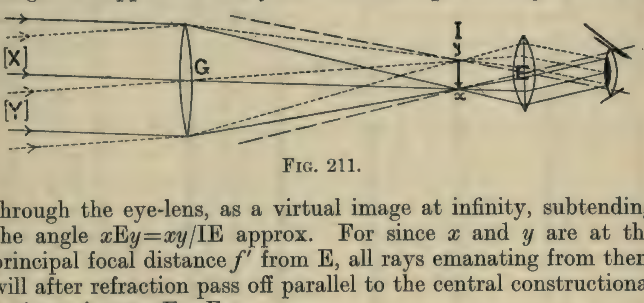


FIG. 211.

through the eye-lens, as a virtual image at infinity, subtending the angle $x\text{E}y = xy/\text{IE}$ approx. For since x and y are at the principal focal distance f' from E, all rays emanating from them will after refraction pass off parallel to the central constructional 'scissors' rays xE, yE.

Now the natural angular diameter of the very distant object is $\angle\text{XGY} = \angle x\text{G}y = xy/\text{GI}$. Hence the ratio of angular sizes (see defn., § 456) is $xy/\text{IE} \div xy/\text{GI} = \text{GI}/\text{IE}$

or, **magnifying power** of a telescope $= \dfrac{\text{focal length of object-glass}}{\text{focal length of eye-piece}}$

* [*Per contra*, a 'telescope arranged for celestial photography' has this eye-piece removed and a plate put in the focal plane. It has returned to the simple camera condition. The photographs are afterwards scrutinized with magnifiers.]

Usually one finds the m.p. by pointing the telescope at a distant brick wall and, keeping both eyes open, counting how many bricks the thickness of one magnified brick appears to cover. This number is the Magnifying Power.

In practice, the object-glass is an achromatic lens, of long focus so as to produce a fair-sized image, and large to let in plenty of light, § 472. Great gain in size of field of view and freedom from colour is obtained by replacing the single eye-lens by a Ramsden or Huyghens eye-piece, § 458, or a solid aplanatic magnifier, Fig. 208 (V).

The image of course appears inverted. This is no disadvantage in **astronomical** work, but is remarkably inconvenient when attempting to follow moving terrestrial objects. For how it can be righted see §§ 466, 467.

§ 460: **Reflecting telescopes.**

The refracting telescope was handicapped for a hundred and fifty years by the chromatic aberration of its object-glass, and

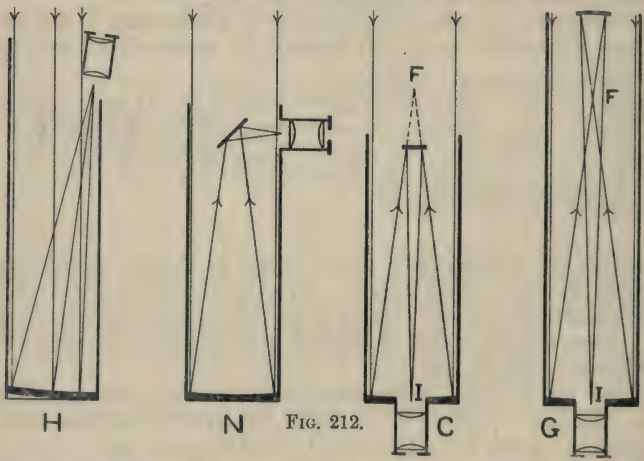


FIG. 212.

before the discovery how to correct this, its reflecting rival had attained to no small size and perfection, for reflection causes no colour troubles. And by trial and skill in the 'fine grinding' and 'figuring' the mirror is made more parabolic and spherical aberration removed also. The mirror is of speculum metal, or nowadays of glass silvered on the front; having only one

surface to make true, the reflecting telescope is much less costly than a refractor, which has four. Several large astronomical reflectors are in use, but distorting variations of temperature affect them more than refractors. [One observer is now engaged in an attempt to keep the whole of his great 5-ft. reflector, in a mountain observatory, at a temperature constant within 2° F.]

The difficulty of the observer's head getting in the light has been got over in various ways :—

HERSCHEL tilted his great 4-ft. mirror so as to bring the image to one edge of the 40-ft. tube, Fig. 212 (H). This rather spoils the image, rendering it astigmatic.

NEWTON placed a small flat mirror at 45° to the axis so as to throw the image to the side of the tube, in which the eye-piece is inserted at right angles. This is perhaps the most common arrangement, Fig. 212 (N).

In the CASSEGRAIN, Fig. 212 (C), a small convex mirror returns the rays, through a hole in the concave, to form the image* in front of an eye-piece placed in the usual telescope position. F and I are conjugate foci of the convex mirror, which is forming a real image I of the 'virtual object' at F, the principal focus of the speculum. This pattern of reflector has several advantages. [Without its eye-piece it is the reflecting analogue of the telephoto lens.]

GREGORY'S, Fig. 212 (G), has a concave small mirror out beyond the image F, of which it forms a second enlarged* image I for examination through a hole in the great mirror. [Without its eye-piece it is the reflecting analogue of the refracting telescope used to project an image. See Note to next paragraph.]

§ 461. **Focussing a telescope.**

A short-sighted person pushes in the eye-piece so that I lies within its focal length, he then sees an image of it (found by the common magnifier construction, Fig. 207) somewhere within his range of vision.

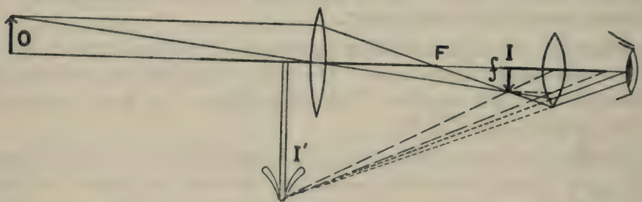
Always, if the object comes nearer, its image retreats from the o.g. and gets inside f' of e.p. as before. For a time the eye tolerates this by accommodating for a nearer virtual image, but soon one

* As drawn, enlarged about 4 diameters ; so that with equal eye-pieces the last two telescopes have 4 times the m.p. of the others.

has to pull back the e.p. and the principal foci of o.g. and e.p. no longer coincide, see Fig. 213.*

As the object still draws nearer e.p. must be drawn back faster till ultimately the foci are separated widely, the real image is actually farther from o.g. and therefore larger than the object itself. *The telescope has become a microscope.*

[NOTE.—For examining the sun without special apparatus e.p. is drawn back till the image is outside f', then an enlarged re-inverted real image forms on a card held a foot or more behind the eye-piece, at the conjugate focal distance. Compare Fig. 216.]



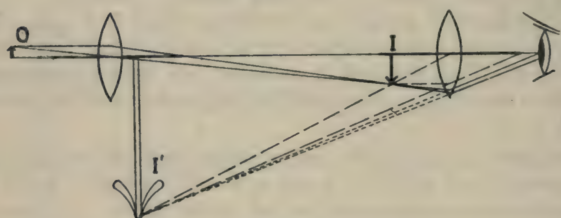


FIG. 213.
FIG. 214.

§ 462. **The Microscope.**

The length of the instrument produced in this way is excessive. It is shortened by making the object-glass now of very short focus, i.e. *two short-focus convex lenses, widely separated, form a compound microscope*, Fig. 214.* The 'object-glass' forms a real inverted enlarged image at or just within the focal distance of the magnifier 'eye-piece'; the eye then sees a further enlarged virtual image of this at some distance within its range of distinct vision.

* *On the drawing of these figures.* Object-glass gives I from O by standard construction. Ditto (broken lines) for eye-lens gives point of virtual I′. Producing the actual rays to meet eye-lens they are refracted and enter eye as if from point I′

The **magnifying power** is evidently the magnification of the first image, multiplied by the m.p. of the e.p. The former = distance a of image behind [emergent principal plane of] o.g. $\div$ distance b of object in front of [entering p.p. of] o.g., and this $=(a/f-1)$, by § 391. The latter is (10 in.$/f'+1$) by § 456.

$$\therefore \text{ m.p. of microscope} = (a/f-1)\times(10 \text{ in.}/f'+1)$$

or roughly in practice with o.g. and Huyghenian e.p. both of short focus, a = length of tube, and (multiplying out)

$$\textbf{m.p.} = \frac{\text{tube length}}{f \text{ of o.g.}} \times \frac{10 \text{ in.}}{f' \text{ of e.p.}}$$

The magnifying power can be measured by the same method as for a simple magnifier, § 456, but a modification is more convenient in practice. The microscope is laid horizontal and is stood on a steady pile of books so that its eye-piece is 10 in. above a paper on the table. A little piece of glass, tinted or lightly smoked on the back to dull the second reflection, is supported at 45° against the eye-lens, Fig. 215. Then an eye looking vertically down sees the paper and pencil through the tinted glass and also the whole microscope field by reflection, as if lying on the paper. A 'stage micrometer'—a very fine scale diamond-ruled on glass to, say, ·001 in.—is focussed by the microscope, and its rulings are traced in pencil on the paper: the average distance between two marks, divided by the actual ·001 in. = the magnifying power.

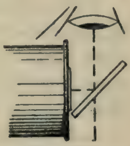


FIG. 215.

This 45° device, which facilitates the making of drawings of microscopic objects, constitutes a **Camera Lucida.** In practice the simple tinted glass does very well, requiring less adjustment than other patterns of the contrivance. The pencil and paper must be brightly lighted.

§ 463. **Focussing a microscope** is effected, not by pulling out the draw-tube, which would alter the m.p., but by moving the whole microscope towards or away from the object.

The focussing gear must be very delicately adjustable (micrometer screw); for by § 392 an m times magnified real image travels along the axis m^2 times as fast as the object; say with a $\frac{1}{6}$ in. o.g. m about 32, $m^2 = 1000$. This real image has to be examined by, say, a 2-in. focus e.p., and the reader can calculate that a motion of $\frac{1}{3}$ in. towards this carries the virtual image right

through the range of distinct vision. Hence a jerk of $\frac{1}{3000}$ in. would throw the object out of focus altogether.

For photomicrography the microscope is focussed back a trifle (to the right), the first image moves (to the left) out in front of the eye-piece's focus, and a real re-inverted enlarged image forms on the plate, Fig. 216. [The faint dots suggest the position of the lenses for ordinary visual use.]

But for a useful makeshift, leave the microscope in focus for your eye at rest, it is then emitting parallel light ; put over it your ordinary camera focussed for ' infinity,' and expose.

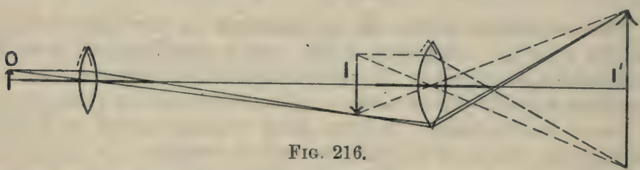


FIG. 216.

§ 464. The object-glass of a microscope is actually an achromatic combination of many lenses and is capable of dealing with rays from the object making very considerable angles with the axis, particularly in high powers. A $\frac{1}{4}$-in. equivalent focus objective should take in a cone of rays spreading as wide as 45° all round the axis. Objectives of the highest powers require to be optically connected to the object by a drop of oil of cedar wood (μ 1·5). This enables the object to send into them rays in such a wide cone that not only would they fill a hemisphere in air, but many would be totally reflected (Fig. 159) from the under-surface of the cover-glass, back into the highly refractive balsam in which the object is contained.

The advantage of this wide-angle illumination has been mentioned in § 453 and will be further considered in § 476.

The sine of the angle the extreme rays make with the axis, multiplied by the refractive index of the medium from which they are received (air 1, oil 1·5), is the **Numerical Aperture** (N.A.) of the objective. *The brightness of the image is proportional to* $(N.A.)^2$ *and the ultimate fineness of definition—much more important than mere magnification—is proportional to N.A.*

A high-power objective of large N.A. is, however, of little use unless adequate illumination is being sent up by a substage condenser of large N.A. Failing this, everything in the dim field is bordered by ' diffraction fringes ' which cross and recross and

give rise to abundance of utterly false detail. To see them at their worst, turn direct sunlight into your microscope with the plane mirror. For low powers the concave mirror provides a cone of light of wide enough angle.

The microscope eye-piece is usually a Huyghenian or else a solid ' compensation ' triplet, Figs. 210, 208 (v).

§ 465. **Cross-wire and micrometer telescopes and microscopes.**

Any object placed in the focal plane of the eye-piece is of course seen along with the image, just as was the grain of the ground glass in § 459.

Here therefore can be placed pointers of all sorts, and ' cross-wires,' the line from whose point of intersection to the optical centre of the object-glass is the instrument's optical axis and a very definite direction indeed. The ' Meridian of Greenwich ' finds its only tangible representation in a vertical spider-line, stretched across the focal plane of the great meridian-circle telescope.

Here again can be placed delicate ' eye-piece micrometers.'

A scale of tenths of a millimetre, diamond-ruled on a glass disc and dropped on the diaphragm inside the Huyghens eye-piece, is a useful addition to any microscope. The field-lens, however, distorts the outer parts of the image a little, and besides, the whole eye-piece is often loose. More reliable and delicate is an arrangement of the finest spider-lines on a little frame, which is traversed across the tube by a micrometer screw and is examined by a Ramsden eye-piece. With such one measures the separation of a double star, or the length and breadth of a bacillus.

The effective value of the scale divisions or the screw turns is found by measuring with them the length of an object of known size viewed through the instrument.

§ 466 : **Erect-image telescopes and microscopes.**

There are three ways of modifying a refracting telescope or microscope so as to obtain a view the right way up :—

I. *The real inverted image formed by the object-glass is examined not with a common eye-piece, but with a compound microscope, which of course gives a re-inverted view of it.*

In pocket, target, etc., telescopes the first draw-joint forms the microscope, having a Huyghens eye-piece at one end and an ' erecting glass ' (the micro-objective ; usually a couple of strong convex lenses with a small stop between) at the other. Sometimes

this is usable as an ordinary microscope, but often it is so perversely designed as to require an object actually on its front lens or even inside it—no trouble to a real image, but impossible to a real object. The re-inverted real image J, Fig. 217, is usually about twice as

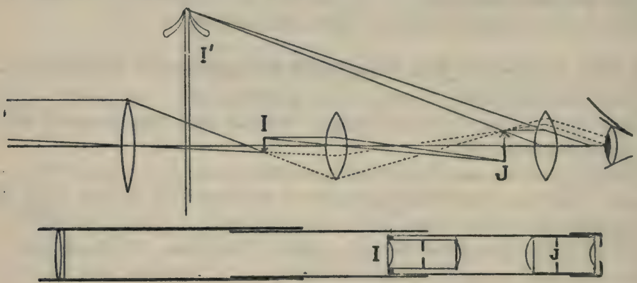


FIG. 217.

big as the first, but by varying the ' microscope ' tube length its size, and hence the m.p. of the whole instrument, can of course be widely altered [' *Pancratic* ' (=all powerful) *Eye-piece*].

In Fig. 217 the solid lines are standard constructions applied to alternate ends (for clearness) of the images I J. The actual paths of the two rays shown from the top of the object have then been dotted in, they enter the eye as if from top of I′. The lower figure in Fig. 217 is a $\frac{1}{3}$-scale section of a serviceable little telescope now on the market. Its m.p. is 10, and the first joint is a handy microscope $\times 20$.

These telescopes are long, are adversely affected by the aberrations of the additional lenses—producing the too familiar haziness of definition—and they are exceedingly particular about exact focussing, § 463.

In **microscopes** the erecting glass is put at the bottom of the draw-tube ; by moving the latter the m.p. can be varied greatly. Those who have attempted dissecting under the microscope will understand the utility of the arrangement.

§ 467 : **Erect-image instruments.** II. *The rays on their way from the object-glass are so bent and folded by repeated reflections that they form an erect image.*

The reflections are total, in right-angled prisms placed at right angles as in Fig. 218. The image is turned right way up by the

first prism, but is now laterally reversed, and has to be turned left for right by the second prism.

These 'Prismatic' instruments are compact, beautifully free

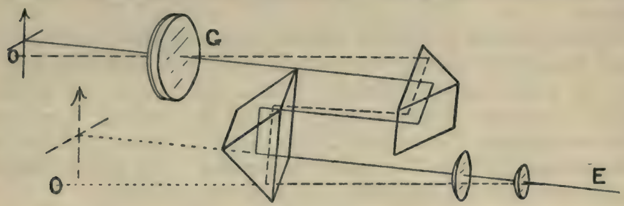


FIG. 218.

from aberration blur, and easy in focussing. As binoculars, especially when with enhanced stereoscopic effect due to wider apart object-glasses, they far excel the older patterns.

§ 468. **Erect-image instruments.** III. *The rays from the object-glass are prevented from coming to any focus till they reach the retina, where of course an inverted image means an upright view.*

This is effected by checking their convergence by a **concave eye-lens.** This was GALILEO's form of telescope.

The path of the rays in the normal condition *when the back foci of o.g. and eye-lens coincide* is shown in Fig. 219. From one end*

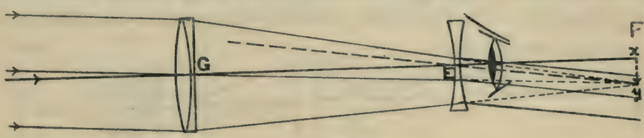


FIG. 219.

of the real image (now dotted because no longer actually formed) a 'scissors' ray has been drawn to E; all (dotted) rays which were coming to a focus at this point are turned off by the concave lens in a bundle parallel to this central ray (§ 377).

They enter the distance-focussed eye with an angular separation $x\mathrm{E}y$ instead of $x\mathrm{G}y$, or $\mathrm{M.P.}=xy/\mathrm{FE}\div xy/\mathrm{FG}=\mathrm{FG}/\mathrm{FE}=$ratio of focal lengths, as before.

They appear to come from a virtual image at infinity. If the object approaches its real image moves out to the right of the

* The other end has been made to lie on an axial ray, and the other rays coming to it are omitted, to save confusion.

foci and the virtual image is formed at nearer and nearer focal distances, conjugate to it, of the eye-lens, until presently the draw-tube has to be pulled out more, unless the user is short-sighted.

The aberrations of the two lenses partly neutralize each other, hence the view is clear and the instrument short, simple, and cheap. But the field is small, especially with higher powers; it all lies inside a circular window (o.g.) looked at through a strong diminishing lens. This sharply limits the m.p., which is 1·5 to 2 in **opera glasses** and 3 to 5 in **marine binoculars**; and it puts microscopes with concave eye-lenses out of court altogether.

§ 469: Simplest of telescopes is Baden Powell's **Unilens.** It is a large weak convex lens held up on the far end of a walking stick. The eye must be relaxed till it focusses already convergent light, i.e. is equivalent to an eye focussed for parallel light with a weak concave lens in front of it. Thus the Unilens is virtually the o.g. of a long Galileo telescope. At 6 ft. a $\frac{1}{3}$-diopter lens would have m.p. 3, the maximum comfortable for most eyes.

The **Telephoto Lens** is a Galileo telescope with the concave lens drawn farther back, so that I would lie within its focal length, when a much enlarged real I′ forms at the conjugate focal distance, by the construction of Fig. 220. Variation of the camera length EI′, and simultaneously the lens separation, widely alters the magnification. The faint dots show the lens position as telescope.

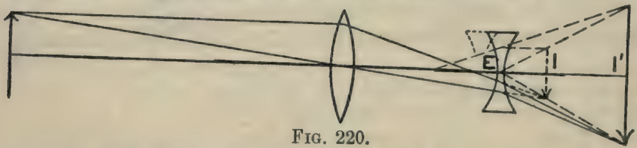


FIG. 220.

§ 470. **The Ophthalmoscope.**

Wishing to look into a deep dark cavity one reflects light into it by a mirror as at A, Fig. 221. Better all-round illumination would be gained from a mirror B with an eye-hole in the middle, and far better still from a concave C, concentrating the lamplight.

Such a concave mirror is the first requisite in **laryngoscopes, ophthalmoscopes, etc.**: it enables most cavities to be examined with only the further aid of a little mirror on a long handle to explore behind corners.

Turned on a friend's eye it soon shows it by no means the black-walled chamber you imagined. In fact, if only our eyes would

open to the enormous aperture of a cat's, they would shine a faint but noticeable golden pink 'in the dark,' i.e. when reflecting a gleam from a light near or behind the observer. [The cat's eye has the faintly reflecting retina backed by a more brilliantly reflecting layer instead of the black 'choroid.']

But the eye is glazed with a lenticular window. If focussed for parallel light this means that the retina illuminated as in Fig. 222 (upper) will be seen clearly by another eye focussed for distance. But if short-sighted, rays from the retina leave the eye converging.* One of a trial series of lenses (concave for short, convex for long sighted eyes) is used to correct this, as shown.

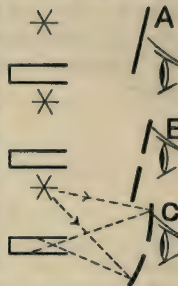


FIG. 221.

If the patient's accommodation is paralysed with belladonna, the instrument quite close up, and the observer's eye truly focussed for distance, this lens is evidently the proper spectacle to enable the patient's eye at rest to see at distance.

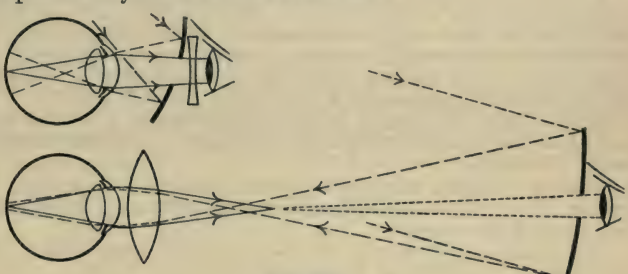


FIG. 222.

A general view of the retina is more easily obtained by the *indirect* use of the ophthalmoscope, Fig. 222 (lower). A mirror about 1-ft. focus forms an image of the lamp in the air 2 or 3 in. before a convex lens, of that focus, held to the patient's eye. This lens makes the dotted rays from the image about parallel, and then the eye converges them on a small patch of retina. Rays from this brightly lit patch pass out through the 'crystalline' and the glass lens and form an aerial inverted magnified real image of

* Toward an image of it. Recollect that if this page is clearly imaged on your retina a sharp image of your retina is likewise being thrown on this page, but of course far too faintly lit for anyone to see.

it near the focal distance of the latter. Focussing his eye for near vision, the observer examines this image through the hole in his mirror. Thus at nearly the same point in space the patient sees the brilliant image of the lamp flame, and the observer, looking the opposite way, sees the image of the patient's illuminated retina.

EXAMPLES.—CHAPTER XLVII

1. A 2-in. focus magnifier is held 1 in. from eye with D_n 9 in. Where must object be? [L]m.

2. Figure the essential parts of a lantern for projecting. If the lens has a focal length of 8 in. and is 15 ft. from the screen, find size of picture of slide 3×3 in. Compare the illuminations of slide and picture. [L.]

3. Where are the cross-wires put in a telescope and why must they be put in any particular position? [L.]

4. Why is a telescope with cross-wires in the eye-piece used in a surveyors' level or a theodolite for observing directions of objects which can be quite easily seen directly? How could you ascertain whether the cross-wires of a telescope intersect on its axis? [L.]

5. Describe with diagram a telescope or a microscope; how ought the eye-piece to be shifted for a long-sighted person? [Ab.]

6. Show that two convex lenses, of focal lengths 1 in. and 2 in., can be used as a simple microscope, a telescope, or a compound microscope. [L]m.

7. Draw diagrams showing the paths of several rays through two convex lenses of 2 cm. focal length combined into (a) a telescope, (b) a compound microscope.

With a number of lenses to choose from, what magnitudes of focal length would you select for these two instruments? [L.]

8. What is meant by the magnifying power (1) of a telescope, (2) of a microscope? How can it be determined experimentally or calculated theoretically? [Ab.]

9. Describe opera glass, and find focal length of lenses of one which is 3 in. long and magnifies 3 times. [Ab.]

10. Describe the principal forms of eye-piece used in telescopes and microscopes, and explain their advantages over a single lens.

11. Explain the ophthalmoscope. [L]m.

12. Describe a combination of lens and concave mirror capable of always reflecting back a bright beam of light to a distant source which moves about anywhere within 10° of the axis of the arrangement. [L.]

Chapter XLVIII

APERTURE IN OPTICAL INSTRUMENTS

'Aperture' has been defined in § 454 as being proportional to the width of the 'window' through which practically plane waves of light enter the optical instrument.

Aperture is of prime importance, for it controls not only the Brightness of illumination, but also the minuteness of definition or the 'Fineness of grain' of the image when it is 'in the best focus' that a good instrument can attain.

The Brightness of Real Images

§ 471: The concentrating action of a lens tempts us to believe that with large enough lenses we might obtain focussed images brighter and brighter without limit. But this, by § 500, would mean hotter and hotter without limit, hotter even than the original source. This has never been done, and experience of all sorts (summed up in the second law of thermodynamics) is against its ever being done. The image cannot be hotter or brighter than the object; one cannot, for instance, 'focus' the heat from a kettle and kindle a fire with it. Figs. 166 and 174 show that as one gets closer to catch stronger radiation, the image over which it has to be spread inevitably gets proportionally *larger*, not brighter nor hotter.

In the camera of § 454 all the light sent from a small patch of the illuminated object to the opening of the lens is concentrated on to a few grains of the sensitive silver bromide on the plate and the rate at which these are acted on is proportional to the square of the 'Aperture' as there defined, viz. (diam. of lens opening $\div$ focal distance)^2, assuming of course that the brightness of the object and the sensitiveness of the plate are unchanging. A longer focus lens of the same aperture, $f/8$ say, would have a larger diaphragm and would collect more light, but would have to spread it over a proportionately larger image.

§ 472 : When a real image is formed on a diffusing white screen it becomes visible in all directions with a brilliance proportional to the light brought up to it per square centimetre, but if it is not formed on a screen the case is different. It now radiates only into a limited cone of directions ; to see it the eye must be placed in this cone, e.g. in the diagram of a Telescope, Fig. 211, the point y of the real image can be seen only by the cone of dotted rays leaving it. An object-glass O of wide aperture (diameter$\div$focal length) widens this cone, and if at length all the light received can be brought into the eye the view will brighten as the aperture widens. All the parallel light, forming a beam of width W, from a patch of the bright surface of a distant object, will enter the eye provided that w, the width of the parallel beam leaving the eye-piece, does not exceed the diameter of the pupil (2 mm. by day, 5 mm. at night). Now $W/w=GI/IE=M.P.$, and when $w=5$ mm., the diameter of the pupil at night, this is called the **Normal Magnifying Power** m of the instrument. Any increase of aperture beyond this, without increase of m.p., widens w wastefully.

In this condition the whole of the pupil is illuminated, and while it receives $(W/w)^2=m^2$ times the light it did without the glass, it is viewing an image m^2 times larger in area, i.e. the image is just as bright as the object, bar slight losses due to reflection and absorption in the glasses.

Suppose the magnification is pushed to a higher M, this involves either a shorter focus eye-lens or a longer focus object-glass, in either case unless the object-glass is enlarged the width of the beam entering the eye is diminished, the whole pupil is not lit up, the eye still receives m^2 times the light but from an image M^2 times the area, the brightness of the image is only $(m/M)^2$ that of the object. [Look at the daylight sky with a telescope of high m.p., and notice how it is darkened.]

Hence **night glasses** should not exceed their normal magnification $m=W/w$; or W the object-glass diameter should be $m\times\cdot5$ cm.=[half the magnifying power] centimetres. Dainty waistcoat-pocket telescopes with small lenses and high m.p. soon fail at dusk. Night glasses increase the apparent size of the object without diminishing its brightness, the impression on the retina is more widespread and more easily perceived, just as a collar is more easily seen than a dropped collar-stud, on a dark morning. And further, everyone who has hunted for faint stars

knows how much more sensitive are the side parts of the retina, on to which the image now spreads, than is the centre point.

§ 473 : The case of a **Star** is different. Stars are so distant that no one has ever been able to magnify their image up into a true disc of light. The image is not, however, a mathematical point, but a patch or 'spurious disc' the size of which is settled by diffraction and is calculable much as in § 474 (=about $2z$). It is therefore inversely proportional to the angular breadth d/F of the cone of light from the object-glass, i.e. it is the same whatever the size of the telescope provided that its 'aperture,' usually $F/16$, is kept the same. This patch is so small that eye-pieces giving 4 or 5 times the normal magnification can be used on it before it perceptibly broadens out. Up to this limit the whole of the star's light that falls on the great object-glass has been poured into the eye as from a point of no appreciable extent, and this therefore appears more brilliant than the star in the ratio (area of o.g./area of eye pupil) = (normal m)^2.

Thus a large telescope collects enough light from very small stars to make them visible. And at high magnifications it also darkens the surrounding sky, and in these ways may enable one to find the brighter stars in daylight.

When there is plenty of light the magnifying power is often pushed far beyond the normal, both in telescopes and microscopes. This is not objectionable until the emergent beam w becomes so narrow that it shows up specks in the eye-piece or eye too prominently.

Second Effect of Aperture. Resolving Power

§ 474 : *The* **Resolving Power** *of an optical instrument is a measure of the fineness of definition of detail in the image presented to the observer's eye, and upon it the value of the instrument depends.** It is quite distinct from Magnifying Power. This latter, beyond a lower limit, merely makes the observation of already 'resolved' detail rather easier. A lens at $f/16$ produces finer detail than a pinhole camera ; it has a greater resolving power. Both pictures can be enlarged afterwards, but that process produces no more detail, it only enlarges what detail happens to exist, and enables one to discriminate between the pictures at a glance.

The Interference experiment of Fig. 116 is realized optically

* Perhaps the glamour of 'high magnifying power' has to pass away before the beginner realizes this.

by using, as sources P and Q, the two reflections of an illuminated slit in two plane mirrors slightly inclined to each other. These slits are not lines, but have an appreciable width, they are like lancet windows, perhaps 1 cm. $\times$ ·025 cm. broad. By reducing the inclination of the mirrors they can be moved closer into one aperture of double width, and then even be superposed. The result is that the alternate light and dark interference bands on the screen, which were very fine and narrow at first, become continuously coarser, until the two luminous sources are superposed, when measurement of their width shows that they may still be regarded as produced by the interference of two point-sources which are the centres of the narrow halves of the 'lancet window.'

With a broader 'window' the light and dark bands are blurred into a uniform illumination, but they are still to be seen at the edges, or at the edges of the shadow of an obstacle lit by the 'window,' i.e. they reappear as soon as any *structure* is being looked at.

In every optical instrument there is a window which is limiting the breadth of the comparatively flat waves of light passing through it (breadth of the nearly parallel beam)—the iris of the eye or of the camera, the rim of the object-glass of a telescope, the square face of a prism, etc.—and the result is that every line in the structure of the thing examined is represented by three or four parallel interference bands, every point becomes a target of light and dark rings. With a narrow aperture these weave into a tangle of false detail, and a plain view can be obtained only when the aperture is wide and the bands, whose distance apart varies inversely as the aperture-width, § 293, are so close together that the eye cannot separate them.

Everyone who has examined a half-tone block with a magnifying glass, or a photographic negative with a microscope, knows perfectly well that these have a mechanically granulated structure. Now, we see that every picture that any optical apparatus can produce has really an 'optically granulated' structure, and unless wide aperture has given a fine optical grain the picture will be as unintelligible as a half-tone block under a pocket lens.

Calling the width of the window-like aperture 2R, Fig. 223, the interfering points will be R apart, and on a screen distant a from them the bright interference bands follow one another at distances $z=la/\mathrm{R}$, where l is the wave length of the light. Bright and dark bands alternate at $\cdot 5la/\mathrm{R}$. See § 293.

Now it is generally accepted that if two sets of bands are superposed so that the bright bands of one set fall on the dark bands of the other set, they will not obliterate each other (the dark band is more diffuse than the bright), but the double set of bright bands will be separated by *just distinguishable* darker lines. If the bright bands are pushed closer than this the dark dividing line disappears. Hence if there are two streams of light—from a double star, for instance—passing through the aperture, and inclined to each other at such an angle that their two interference systems are superposed in the position mentioned, i.e. at an angle $\cdot 5la/\mathrm{R} \div$ distance of screen from aperture $= \cdot 5la/\mathrm{R}a = \mathbf{\cdot 5}\boldsymbol{l}/\mathbf{R}$ radians, it will be just possible to distinguish that they came from separate sources—the double star will be 'resolved.' This angle will be called the **minimum angle.**

FIG. 223.

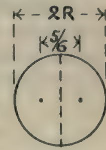


FIG. 224.

With a **Circular Aperture,** Fig. 224, taking as interfering sources the centres of gravity of the two semicircles $8\mathrm{R}/3\pi =$ very nearly $5\mathrm{R}/6$ apart, the **minimum angle** is $\cdot 5l \div 5\mathrm{R}/6 = \mathbf{\cdot 6}\boldsymbol{l}/\mathbf{R}$ radians, a result agreeing very closely with that of the most elaborate calculations.

RESOLVING POWER OF VARIOUS INSTRUMENTS

§ 475 : **The Eye.** Taking R of pupil as about ·15 cm. ($\frac{1}{16}$ in.), and l of the brightest part (the yellow-green) of white light being ·00005 cm. ($\frac{1}{50000}$ in.) these give the minimum angle $\cdot 6 \times \cdot 00005/\cdot 15 = \cdot 0002$ radian = a length of ·005 cm. at 25 cm. from the eye ($\frac{1}{500}$ in. at 10 in.). This then is the utmost fineness of detail that a perfect eye could perceive in any object. But the eye has not the rigid perfection of fine glass lenses, and at best cannot distinguish detail closer than ·01 cm. ($\frac{1}{250}$ in.) Now the dark and bright interference bands which build up the picture the eye sees are only at the calculated ·005 cm. apart; one strong pair runs along every edge in view, they are too close to distinguish as such, and the reader will doubtless be surprised at the statement that the most perfectly sharp outlines he sees against the sky are really bordered by two or

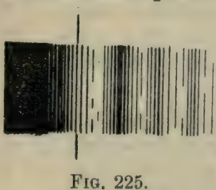


FIG. 225.

three bands like Fig. 225 (where the actual position of the edge is indicated by the longer line).

Telescope. The circular rim of the object-glass limits the entering beam. The minimum angle between details which a pocket telescope with a 1-in. o.g. can distinguish is hence $\cdot 6 \times \cdot 00002$ in. $\div \cdot 5$ in. $= \cdot 000024$ radian $= \cdot 000024 \times 180 \times 60 \times 60 \div \pi$ $=5$ sec. of arc. A great 42-in. refractor can resolve double stars only $\frac{1}{42}$ as far apart $= \cdot 18$ sec., or details in the moon, 240,000 miles away, about a furlong apart. No matter what magnifying power is used this is the limit of visual telescopic resolving power at the present day.

§ 476: **Microscope.** Removing the eye-piece from either telescope or microscope, one sees the back lens of the object-glass as a round illuminated window which is sending light up the tube. The light actually entered the telescope as a [plane wave] parallel beam of this size: in the microscope the front lenses of the objective have dealt with a cone of light from the object diverging at semi-angle α (see § 464) and have produced a parallel beam of the observed size, which the last lens is dealing with just like a telescope o.g. With the usual short-focus objective the object is at practically the principal focal distance f from the first principal plane, at which therefore the cone of light has spread to have a radius $R=f \sin \alpha$, and this is the radius of the illuminated window seen in the second principal plane. Hence the minimum angle $\cdot 6l/R = \cdot 6l/f \sin \alpha$. The distance apart of points in the object which send light at this angle to the lens, distant f, is evidently $f \times$ angle $= f \times \cdot 6l/f \sin \alpha = \cdot 6l/\sin \alpha$.

If the object is immersed in oil, however, a ray making a small angle with the normal will make a μ times greater angle when it emerges into the air (by Snell's law, and putting small angles = their sines). Hence the minimum angle in oil is only $1/\mu$ the above, and the minimum distance visible $= \cdot 6l/\mu \sin \alpha$. This denominator has been defined in § 464 as the Numerical Aperture of the objective, therefore

The smallest distance apart of observable detail under the microscope is six-tenths the wave length of the light divided by the Numerical Aperture of the objective; or lines per inch = N.A. $\div \cdot 6$ w.l.

e.g. a $\frac{2}{3}$-in. o.g. of $\cdot 2$ N.A. should resolve 16,700 lines per inch.

a $\frac{1}{6}$-in. of $\cdot 85$ N.A. should resolve 65,000 lines per inch.

a $\frac{1}{12}$-in. oil-immersion of 1·3 N.A. should resolve 108,000 lines per inch.

This assumes that the whole of the back lens is illuminated; with inadequate substage arrangements this may not be the case and the resolving power suffers—it is very striking how the uniform tint of a diatom will suddenly break up into a pattern of lines and dots as the substage condenser is brought up into focus. On the other hand, by stopping out central rays and illuminating the outer margin of the lens more strongly, the 'interfering centres' can be moved farther apart and the resolving power increased, for special purposes.

Light of shorter wave length should reveal proportionally finer detail; in the **ultra-violet microscope** photographs are obtained with radiation of half the wave length of the usual yellow-green and showing double the detail.

Quite distinct is the Ultra-microscope contrivance of § 428. This will disclose the existence of particles of diameter only one-fortieth the minimum distance of microscopic separation. But it can give them no shape, and cannot distinguish two particles at less than this distance.

§ 477: **Prism spectroscope.** Taking for simplicity in the calculation a *thin* prism, the width 2R of the rectangular beam of light entering the telescope is practically the width of the prism, Fig. 226. The base has a thickness T $=2R\times$ angle A, hence $2R=T/A$. Therefore the minimum alteration in deviation observable $=\cdot 5l/R$ $=lA/T$.

FIG. 226.

The whole deviation D for light of wave length l is $D=(\mu-1)A$ and D' for w.l. $l'=(\mu'-1)A$; subtracting we get the change of deviation corresponding to a change of wave length from l to l',

$$D-D'=(\mu-\mu')A.$$

Making this $D-D'$ the minimum change observable, we have $lA/T=(\mu-\mu')A$, whence

$$\frac{l-l'}{l}=\frac{1}{T\dfrac{\mu-\mu'}{l-l'}}$$

The left-hand side is the smallest change of wave length the prism can reveal, expressed as a fraction of the w.l. under observation. $(\mu-\mu')/(l-l')$ is the rate at which the refractive index changes with wave length; hence *The smallest observable percentage change of wave length is inversely proportional to the thickness of the base of the prism multiplied by the dispersive power of its glass.*

e.g. the twin sodium lines have wave lengths ·00005889 and ·00005895 cm., hence $(l-l')/l=6/5892=$about $1/1000$. Ordinary flint glass has $\mu=1{\cdot}617$ for l (red) ·0000656 and $\mu'=1{\cdot}635$ for l' (blue) ·0000486, hence $(\mu-\mu')/(l-l')=\cdot018/\cdot000017=1060$. Hence the smallest flint prism that can split the yellow sodium line into its twin component lines has a thickness of base given by

$$\frac{1}{1000}=\frac{1}{\text{T}\times 1060} \text{ or T=nearly 1 cm.}$$

The reader fond of trigonometry who will work this calculation through for the usual 60° prism will arrive at precisely the same expression. The resolving power, i.e. the fineness of definition of the spectrum lines, depends only on the difference in thickness of dispersive glass that the extreme right and left rays pass through before entering the telescope, and it does not matter whether this is contained in one large prism or in several little ones, or what their angles may be. Here again, little spectroscopes may show as long a spectrum as big ones, but it can only be comparatively fuzzy and ill-defined.

§ 478 : The principle of the **Diffraction Grating** has been shortly explained in § 296. A piece of fine wire gauze, for instance, placed in a beam of light, will break up the luminous disturbance that falls on it and will send out waves of light of various lengths in various oblique directions.

AB, Fig. 227, is the distance from centre to centre of two transparent spaces, of which there are n per cm. A plane light wave falls flat on the grating so that A and B vibrate in the same phase; then if BC=a complete wave length [or m complete wave lengths] A and C vibrate in the same phase and are points on a possible wave front. A telescope whose axis is parallel to BC will collect all these from all pairs of apertures in the grating and focus them into a bright spectrum line of wave length BC [or $1/m$ of BC]$=l$. A shorter wave length BC′ will focus into a bright line visible in the direction BC′, and so on.

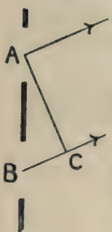


FIG. 227.

$$\text{BC}=\text{AB } \textit{sine } \text{CAB}=(1/n) \text{ cm.}\times \textit{sine } \text{CAB}=l$$
$$\text{or } \textit{sine } (\textit{deviation})=nl.$$

Thus looking through the regularly woven silk of an umbrella at a distant street-lamp, there is seen running across both warp

and weft a series of spectra, blue ends (short l) nearest the light, and successive spectra correspond to $BC=l$, $BC=2l$, etc.

For scientific purposes this very rough sort of grating is superseded by plates, either transparent or reflecting, on which 15,000 lines per inch are ruled with the utmost attainable accuracy. The largest gratings hitherto ruled have about 75,000 lines, covering a space about 5 in. $\times$ 2 in. on a speculum metal mirror.

Resolving power of grating.

The apparent width of the grating seen in direction BC is nearly its whole actual width if deviation CAB does not exceed about 25°.

$$\therefore \text{ Minimum Angle} = \cdot 5l/R = l/\text{whole width of grating.}$$

Now sine $CAB=ln$, and for deviations below 25° angles in circular measure are nearly the same as their sines, or Deviation$=ln$.

$$\therefore \text{ Difference of deviations for waves } l \text{ and } l' = (l-l')n.$$

Putting this angular separation equal to the minimum observable angle, $(l-l')n = l/\text{whole width of grating.}$

$$\therefore \frac{l-l'}{l} = \frac{1}{n \text{ lines per cm.} \times \text{cm. width}} = \frac{1}{\text{total no. lines in grating}}$$

e.g. to separate the sodium lines a grating of 1000 lines would suffice, and the largest gratings define about 75 times more clearly than this.

CHAPTER XLIX

SPEED OF LIGHT

GALILEO endeavoured to ascertain the speed of travel of light by stationing two observers with dark lanterns a long way apart, B to uncover his light when he saw A's opened, and A to judge the interval between opening his own and seeing B's. But the speed of light far exceeds that of sound, and we know now that the uncertain small results in this experiment measured only 'personal equation,' in this case a double interval between the eye seeing a signal and the hand making an effective response.

It was by utilizing the great distances of inter-planetary space that the speed of light was first measured.

§ 479. **Römer's method.**

The earth and the planet Jupiter revolve round the sun in nearly the same plane, Jupiter taking twelve years to complete his year. In Fig. 228 the lower line joins their relative positions

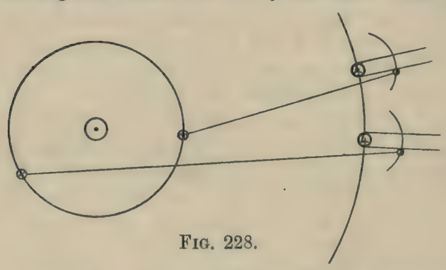


FIG. 228.

four or five months later than the upper pair. Again nearly in the same plane, four well-known satellites revolve round Jupiter—four tiny points of light that are the first test of a field-glass—so close that three are eclipsed every revolution, and so rapidly on account of his vast mass that their plunge into his shadow is complete in a very few seconds. In fact, the Jovian system is a celestial

timekeeper made use of for checking ships' clocks on the seas.

But it was observed that during a month (lower positions) when Jupiter was a morning or evening star, the eclipses lagged several minutes behind the times predicted from observations in months when he was high in the sky at midnight.

At length the Danish astronomer Römer gave (ca. 1675) the explanation that in the former case the earth was farther from Jupiter, and that the lag represented the time necessary for light to travel the extra distance. Calculations made on this assumption showed that light averages nearly 1000 seconds (996) to travel the whole breadth of the earth's orbit, which we now know to average twice 92 million miles. This gives a speed of 185,000 miles per second.

But this measurement depends on the sun's distance, which has to be computed from astronomical observations of great difficulty and complexity, and direct terrestrial methods are desirable.

§ 480. **Fizeau's method.**

The trouble in Galileo's method was the slowness of the experimenters. Fizeau replaced A's hand and shutter by a rotating cog-wheel, which gave a succession of shutters, and B by a mirror, and so evolved a method exactly analogous to that of Sedley Taylor for the speed of sound, § 308.

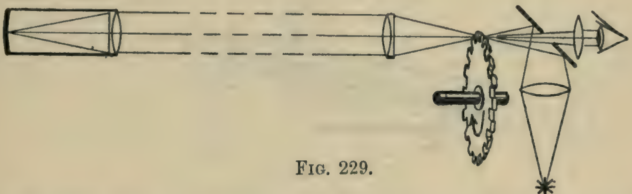


Fig. 229.

Fig. 229 shows a cog-wheel which had 720 square teeth separated by 720 spaces, teeth and spaces being of equal width. From a limelight (the asterisk) light is sent through a lens and is then reflected by the inclined mirror to form among the teeth a bright image which lies at the principal focus of a second lens. 'Parallel' light therefore travels hence several miles to the reflector on the left, returns to the second lens, and is formed by it into a return image among the teeth. This is inspected through a hole in the inclined mirror with a magnifying eye-lens. The teeth of the wheel

are bevelled and highly polished so that the outgoing illumination which falls on them as they pass is thrown away and not reflected back to trouble the eye.

Now if the wheel turns at a certain speed it will happen that the flash sent out through one of its gaps travels to the reflector and back while a tooth moves in and blocks up the sending gap. At this speed the observer sees no return image at all. Speeding up the wheel the image reappears, at twice the speed it reaches a maximum brightness, for the next *gap* has moved into the stead of the sending gap, at 3 times it disappears, at 4 times is bright again, and so on. From a series of speeds taken like this the speed which just brings the first tooth over the gap can be accurately found; if this is n turns per second the time occupied is $1/1440n$ second and in this time light has travelled to the reflector and back, a distance 2D.

$$\therefore \text{Speed} = \text{distance} \div \text{time} = 2D \times 1440n.$$

$$D \text{ was } 6{\cdot}4 \text{ miles, } n \text{ revs.} \quad \therefore V = 185{,}400 \text{ m./sec.}$$

§ 481: **Foucault's method.**

In this a spinning mirror on the left of Fig. 230, lit with 'parallel light' from a collimator (the tube of which is bent at a right angle, to get the lamp out of the observer's way), sends once in a revolution a flash to a distant reflector, on the right.

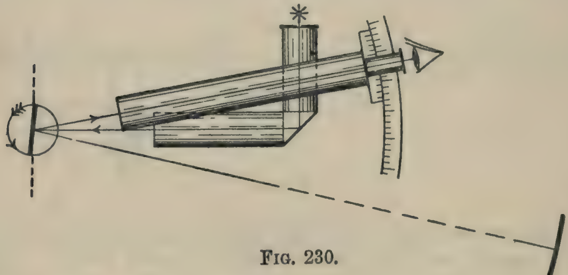


FIG. 230.

When this returns, the mirror, spinning at perhaps 1000 revs. per sec., has turned through a small angle, say into the dotted line, and reflects it, not now in the original direction, but into a telescope (lying just on top of the collimator) where the angle between collimator and telescope $= a^\circ =$ twice the angle turned through by the mirror, § 362.

If n revs. are made per sec. $\frac{1}{2}a$ is turned through in $a/2\times 360^\circ \times n$ sec. and in this time the light travels to the reflector and back, 2D.

$$\therefore V=2D\div a/2\times 360\times n=1440nD\div a.$$

Foucault placed tubes of water between the mirrors and found that light travelled in the water only $\frac{3}{4}$ as fast as in air, as the Wave Theory requires from its refractive index, which is $\frac{4}{3}$.

As the mean of all recent experiments it may be taken that the velocity of light is 3×10^{10} cm. or 186,000 miles per sec. in vacuo (and practically as much in air).

EXAMPLES.—CHAPTER XLIX

1. Describe some method by which the velocity of light has been determined. How does the velocity depend on the intensity of the light? [L.]

2. Show relative positions of Earth, Sun, and Jupiter when intervals between satellites' eclipses greatest and least. Explain the difference. [M.]

3. Describe either Fizeau's or Foucault's method of determining the velocity of light. Prove, by theoretical considerations, that refractive index of a substance is ratio of velocity in air to velocity in substance. [M.]

4. Describe the revolving-mirror method of measuring the velocity of light in air or other media. The fixed mirror was 3 km. from the revolving mirror, which made 500 revs. per sec. Angular deviation of return ray was 7° 12′; calculate velocity. [L.]

5. The return ray makes an angle of 18° with its original direction. The distance between the two mirrors is 10^6 cm. and the rotating mirror is making 375 revs. per sec. Calculate velocity. [L.]

6. Describe Fizeau's method for the velocity of light; he found light reflected from mirror 8663 m. distant was eclipsed by a wheel of 720 teeth at 12·6 revs./sec. Calculate velocity. [St. A]m.

Chapter L

POLARIZED LIGHT

We come to a property of light waves that compels a sharp distinction to be drawn between the motion of the particles in them and in sound waves. Of two beams of light, perfectly indistinguishable to the eye, one may pass unhindered through certain pieces of clear colourless spar which quite stop the other. A glossy surface will always reflect one but may blot out the other, or reflect it feebly, or fully, according to position.

Such light is said to be **polarized.** Nothing like it occurs in Sound.

§ 482. Light vibrations transverse.

Imagine a stick and some vertical palings. Held lengthways it can *always* be pushed through the fence, but held crossways in the middle it will go through when parallel to the palings but be bounced back when horizontal.

Let the stick represent the to-and-fro track of a particle taking part in a travelling wave motion. In the first case the vibration is *longitudinal* as we know it in sound waves. In both the other cases it is *transverse* (Fig. 112, T) and what happens resembles the effects of polarization described above. It is concluded that the vibrations in light waves are transverse, each particle being confined to its own plane perpendicular to the direction of travel of the light.

In ordinary light it can vibrate in that plane in lines and ellipses wandering in all directions in turn ; anywhere in planes cutting the paper perpendicularly in the upright diameters of Fig. 112.

In plane polarized light it is confined to one fixed direction in that plane, e.g. in the upright diameters of the figure.

§ 483. Passage of light through a crystal.

Suppose some shot set bouncing to and fro across a circular pipe, Fig. 231 (upper). Each can continue to bounce along the

diameter it starts in, because it hits the wall perpendicularly at each end. In an *elliptical* pipe, Fig. 231 (lower), however, it is usually flung back a different way at each bounce and *the only two directions in which vibration can continue permanently are the long and short axes of the ellipse* which are perpendicular to the walls at their ends, and are *at right angles to each other*.

Instead of a sudden blow from a rigid wall it is not difficult to imagine elastic forces applied gradually, with the same result.

Now in a **crystal** (except *cubic*) we know the elasticity differs in different directions.

A ray of ordinary light passing through a crystal splits into two. These two rays when separated are found to be plane polarized, and their vibrations are at right angles.

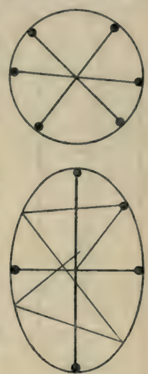


FIG. 231.

§ 484. **Double Refraction.**

They tend to separate themselves, for controlled by unequal elasticities they travel at unequal speeds in the crystal, i.e. are unequally refracted and usually follow different tracks. This *double refraction* is best seen in Iceland spar (calcite), a cleavage piece * of which lying over Fig. 231 produced Fig. 232.

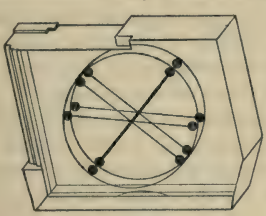


FIG. 232.

As the flat piece is rotated on the page one image—the 'ordinary'—remains fixed, but the 'extraordinary' moves round it and does not obey the first law of refraction.

A prismatic rock-crystal (quartz) held to the eye shows two little overlapping spectra of a flame.

To get a single beam of plane polarized light.

From a large piece of spar the two polarized beams, each half as bright as the original beam, will emerge quite separated, but large spar is hard to get.

The **Nicol Prism** is our best means. It is shown in section in Fig.

* Little bits are cheap at any mineralogist's, or can often be broken out of chemical laboratory stuff.

233. A long 'rhomb' of spar is sawn across very obliquely, polished, and re-cemented with Canada balsam. Now μ balsam $= 1{\cdot}55$ and μ calcite, ordinary ray, $= 1{\cdot}6$, therefore when this ray tries to pass very obliquely into the optically lighter balsam it is totally reflected and thrown aside, to be ultimately absorbed in black varnish on the side of the prism. But the extraordinary ray, μ calcite $1{\cdot}5$, passes through unaffected, for the balsam is optically the denser now.

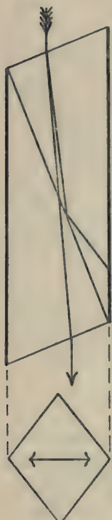


FIG. 233.

FIG. 234.

Schorl or Tourmaline is a dark mineral which absorbs one of the vibrations much more than the other. Therefore if ordinary light falls on a $\frac{1}{8}$-in. slice cut lengthwise from a schorl crystal the dim brown or green light that does get through is plane polarized, Fig. 234.

Most other crystals are but feebly doubly refracting, they polarize the two beams, but fail to separate them to any extent.

In all crystals there is an *optic axial direction* in which no double refraction occurs. It is parallel to the length of a rock crystal; it enters the blunt corner of a calcite rhomb symmetrically to the three faces. Evidently quartz (pebble) for spectacles ought to be sliced up straight across the crystal. Gypsum, sugar, etc., have two optic axial directions. Rock-salt and fluor-spar are cubic and do not doubly refract.

§ 485. **Polarization by reflection.**

There is another quite different way of polarizing light. Light reflected obliquely from any glossy surface (but not from metals) is more or less polarized, and at a particular *Polarizing Angle (of reflection) light reflected from a perfectly clean surface is wholly plane polarized and is vibrating parallel to the surface.* Meanwhile most of the light plunges into the surface and contains a corresponding excess of perpendicular vibration, and *the transmitted*

light is therefore partially polarized. Players of 'ducks and drakes' will easily remember all this.

Tan (polarizing angle) $= \mu$. For water it is 53°, glass $57\frac{1}{2}$°.

Inky water, a dark glass, or a shiny black book laid on the window-sill and looked at, at about these angles from the vertical, makes a splendid polarizer. So does a stack of glass plates, while a dozen microscope slides cleaned up, stuck in a bundle at nearly 60° in a square card tube, and looked through, does as well as a nicol. The reader can puzzle out the virtue of a *bundle.*

§ 486. Now if this polarized light meets a second polarizing arrangement of any sort (called the 'analyser') it will (a) continue unchecked if the direction of possible vibration in the analyser is the same as its own, (b) be dimmed if they are inclined, and (c) be stopped if they are at right angles. A motion has no component at right angles to itself.

For instance, light is reflected again from a second plate when parallel to that which polarized it, but not when turned through

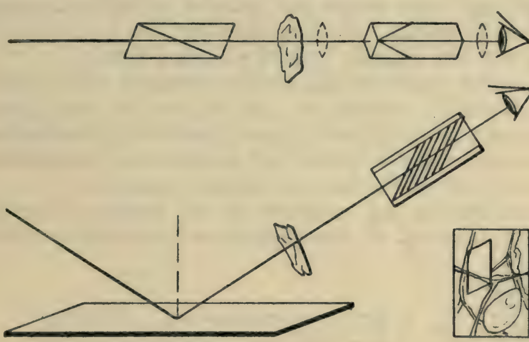


Fig. 235.

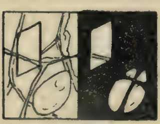


Fig. 236.

90° on the ray as axis. It passes *through* a bundle when perpendicular but not when parallel to the first reflector. 'Crossing' tourmalines or nicols blackens the field of view, Fig. 234. In Fig. 235 at the top is a pair of nicols 'crossed'; below is a polarizing plate with 'bundle' extinguishing reflected light.

§ 487. **'Depolarizing' effect of thin crystalline fragments.**

A piece of a crystal held between 'crossed nicols' appears bright in the dark field. (Try chips of mica between your inky water and analysing bundle, as in Fig. 235.)

The crystal has to split the incident vibration by the usual parallelogram law into two components in its own possible directions of vibration. One component travels faster, it reaches the other side a fraction of a wave ahead of its companion, there is a phase difference, and now when these rays meet the analyser they do leave a component in its direction, i.e. light passes through.

[Cf. Fig. 108, the straight line in the corner has changed into one of the ellipses, which has breadth parallel to the line in the opposite corner, the analyser, perpendicular to the polarizer.]

Thin crystal plates show soap-bubble colours when one colour has gained a whole wave and is extinct again, but thick plates, like thick films, are not coloured [cf. § 427].

§ 488. **Uses of polarized light.**

Thus nicols applied to the microscope are invaluable to the mineralogist, and they enable the biologist to pick out under the microscope crystals, starch grains, etc., in a smother of tissue (Fig. 236 left, ordinary; right, polarized light).

All transparent solids become doubly-refracting under *strain*. Light will gradually reappear in a bit of glass as you squeeze it in pincers in your dark field. Hence glass intended for optical purposes is examined in polarized light to detect any temperature strains, etc., which would warp the finished surfaces.

Certain substances, e.g. solutions of the sugars or tartaric acids, the terpenes and all that contain an 'asymmetric carbon

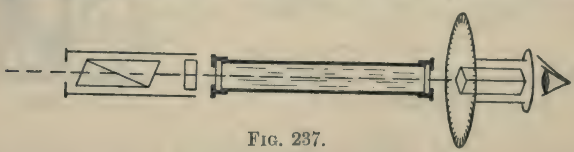


FIG. 237.

atom,' possess the property of *rotating the plane of polarization*, i.e. as the polarized light travels through them, the direction of vibration in it slews round, through an angle to right or left characteristic of the (substance × quantity passed through).

In **polarimeters** the clarified solution is put in a glass-ended tube 20 cm. long, between crossed nicols, Fig. 237; light enters the polarizing nicol on the left and the angle the analyser, mounted in a graduated circle, has to be turned through to restore darkness gives a means of estimating the contained Sugar, say (the substance for which they are largely used). Since the deepest darkness is hard to decide upon, they contain also a crystal device (e.g. a 'biquartz') and the adjustment is to equate the dimness of two semicircular halves of the little field of view in sodium light, or in daylight to bring both from red and blue forget-me-not hues to one 'sensitive' lavender tint.

EXAMPLES.—CHAPTER L

1. Contrast the phenomena observed in the propagation of light and sound. [M.]

2. How can polarized light be produced? State its distinctive properties and some of its uses. [L]m.

3. Describe a Nicol prism, and trace the path of a ray of light which enters it. [M.]

4. What is the peculiarity of sugar solution as regards its action on light? Describe an instrument which utilizes this.

Chapter LI

RADIATION

By **Radiation** is meant the transmission of energy from place to place without the aid of intervening matter.

It is the only process by which energy in the form visible to us as Light can travel; it is one of the three ways in which Heat moves; the radiation of electro-magnetic energy is employed nowadays in 'wireless telegraphy.'

§ 489. In Chapters XXXVIII and XXXIX we have studied the transmission of Light in straight lines, its reflection, refraction, absorption, etc. It can be shown that Radiant Heat behaves very much like Light. That it travels across empty space and in the same straight paths as light, that it can pass through some material substances, and that it cannot pass through others, is tacitly admitted by everyone who pulls down the blind for protection from the sun's fierceness.

The radiation-catching-and-measuring contrivances to be described below will tell us that the intensity of heat received from a small hot body is inversely proportional to the square of the distance from it.

When the conjugate foci for light of a good-sized concave mirror have been found it is easy to show that they are also conjugate foci for radiant heat. For a thermometer, with its bulb held at one, soon begins to rise when there is held at the other a burning match or even a knob of metal only 'black-hot' or a test-tube of boiling water. There is the same need for accurate shape of the reflecting surface as with light, but not for the same high polish.

Lenses refract Light and Radiant Heat both very much alike, though not with exact correspondence, for here there is the possibility of a difference in refractive index and absorbing power of the material for the different sorts of radiation. The bright image of the sun formed by a burning glass is practically its best focus of heat; its burning power is very little diminished

by a red glass held in front, though the light is reduced. Tyndall filled a large hollow convex lens (made of two clock glasses) with a black strong solution of iodine in carbon disulphide. This focussed solar radiation and raised platinum-foil to white heat, yet when he placed his eye at this focus for a moment (taking precautions against superficial burning) there was no sensation of light coming from the black surface. The liquid lens happened to be opaque to visible light but transmitted a 'dark heat' and focussed it in the usual optical way.

Tyndall's experiment is an instance of the conversion of a trifle of the heat-energy into visible light-energy of the incandescent platinum. Conversely, when light falls on a 'dead-black' surface it disappears, but the surface is warmed. In fact, the most general way of studying radiant energy is by its heating effect when caught or 'absorbed.'

§ 490. If a piece of ice is put in one focus of the concave mirror mentioned above, the thermometer goes down. An iced drink in a vacuum-flask, which protects it from all thermal effects except radiation, slowly loses its chill. These are radiations of cold out from the cold body, but may equally be regarded as radiations of heat back from the warmer thermometer, or in from the warmer walls, and this is the way we prefer to look at it (cf. § 155). That not only hot bodies, but bodies at quite ordinary temperatures, can radiate heat under suitable circumstances, is indisputable; for put them in a cold place—outdoors, say, on a clear night when they can radiate to the cold vault of interstellar space—and they begin to get colder forthwith.

§ 491. Does the radiation which has resulted in this loss of heat start only when the body is placed among colder surroundings? Does the laying of a bit of ice near a thermometer excite the bulb to begin radiating in the direction of the ice only?

Is it not much more likely, as suggested by **Prévost** (1792) in his **Theory of Exchanges of Radiation**, that everything is radiating all the time, at a rate dependent on its own temperature—the faster the higher the temperature—and not at all on that of its surroundings? And the surroundings also are radiating at a rate dependent on their temperature. But all the time all the surfaces are also absorbing radiation that happens to fall on them,* and the result is that if a body is entirely surrounded by other objects

* [Compare the action of a liquid surface, which both emits and receives vapour molecules.]

—walls, clouds, etc.—at the same temperature as itself, it gets as much as it gives, and remains at the same temperature. But if in some directions there are colder surfaces in view—ice, clear sky, etc.—it loses more radiant energy in these directions than is returned to it, and it cools. Then part of the surroundings will be warmer and part colder than itself, it will gradually settle down to an intermediate temperature, at which, while radiating in all directions alike at the rate appropriate to its temperature, it will absorb just enough additional radiation from the warmer aspect to make up the deficiency in the return from the colder. Incidentally it forms a temporary resting-place for the balance of radiant energy passing from the hotter to the colder places around.

§ 492. For the above argument to hold good it is evidently necessary that the **Radiating Power** (i.e. the amount of energy radiated per square centimetre per second) of a surface at any particular temperature must be precisely equal to its **Absorbing Power** for radiation sent from other surfaces at the same temperature.

At high temperatures the radiating power is enhanced, and we shall see later that the radiation emitted is not only increased in quantity but also altered in quality, e.g. it may begin affecting the eye as red light, as well as warming the whole face more.

The equality of powers demanded above therefore means that *the rate of radiation from a surface is equal to the rate at which it absorbs radiation of the same temperature-quality.*

§ 493. Radiation falling on a surface may be disposed of in three ways :—

I. Part of it is **reflected** back ; regularly from a highly polished surface, diffusedly from a rougher one. About 80 % of the incident *light*, for instance, is reflected from polished silver or from quicksilver, 'frosted silver' reflects as much diffusedly, white paper 70 %, dark cloth, earth, etc., 5 to 25 %. And the cook knows that the tin interior of a Dutch oven reflects the fire-heat to the revolving fowl even better than it does the fire-light, but that it must be kept clean and bright, i.e. a fair reflector of light, or it will lose its roasting efficiency.

II. The remainder is partly or wholly **absorbed.** Evidently if a large proportion has been reflected there is not much left to be absorbed. A good reflector is therefore a poor absorber

of radiation. And since absorbing and radiating powers at any particular temperature are equal, the *Radiating Power of a good reflector is much less than that of a good absorber when both are at the same temperature.*

We may fairly expect therefore that a white tile with a dark figure on it, or a piece of platinum foil with a spot of iron rust (where an iron salt has been ignited on it) will show a bright spot on a dark less-radiant ground when maintained at a bright red heat, the optically more absorbent surface becoming visibly the better radiator ; and this is actually the case. But seeing that radiation differs so much in quality according to the temperature of its source, does it follow that a good reflector of daylight, which is the radiation from a sun at 6000°, will be necessarily a good reflector and therefore poor radiator at low temperatures—will a copper kettle radiate* any the less warmth because it happens to be brilliantly polished ?

Very often it may be said that once a good reflector always a good reflector, and once a good radiator (absorber) always a good radiator (absorber), but different materials do offer remarkable differences in the way they deal with different sorts of radiation. A hardly visible film of lacquer will treble the rate of radiation of hundred-degree warmth from the bright kettle, or will increase the rate at which poker and tongs warm up as they lie in the hearth ; the shining bulb of a thermometer heats faster in sunlight than the brilliance of the quicksilver inside the glass would lead us to expect, transparent quartz and fluor-spar reflect certain portions of the infra-red spectrum almost totally, while polished silver reflects the ultra-violet only partially.

III. The remainder, that is neither reflected nor absorbed, filters through or is **transmitted**, the material being 'transparent' (*light*) or 'diathermanous' (*heat*) to that particular sort of radiation.

It is here that the most striking differences are noticeable. The selective transmission of visible radiation, which gives rise to colour, has been dealt with in § 420. There is a similar selection for the dark radiations of the infra-red or the ultra-violet. Thus a 6-in. trough of clear water is frequently employed to filter out the heat (infra-red) from an arc lamp and prevent damage to the micro-slide, etc., which it is desired to project on the screen. Carbon disulphide, just as transparent to light, would let the

* Not the total rate of cooling, see § 175.

heat through freely. An aqueous solution of black dye can stop everything but a trace of radiant heat, being both opaque and 'athermanous': the diathermancy of an opaque solution in carbon disulphide has been mentioned in § 489. The absorbed radiation warms the water, while the other liquid of course remains cool.

§ 494. Easily performed experiments are these :—

(1) Holding one hand a few inches in front of a bunsen flame, open and close the air-holes of the burner. You will feel that the luminous flame sends more warmth to the hand than the non-luminous flame, in spite indeed of the less perfect combustion giving it a lower temperature (1500° instead of 1750°). The luminous flame is nearly opaque, i.e. it absorbs light and heat strongly, hence it also radiates strongly. The 'atmospheric' flame is nearly transparent, i.e. it absorbs but little, consequently it can radiate but little. In fire-place gas-stoves an opaque solid, asbestos, is heated and radiates much more than could the hotter clear flame alone. That transparent diathermanous gases possess no radiating power is easily observed by looking through a white-hot tube, the air inside is as invisible as ever; and the air near the carbons of the electric arc remains quite clear.

(2) A transparent bead of fused borax remains clear and almost invisible while the encircling wire of opaque platinum is glowing red. The solution in it of a trace of copper makes it at once a nearly opaque fiery-red mass, which cools to a glass partially transparent, but greenish, i.e. absorbing just that red light which it radiates vigorously when heated.

(3) Using a thermopile (see below) facing a vessel of hot water, it is easy to show that thin opaque ebonite obstructs radiant heat less than does clear glass.

The Quantitative Study of Radiation

§ 495. **Radiation meters.**—To study radiation quantitatively we must first have some means of catching and completely absorbing radiation of all sorts. Lampblack (the fine soot of burning oil) is much the best absorbent, and when properly applied to a surface enables it to absorb perhaps 99 % of the incident light and heat. A patch of sunshine on the sooty black of a fire-place is quite plainly visible, however, and shows that the absorption is not total. A surface covered with a deep velvety pile is better, as the radiation is scattered by sidewise reflections

before it can reach the flat surface which might have reflected it back to its source, but there is a difficulty in measuring the heating of the velvety mass and usually we have to be content with the lampblacked flat surface.

The oldest way of detecting the warming of the black due to the radiant heat and light it absorbs is by spreading it on the bulb of a little **air thermometer**, conveniently the **differential** pattern of Fig. 238, in which an index-thread of liquid moves in the narrow stem between the black bulb and a second bulb kept out of the way and cool. Ether and its saturated vapour make more sensitive filling for the instrument.

FIG. 238.

A more accurate and convenient means is by measuring the current of electricity produced when the junction of two different metals is heated. The current-measuring galvanometer is a fairly delicate one such as described in § 611 ; the **thermo-electric junction** calls for description here. Bismuth and type-metal are the two metals which show the greatest effect, but as the heating may be very small the effect is often magnified by having 50 or 100 soldered junctions in succession, and pairs of little bars are packed (with varnish or mica insulation, shown black) into a block perhaps an inch cube, called a thermo-electric pile or **Thermopile**, arranged as in Fig. 239. The current zigzags through

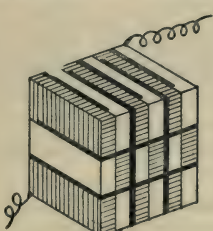


FIG. 239.

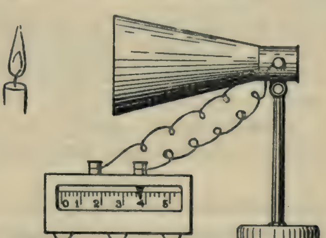


FIG. 240.

the pile, being driven always from bismuth to 'antimony' at the warm junctions (which are gathered at the face of the pile and blacked over, while the inevitable alternate antimony-bismuth solderings are kept at the back in the cool). The pile is embedded in plaster and mounted with a funnel in front to protect it from stray sideways radiation. Fig. 240 shows it

receiving radiation from a candle and actuating a commercial micro-ammeter.

This massive pile of badly conducting brittle materials is being superseded by a more modern thermopile in which a thin wire is built up of half-inch lengths of copper and nickel-copper alloy ('eureka,' § 619), hard-soldered together, and the solderings beaten thin and blacked. The wire is twisted into figures of eight and only the middle row of junctions is exposed through a slit in a sheltering wall of cork to the radiation, Fig. 241.

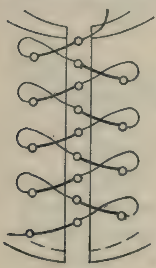


Fig. 241.

In Boys's Radio-micrometer the single bismuth-antimony junction was directly attached to the moving coil (of 1 turn) of the galvanometer, which would deflect visibly for the heat from a candle 100 ft. away.

The **Bolometer** is a platinum resistance thermometer (§ 628) in which one of two very slender strips ($1 \times \cdot 05 \times \cdot 0004$ cm.) of blackened platinum, stretched side by side, is exposed to the radiation. The strips are 'in opposite arms' of a Wheatstone-bridge arrangement (§ 627) and the additional warming of the exposed strip increases its electrical resistance, upsets the electrical balance, and causes a proportional galvanometer deflection.

Over this instrument a spectrum with its lines parallel to the strip can be slowly moved, and the photographic record of galvanometer deflections gives an exact measure of the intensities of every bright and dark line. And the bolometer is so sensitive that it has measured, within a few per cent, the heat value of the radiations collected by a large telescope from various of the fixed stars.

Another radiation measurer is the **Radiometer** of Crookes. The little spinner with its four aluminium vanes, each bright on the forward side and black on the back, turning slowly in its vacuous globe in the daylight, or buzzing round at a great rate in the sunshine, is a familiar ornament of the optician's window. More delicately, for measurements, the vane hangs by a quartz fibre, and radiation falling on the blackened side deflects it to a small extent measured by attached mirror and lamp-and-scale. The action of the instrument is that the blackened surface becomes heated, and warms the air molecules that strike it, i.e. increases

their speed, and the recoil as they are driven off faster pushes the plate backwards.

§ 496. **Relation between radiation and temperature.**

It was pointed out in § 172 that Newton's Law of Cooling was derived from experiments on cooling in a draught, and in § 175 that this air convection accounted for at least seven times as much loss of heat as did pure radiation, at moderate warmths. Strangely misconceived attempts were made, however, to apply this law to pure radiation. As a matter of fact, for quite small differences of temperature, the law is a mathematical 'first approximation' to any number of possible true laws. Between 15° and 30° C. say, when the starting-point is −273° C., there would be cause for surprise if it did not hold fairly well, but pushed to an extreme it led to such outrageous results as a temperature of 7,000,000° C. for the sun!

Various experimenters cooled hot bulbs, electric wires, etc., in more or less defective vacua, and devised formulæ to express the dependence of radiating power on temperature, but these formulæ were all empirical, i.e. they fitted the results of a particular set of experiments but nobody could find any theoretical basis for them.

It was left to Stefan in 1879 to observe that an old result of Tyndall's, that a platinum wire at 1200° C. radiated 11·7 times faster than one at 525° C., agreed with what is now called **Stefan's Law—The rate of radiation from a fully radiating surface is proportional to the fourth power of its absolute temperature.**

For $(1200+273)^4/(525+273)^4 = 11{\cdot}6$.

§ 497: Experiments made to test this law were rather contradictory at first, but we now know that the difficulty lay in getting a **'full' radiator.** This is a surface best described perhaps by its converse property, that it is a complete absorber of all kinds of radiation, reflecting none; a **'perfectly black body.'** Hence it would give out every sort of radiation in full proportion according to its temperature, without bias or selection of any particular sort.

We have seen that lampblack is not 'perfectly black.' But the tiny deep cavities between the fibres of velvet are dark, a keyhole is always dark, the pupil of the eye is black, the bunghole of an empty barrel appears utterly black, even if the barrel had contained white lead. That is, a small hole in the side of a large

closed cavity acts as a perfectly black body. Conversely it acts as a full radiator when the walls of the cavity are kept at the same temperature all over.

The radiation escaping from the hole comes from the opposite wall of the oven; whatever of full radiation this wall cannot emit because of its defective radiating power, it reflects diffusely from the radiation falling on it from the rest of the walls. A mass of glass inside the oven, between radiating partially on its own account, and reflecting and transmitting radiation from the walls, would likewise send full radiation out of the small hole.

Indeed, things become indistinguishable when inside a closed cavity with walls of uniform temperature. In a long-closed room, without lamps or heaters—a closed chamber at 285° A.—the eye is useless and the hand cannot detect warmth or coolness radiated from anything, one has to depend on other senses. Introduce a closed stove (few hundred degrees) or a lamp (1500°), or open a shutter to daylight (diffused sunlight 6000°) and temperature-sense and sight regain their usefulness. In the depths of a fire only the closest scrutiny can detect nails, bits of white crockery, or flakes of transparent mica when shut in on nearly all sides by the glowing coals that themselves have lost their outlines.

If it is not sufficiently evident that all the contents soon settle down to the temperature of the walls of the cavity consult § 558, reading ' temperature ' in place of ' potential ' and ' lines of flow of heat ' instead of ' electric lines.'

Lümmer and Pringsheim therefore constructed ' black bodies ' on this principle. One was an oven of thick copper with a small hole in one side and heated by steam or flame gases. Another was a long porcelain tube wrapped round with nickel wire; an electric current through this heated the tube and a contained lump of porcelain to bright redness, the glowing interior was viewed through a hole in one end. For the highest temperatures the tube was of carbon and the current passed from end to end through its walls, heating them and a central carbon lump to brilliant incandescence.

A bolometer faced the aperture, and the results, between 100° and 1535°, tallied with Stefan's Law, with an extreme discrepancy of only 3°.

Theoretical proofs, thermal and electrical, have also been given for the Law, and it is now established.

§ 498: In these experiments, as was expected, the radiation received from the small aperture was found to vary inversely as the square of the distance. But the image of the (comparatively distant) hole formed by a particular concave mirror will always have the same intensity (§ 471), for its area also varies as $1/d^2$ and the two effects cancel. In the **Féry Radiation Pyrometer** a gilded concave mirror about 6 cm. diam. and 8 cm. focal length forms the image upon either a thermo-electric junction or the miniature compound spiral of Fig. 62. In use the pyrometer is set up to face the 2-in. hole in the wall of kiln or furnace and the sensitive disc at the focus is examined through an eye-piece perforating the middle of the mirror (the instrument looks like a diminutive Gregorian telescope) to see that the image entirely covers it. Then the whole disc is receiving radiation proportional to (effective temperature °A.)^4 and the pointer reading is quite independent of distance, giving the radiator's effective temperature whether e.g. peering into a pottery kiln or pointing at the sun.

How near these effective temperatures are to the actual temperatures depends on how nearly the surface approximates to the 'black body' or 'full radiator' in its radiating character. Kilns and furnaces do so admirably, but a gas mantle, which reflects white light when cold, emits very 'selectively' an undue proportion of visible radiation, and its temperature would not be accurately obtained. And the cool dark absorption bands in the spectrum of the solar radiation show that it is not 'full.'

§ 499. Thus far Radiation as a whole, now to analyse it.

Discarding glass as too absorbent, a spectrometer is built up of lenses and prisms of fluor-spar, or better, rock-salt, the most transparent and diathermanous of solids. Or else a diffraction grating ruled on a concave metal mirror is used to produce sharply focussed spectra, and lenses are dispensed with altogether.

The spectrum now extends far beyond the visible part (wave length ·76 micron * red to ·38 micron violet) up through the Ultra-Violet where it is photographable to wave length ·2 micron, or source, grating and all in vacuo to wave length ·1 micron, and down through the Infra-Red, the old 'Dark Heat.' Ordinary photo plates are sensitive to blue, violet, and ultra-violet, but notoriously blind to green, yellow, and red. Dyed with certain dyes, however, many commercial 'chromatic' plates now respond to the red, and Abney, and later, Wood, have prepared and used

* A micron is a thousandth of a millimetre.

plates sensitive down to w.l. 2·3 microns. Below this, and always, there is the invaluable bolometer, § 495.

§ 500. As everyone knows, radiation of ‘ dark heat ’ begins at low temperatures and only spreads into the visible region of the spectrum at a ‘ red heat,’ about 500° C. In the spectrum of fire-redness, 600° to 1200° C., green asserts itself, above this blue and violet and then ultra-violet come in, and at the highest temperatures of limelight and arc we get a fair imitation of noonday whiteness.

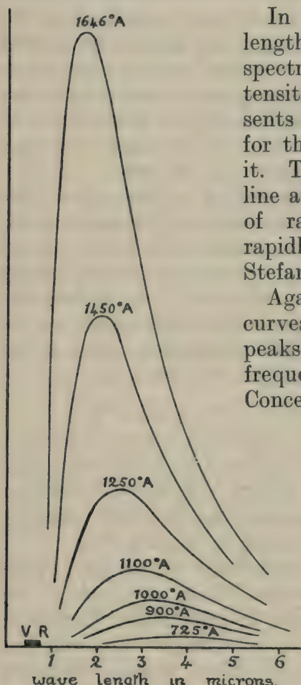


Fig. 242.

In Fig. 242 the abscissæ are wave lengths [VR shows extent of visible spectrum] and the ordinates are the intensities of radiation. Each curve represents the composition of ‘ full radiation ’ for the absolute temperature marked on it. The areas between curve and base-line are proportional to the total amount of radiation, and show plainly how rapidly this increases, in accordance with Stefan’s fourth-power law.

Again, as the temperature rises, the curves acquire sharper peaks, and these peaks are moved towards the higher-frequency shorter wave length violet. Concerning this there is another law, known as **Wien’s Law—In full radiation the frequency of vibration of the radiation which is being emitted in greatest quantity is proportional to the absolute temperature.** Or

$[\text{wave length}]_{max.} \propto 1/T^{\circ A}$

And from Wien’s law and Stefan’s it follows that *the intensity of the maximum-intensity radiation* (*height of the peak*) *is proportional to the fifth power of the absolute temperature.*

As the peak travels violet-ward with rise of temperature the colour of the light changes ; ‘ red,’ ‘ cherry-red,’ etc., are familiar in judging temperature. In the **Wanner Pyrometer** a tiny glow

lamp is held in front of the glowing background and the lamp current regulated till filament matches background in tint and brilliance and virtually disappears. The attached ammeter is graduated to read temperature direct.

If the positions of the peaks can be determined, Wien's law gives the relative temperatures. In this way an estimate of 6200° A. has been made for the solar temperature, and 23,000° for the 'variable star' Algol (with radiation 40 times as intense as the sun's).

§ 501: Modern lamps afford instances of the practical bearing of this. The trouble with candles, flat gas flames, and carbon-filament electric lamps has always been that their temperature is so low (1500° C.) that 99 % of the radiation comes off as invisible heat (see Fig. 242) and only 1 % is visible light. In regenerative gas burners the air was heated as it entered, the flame was hotter and whiter, and the luminous efficiency was nearly doubled. The acetylene flame is likewise hotter, whiter, and more efficient.

The 'intrinsic brilliance,' i.e. the candle-power per square centimetre of radiant surface, is 86 for a carbon glow lamp: for the carbons of the arc, which are at double the absolute temperature, it is about 2^5 times as much, or 3000. Unfortunately the maximum emission is still of invisibly long wave length and the luminous efficiency is only 4 times as great. [The luminous efficiency of a full radiator should be 1 % at 1500° C., 3 % at 2000° C., and 30 % at 4000° C.] The intrinsic brilliance of the sun (6200° A.) is 90,000. That of a candle is only about ·66, and of acetylene gas 6. The candle flame is as hot as the carbon filament, hence if its light is due to incandescent carbon particles these occupy only 1 % of the flame, if due to gaseous radiation it shows what poor radiators gases are at best.

§ 502: The tungsten-filament lamp is at 1900° C. but its efficiency, 3·5 %, exceeds the 3 % of a 'full' radiator at 2000° C. Hence we must conclude that it radiates to some extent 'selectively.' A more marked instance of **Selective Radiation** is the gas mantle, which below 1700° C., with an intrinsic brilliance far less than the tungsten wire's, yet gives a much whiter light. The 'rare earths' of which it is made characteristically give bright band spectra, almost as do vapours (§ 415) and the gas mantle has been developed to have a broad 'bright band' from yellow to violet in which it concentrates a much greater proportion of its total radiation than mere temperature would warrant.

Carrying the principle to an extreme, the selective orange and green radiation of the glowing vapours of sodium and calcium is used in ' flame arcs ' and doubles their luminous efficiency, and mercury vapour is an equally economical green-and-blue radiator in mercury-arc-lamps ($\frac{1}{4}$ watt per c.p.).

Per contra, sunlight is not so rich in ultra-violet as it should be at 6200° A. The atmosphere absorbs much of it: in clear weather and on mountains there is notably more and it helps cause sunburn and snow-blindness.

§ 503. The infra-red spectrum must of course extend to and below the radiation which according to Prévost's theory is constantly going on from everything at ordinary temperatures. Looked at in this way, the idea that we and all around us are constantly emitting portions of infra-red spectrum is a little startling ; why all this rushing about of radiant energy ?

But does one realize the vast outpourings of radiation when it comes to a question of maintaining a little illumination, say ? Stand on a Thames bridge at night, look at the rows of lamps and think of the hundreds of horse-power working to maintain their radiance—and the end of it ?—a little unheeded warmth, a faint glimmer over a few acres of river beneath, and the dullest brown tint to the darkness overhead. Think of the coal and oil trade of the country, half of it goes to keep a few million little boxes, called rooms, carefully contrived with non-conducting walls and the minimum of ventilation, a few degrees warmer than the weather for half the year, and to give them about the least illumination our marvellously sensitive eyes can work with for a few hours a day.

Then think of the power that lights the country-side with many thousand times that brilliance, which combats, for the whole earth, the cold of interstellar space, and reflect that the heritage of every child born into the world is 200,000 horse-power for life.

The enormous increase expressed by the fourth-power law is well exemplified by this, that a little disc at 6200° A. putting in its appearance in the sky for a few hours a day, is able to renew all the radiation that the earth's surface, at about 290° A., loses to the whole cold vault of sky in the twenty-four hours.

Indeed, the radiation, low down in the infra-red, from objects of ordinary temperatures, is comparatively so slight that it is only by utilizing selective line-radiation from a silica-glass mercury lamp that Rubens has been able to get beyond wave length

·1 mm. to wave length ·3 mm.—only ten times shorter than has been produced electro-magnetically.

Rubens studied, *inter alia*, radiation of wave length 8·85 microns, which he was able to pick out by repeated reflections from quartz, suddenly very ‘ athermanous ’ at this wave length. It is said that much of the earth's radiation back into space has a wave length near this, and that a millimetre thickness of water obstructs it greatly.

§ 504. **Theory of the Greenhouse.** Clear glass is transparent to visible radiations but opaque to almost all others. Hence the sun's radiation, largely of visible frequencies, passes through a greenhouse roof and falls upon and warms plants, soil, etc., beneath. Thereupon these emit the low infra-red radiations appropriate to their temperature, and these radiations are trapped by the glass, which either absorbs or reflects them. Rock-salt, on the other hand, would transmit them freely. Now Wood has recently exposed to the sun miniature greenhouses, roofed with glass and with rock-salt, and has found no significant difference of temperature between them. Evidently there is some common action greatly outweighing any trapping effect.

We saw in § 175 that at moderate temperatures seven-eighths of the loss of heat was not due to radiation at all, but to convection. If convection currents are hindered from getting away the rate of loss of heat is vastly reduced. It is the imperfect ventilation of a greenhouse then that amply accounts for its contents retaining a higher temperature than that in the free air outside.

§ 505 : **Steam power from sunlight.** Temperatures higher than are desirable in a greenhouse are frequently attained by objects outside. On a calm sunny day a painted garden seat gets too hot to sit down on. The gardener whitewashes the greenhouse roof in summer to reflect the sunlight and save the plants from scorching.

Consider a square centimetre of ‘ black ’ surface directly facing the sun. Experiments have shown that it receives about 2 calories per minute. It rises in temperature until convection and radiation carry away the same amount, when its temperature becomes stationary. If quite protected from convection it rises to a much higher temperature at which it *radiates* 2 calories per minute to a hemisphere of sky, etc. Applying Stefan's law (and ignoring the small t^4 of sky) this temperature can be worked out as about 150° C.

In experiments in progress in California water is irrigated over the black floor of a large shallow tank; a double glass roof transmits the solar radiation and prevents convection loss. The very hot water flows off into a turbine where it boils under reduced pressure. The steam is condensed in cooler water from a large underground tank and the contents of this are irrigated at night so as to cool by radiation and, if permissible, evaporation [or, for pumping plant, the well water is used]. In another scheme the hot water flows over pipes containing liquid sulphur dioxide under pressure, this boils and drives an engine and is condensed again in cooling pipes kept in the shade and in a fan draught. 20 h.p. and more has been obtained.

In older experiments a huge built-up parabolic mirror reflected the sunlight on to a little boiler at the focus. Practically, scores of suns shine on the boiler, and it soon supplies high-pressure steam.

§ 506: **Mean temperatures of the planets.** It has been stated above that a black surface exposed normally to the sun's rays can rise to 150° C. = 423° A. Poynting has similarly calculated that the whole earth's surface, taking into account the varying obliquity of the sun's rays due to latitude and time of day, and the absence of the sun by night, should have an average annual temperature about 290° A., and this is not many degrees from the truth. Similarly the temperature of Venus comes to 343° A. = 70° C., and that of Mars to 237° A. = −36° C.

§ 507: **Radiation pressure.** The pressure exerted on a surface by a stream of energy falling on it has been mentioned in § 290. The pressure exerted by a strong beam of light has been measured, though it is exceedingly small. A radiating body will of course experience a reaction pressure. No resultant force acts on a body radiating, and being radiated to, equally in all directions, but two hot bodies among cooler surroundings will repel each other.

Consider a swarm of meteorites approaching the sun. Particles are attracted to one another by gravitation, but are kept apart by the sunlight reflected from one to the other, and by their radiations as they absorb the solar heat. They are attracted to the sun with a gravitational force proportional to their masses ($\propto$ diam.^{3}) but repelled from it with a radiation force proportional to their areas ($\propto$ diam.^{2}). Below a certain small diameter the repulsions must exceed the attractions, and the smaller dust is driven out of the swarm and repelled directly away from the sun in a broad streaming comet's tail.

EXAMPLES.—CHAPTER LI

1. Explain Prévost's theory of exchanges of radiation. Consider the special precautions required to get the correct temperature of the air or any transparent medium.

2. Upon what does the rate at which a body radiates heat depend? Prove from general principles that the radiating power of a body is equal to its absorbing power, and describe an experiment which illustrates this relation. [L.]

3. Describe an experiment to show that the sums of the emitting and reflecting powers of different surfaces for heat radiation are equal. Distinguish between the absorbing power of a surface and the absorbing power of the interior of a solid. [L.]

4. How can it be shown experimentally that the heat radiation from a hot body obeys the same laws of reflection and refraction as light? [L.]

5. Mention three facts bearing upon the similarity in character between light and radiant heat. Describe generally the change in character of the radiation from a body, as it is raised from the ordinary temperature to white heat. [L.]

6. How would you show that the amount of heat radiation received from a surface depends (a) upon the nature of the surface, (b) upon its temperature, (c) upon the distance of the receiver? [L.]

7. Why on a frosty night is it often colder in the valley than on the neighbouring hill-sides? [L]m.

8. How would you show that a large amount of the energy radiated by a gas flame consists of non-luminous heat rays, and how would you measure the percentage stopped by a sheet of glass? [L.]

9. What experiments would determine whether a glass or a 'pebble' (quartz) spectacle lens absorbed less radiation from a body below red-heat? [L]m.

10. By what experiments would you show that the radiation from an electric arc extends beyond both ends of the visible spectrum? In what respects do these invisible radiations differ from the visible? [L]m.

11. How would you test whether a source of light is rich in ultra-violet rays? Why does sunlight vary in its content of ultra-violet from day to day? [L]m.

12. What difference will there be in the general shape of the curves of radiation of a black body according as they are drawn to a wave length or a frequency base-line?

13. Describe some form of radiation pyrometer for measuring very high temperatures. How would you propose to test the accuracy of such an instrument? [L.]

MAGNETISM

Chapter LII

MAGNETS AND MAGNETIC MATERIALS

§ 508. It must have been a Palæolithic discovery that there was a sort of black stone that had the power of attracting and holding little fragments of itself. The name by which we now know this rich ore of iron—Magnetite—appears to be derived from a locality so prolific in minerals as to have conferred its name also on two others (magnesia, manganese). In Britain it was the **Lodestone** because it led fragments to itself, or perhaps because a rod of it hung by a hair would turn and point northwards, towards the steadfast leading- or lode-star of the mariner.

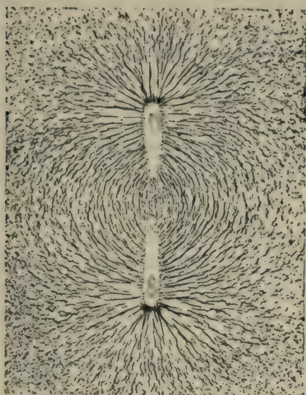
Steel rubbed by the lodestone acquired its powers, and being more workable than the hard brittle stone must soon have replaced it in the **Mariner's Compass,** a specimen of which was brought to Europe from China in 1260.

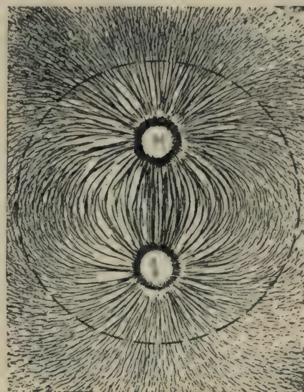
Examining the attractive power of a Magnet—the lodestone, or a piece of steel rubbed with it—one finds it concentrated in parts called **Poles**, whereto iron nails and filings thickly cling and toy compasses vigorously point. There is usually a strong pole near each end, but there may be others, called **consequent poles,** anywhere, often where the steel has been casually touched by the magnet.

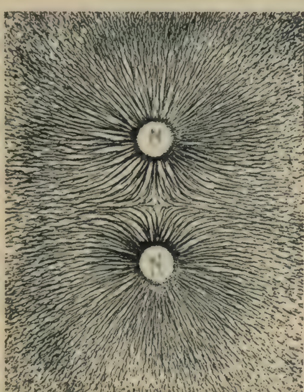
§ 509. **'North' and 'South' poles.** Taking henceforward the usual steel magnet with a pole near each end, it will be found that while both attract iron (and if movable are attracted towards it, by the third law of motion), one will attract and the other repel one end of another magnet. They are evidently of opposite 'polarity' and are distinguished as North and South.

The bar shows no signs of magnetism at its middle, and one would expect that breaking it there would leave the one original

pole on each half. But new poles instantly develop at the broken ends—opposite poles, for the ends cling together—and each half







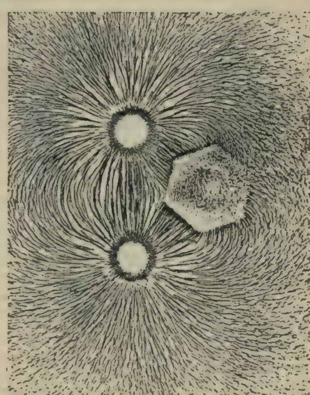


FIG. 243. FIG. 244.
FIG. 245. FIG. 246.

is now a complete magnet. In fact a piece of magnetite, or of steel hardened to brittleness and magnetized, can be powdered up and every particle will of itself cling to iron, being a complete little magnet.

Apparently the stream of magnetism runs right through the bar, but only shows itself where it enters and leaves.

In agreement with this is the fact that *the amounts of North and South magnetism in a magnet are always equal to each other*, whether collected into two poles or scattered among several consequent poles. For a magnet set afloat on a cork turns and sets itself N. and S. and makes no further movement, whereas if one of its magnetic charges were stronger than the other, the action of the great magnet, Earth, on that charge would be greater, and would drag the magnet bodily along north or south.

§ 510. We can follow the stream of magnetism as it spreads out *from the North pole* into the surrounding space—the magnet's 'field'—where it gives rise to all the various magnetic actions. Fine iron filings are sprinkled on a card laid on the magnet, the card is gently tapped and the filings arrange themselves in lines which are stream-lines of the magnetic flow in that particular plane in which the card cuts the magnetic field. In the photograph Fig. 243 the card is lying on a bar magnet and in Fig. 244 on the poles of a vertical 'horseshoe' electro-magnet.

The stream that flows in at the S. pole is just the stream that left the N. pole ;* we have already found it convenient to think of the stream as continuous right through the steel ; it follows then that each magnetic line is a closed endless loop.

NOTICE *that the lines never cross one another*, for that would mean flowing in two directions at once at the same place.

Fig. 245 shows two N. poles, each sending out its own streams of lines; the two sets never mix.

In Fig. 246 an iron nut has been placed in the field, notice how the lines bend round and crowd into it, evidently they find it easier to run through iron than through air, so much easier that the filings show hardly any flow on the card (in the air) just above the nut.

§ 511. **Magnetic shielding.** Few emerge into the hollow middle of the nut, it is easier to run round in the iron than to jump across, the thick iron shell shields the space inside it from outside magnetic influence.

This plan of surrounding a space with thick iron walls is the only known means of keeping out external magnetic influence. Delicate galvanometers sometimes have to be protected from

* All north poles and north polarity drift down the stream. South poles and south polarity 'travel up against it.'

electrical machinery by a close-fitting jacket of 'soft' iron; a small dynamo-room was prevented from interfering with a magnetic observatory by building round it a double wall filled with scrap iron, but really effective shielding on the large scale demands too much iron. The conning tower of a battleship is a poor place for a binnacle, but a quarter or more of the earth's magnetic force still pervades it.

§ 512. **Magnetization by Induction.** The lines crowding in and out of the iron give it the appearance of a magnet; this is still better seen in Fig. 247, where a wrought-iron bar has been placed in the magnetic field. And by trial we find that for the time being it *is* a magnet. An iron nail, for instance, held with one end near a magnet pole, will pick up pen-nibs, etc., on its far end, though they fall when the magnet is removed. A yard of wire rope will carry the magnetic stream round from a magnet to a compass which previously was but little affected.

These things are **magnetized by 'induction,'** magnetization is induced in them.

This explains how a magnet pole (N., say) which attracts a S. and repels a N. pole and should presumably have no effect on

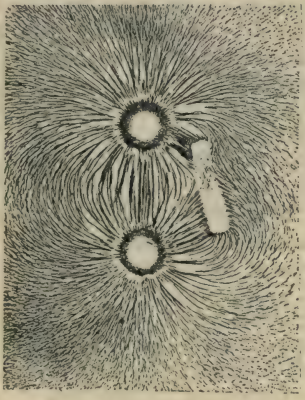


FIG. 247.

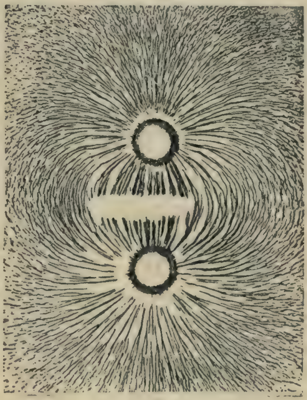


FIG. 248.

a neutral body, yet always attracts ordinary unmagnetized iron. Lines from the pole crowd into the iron, inducing an opposite pole, and these two poles attract each other.

Every little filing in the field becomes an 'induced magnet' and sets itself head to tail with its neighbours, hence the continuous dark lines of them.

The Earth is a magnet, and all iron upon it is more or less magnetized by its induction. In this country its magnetic lines run steeply downwards slightly west of N.; holding a poker N. and S., or vertical, or best in the described direction, and hammering it, it will acquire and retain magnetism enough to quickly affect a compass and perhaps to pick up filings.

Hammering always seems to help, one says vibration 'shakes up the molecules' as tapping shook up the filings, easing their friction on the card and enabling them to turn into line. Vibration in a contrary field reduces magnetization. Steel ships built N. and S. are a magnetic nuisance to their navigators until a year's roundabout voyaging has shaken out most of their acquired magnetism.

But iron placed right across the lines as in Fig. 248 (e.g. a girder east and west) does not get magnetized. For the lines would gain nothing by turning quite at right angles to run along it, and the extra facility of traversing its small thickness is not sufficient inducement to bring many lines out of their short direct courses, i.e. the lines do not crowd in, but pay little more heed to it than to a bit of wood; it shows no magnetic difference from its surroundings, it is not perceptibly magnetized. It is impracticable, for instance, to permanently magnetize a thin steel plate to have one face all N. and the other all S. pole; it is difficult to strongly magnetize a bicycle ball; it is many hundred times easier to magnetize a rod lengthwise than crosswise, whatever direction the poker be held in when hammered its poles will be near its ends and its magnetization practically parallel to its length.

§ 513. **Methods of magnetizing.** *All* magnetization is effected by induction. The commonest process, that of stroking the steel from end to end, always one way, with a magnet pole, is simply exposing every particle of it in succession to a strong field. If instead of the solid a tube full of steel filings be used, they can be seen turning to point to the pole as it passes: it coaxes them all down one way, and the tube (or rod) showing the sum total of all these little (molecular) magnets exhibits on the end at which the magnet left, opposite polarity to the stroking pole.

[Subsequently shaking up the tube jumbles the steel filings together and obliterates their united effect, though each may remain magnetized.]

In the extra work you do in separating the inducing from the induced pole is the source of the magnetized steel's store of potential energy which enables it to turn compass cards, move iron, or drag itself towards it.

The objection to this way of propagating magnetization is that the outer layers act as magnetic shields to the inner, and a bar ¼ in. thick will have a core of scarcely magnetized steel. Hence it used to be the practice to build up large magnets as bundles of thin strips, each separately magnetized.

Now that large electro-magnets are available a better way is to place the bar to join their poles (packing in any gap with lumps of iron) and tap it. The intense magnetic stream flows through the bar and thoroughly magnetizes it.

Magnetization by circulating electric currents must be deferred till later, we may only mention here that strong local magnetization in native magnetite (constituting it lodestone) is usually ascribed to lightning having struck near it, for the great bulk of the ore, though magnetizable, is not naturally magnetized.

§ 514: **Magnetic Permeability**. The ratio of the number of lines which flow through 1 sq. cm. cross-section of a long rod of iron, etc., placed along their natural course, to the number flowing if the iron were not there, is called the **permeability,** P, of the material.

The lines referred to are 'unit' lines, to be defined in § 525. Their number per square centimetre in the iron is called the 'density of induction' and in air is the magnetizing 'field strength.' Thus *Permeability = induction density ÷ field strength.*

Magnetizable substances therefore possess a permeability greater than 1. Some average values (for an 'induction density' about 5000) are as follows :—

Soft cast iron (for dynamos)	500
Soft malleable iron	1000 to 2000
Cobalt	150
Heusler's alloy (Mn.Al.2Cu) annealed	160
Manganese steel (hard white for tramway points)	1·5
Steel, pianoforte wire (retentive)	160
,, 'glass hard' (retains much)	80
Chilled cast iron ,, ,,	50
Nickel ,, ,,	200
Magnetite ,, ,,	4

§ 515: **Para- and dia-magnetic substances.** The very intense magnetic fields obtainable between the pointed pole-pieces of a great electro-magnet disclose a feeble magnetic activity in almost

all substances. Those that behave like iron, having a permeability greater than 1, have been called **paramagnetic,** but there are others that behave oppositely and have a permeability less than 1, these are **diamagnetic.** Some permeabilities are :—

(Para)magnetic—

Air (compared with vacuum)	1·0000004
Oxygen	1·000002
Liquid air (runs up tube to strong poles) . .	1·0025
Ferric salt solutions (·1 gm. Fe per c.c.) . .	1·00033
Ferrous ,, ,, ,, ,, . .	1·00026
Ferro- and ferri-cyanides	nil
Igneous rocks (containing disseminated magnetite)	·0001 to ·036

Diamagnetic—

Water	1 − ·00001
Bismuth	1 − ·0002
Some bismuth-tin alloys (greatest known) . .	1 − ·002

The little rod of bismuth, since it refuses to pass the magnetic stream as readily as air, gets turned aside into the position where it will cause the least hindrance, i.e. (cf. Fig. 248) it sets across (*dia-*) the field, placing its ends in the weaker outer parts.

§ 516. **Temporary and permanent magnetization.**

In soft malleable iron magnetization is easily induced, P averaging 1000 to 2000, but it vanishes immediately the magnetizing influence is removed (except see below). This power of quickly acquiring and losing magnetism is made great use of in electro-magnets.

In hard cast iron, chilled cast iron, and tool steel, especially when very hard, magnetization is far less easily induced, P averaging 80 to 160, but now a large proportion of it is retained ‘ permanently ’ ; though warming, knocking about, the proximity of contrary magnets, etc., gradually enfeeble this permanent residue.

If, however, provision is made for the magnetic stream to circulate entirely through plenty of iron the magnetization is retained much better. The soft-iron ‘ armature ’ or ‘ keeper ’ should be left on a horseshoe magnet, and bar magnets are best kept in pairs, opposite ways, with keepers on both ends. Even a soft-iron electro-magnet may refuse to let go of a thick armature to which it has once actually stuck, until a slight blow or vibration causes separation and demagnetization. A piece of paper between poles and armature usually interposes enough ‘ air gap ’ in the iron circuit to prevent this unreliable action.

The explanation is this, the armature gathers into itself most of the lines from the poles and leaves but few to spread in the air and proclaim their existence. Now these lines were running back past the magnet in the opposite direction to the stream of magnetism inside it, i.e. they tended to demagnetize it: the armature therefore prevents this suicidal tendency.

Iron in the field not only gathers together the magnetic stream but actually increases the total flow, having made the circulation so much easier. Electrical engineers therefore build their machines of massive soft iron with as little air gap as possible, and save much of the power otherwise required in the 'field-magnet.'

§ 517: **Temperature.** At a red heat iron is more easily magnetizable, but at 780°, a 'cherry red,' it suddenly loses all magnetic properties. Permanent magnetization always diminishes as the temperature rises [$\frac{1}{3000}$ to $\frac{1}{300}$ part per degree C. according to the magnet] and quite disappears at the same temperature.

Heating to redness is therefore sometimes used to demagnetize specimens, but they must afterwards be placed magnetic east and west, or they will pick up no little magnetization from the earth as they cool through the temperatures of high permeability.

Magnetite likewise demagnetizes at bright redness, cobalt at 1100° and nickel at only 320° C. A curious nickel steel demagnetizes at 600° and has to be frozen before again becoming magnetic at all.

§ 518: **Magnetization Curves.**

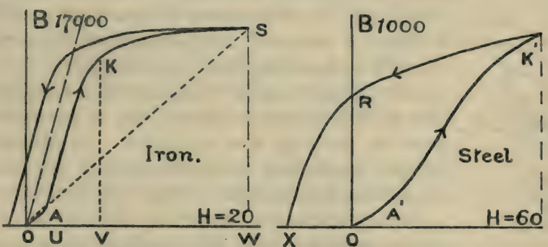


FIG. 249.

If a bar of **soft iron** is subjected to a gradually increasing field its magnetization increases in three distinct stages, Fig. 249, where B = induction density, H = magnetizing field.

From O to A the magnetization is proportional to the field strength, P is constant, but small, and when the field is removed the specimen immediately and perfectly demagnetizes.

From A to K the magnetization increases enormously, the apparent permeability increasing from its steady value AU/OU to a maximum KV/OV.

Further increase in field strength evokes only slight response from the specimen, which presently becomes practically **'saturated.'** The permeability SW/OW gradually diminishes till at very intense fields it is less than a hundredth its value at K. It is true that beyond the 'knee' K the number of lines still increases much faster than if no iron were present, but it is becoming disproportionately expensive to maintain the magnetizing field and in machinery the magnetization is not pushed beyond K.

With very long bars, or rings, as the field is reduced, the magnetism remains greater than at the same field on the way up, but this is less noticeable with short specimens (§ 516). The long specimen approximates to the closed magnetic circuit of § 516; the same diagram serves, but all the ordinates are *inclined* as the dotted line, which shows a 3 or 4 times greater remanent magnetism at zero field (see below).

The magnetization of **hard steel** rises far less rapidly, and the A′ and K′ bends are smoothly rounded. Saturation demands a field far beyond the diagram, and yet means much less magnetization than in iron.

Returning, as the field diminishes, the magnetization falls only slowly, so that at R where the field is zero, there is still left the *Permanent Magnetization* OR, and it takes a reversed field strength OX to remove this. OX is called the 'coercive force.'

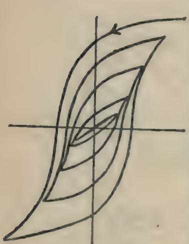


Fig. 250.

This 'Hysteresis' or 'sticking' of the magnetism, like solid friction, causes a loss of energy which goes to heat up the specimen. In alternating-current machinery it absorbs 10 h.p. per ton with the softest iron, and steel would be ten times worse.

Demagnetization cannot be effected in practice by the reversed field OX, because the curve is so steep at X that the least overrunning puts in an appreciable reversed magnetization. Demagnetization is effected by reversing the field again and again, meanwhile gradually weakening it. The effect is to carry the

magnetization round in a 'cycle' like Fig. 250 (the bottom half of which is a replica of the top, for reversed current) which gradually shrinks up to the point O. Thus a specimen is put inside a coil in which flows a current frequently alternated while gradually weakened. Or a watch that has become magnetized, and either gains or sticks, can be cured by holding near a dynamo-magnet and turning over and over, meanwhile very gradually withdrawing it to a greater distance.

§ 519: **Ewing's development of the molecular magnet theory.** On the assumption that individual molecules are permanently magnetized Ewing has suggested that they act like a swarm of

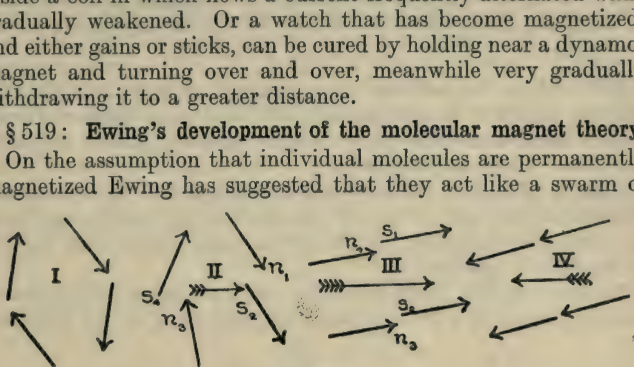


FIG. 251.

little compass needles, of which it suffices to consider four. At first, in the absence of external field, these are settled as in Fig. 251 (i), under their mutual attraction, head to tail in a closed ring which produces no magnetic effects outside.

A weak field in the direction of the arrow persuades them to move apart a little to follow it as in (ii); removed, they return at once to the original 'unmagnetized' position.

A stronger field presently draws the poles n_1s_2, n_3s_4 so far apart, enfeebling their mutual attraction, that the 4 needles become unstable and swing round as in (iii); all point the same way, imitating a magnetized specimen, K, Fig. 249.

Further increase of field can now only separate s_1n_4, s_2n_3 a little as it pulls the needles into line with itself: the specimen is saturated.

Removing the field, the needles remain almost as in (iii) under their mutual attraction: the specimen remains permanently magnetized.

A reverse field of some strength will be necessary to upset this stable arrangement, and when it does so the needles are unlikely to return to the half-and-half position (i), but to all swing round through X into (iv).

The best way to get them back to (i) is to stir them round

violently with a magnet, then removing it and leaving them to quiet down (*a diminishing alternating field*).

[The magnetization of the molecule may be due to the revolution of an electron inside it, equivalent to a magnet perpendicular to its orbit. Then the orbit can tilt without the molecule moving as a whole.]

§ 520. Now the portions of the magnetic lines outside the iron may be regarded in another way, and that is as **lines of Force,** elastic lines on the stretch, tending to shorten, pulling together the pieces of iron they connect.

Thus in Fig. 246 the iron nut is being pulled by two dense bundles of lines, and in Fig. 247 the bar is being pulled round into line with the two poles. In the actual experiments these had to be fastened in position.

Each line in Fig. 252 (drawn for me in an examination) represents the track of a little compass stepped along in the direction

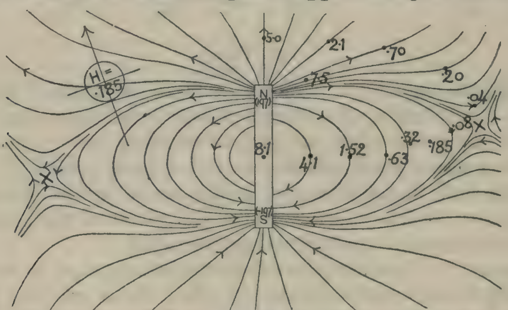


Fig. 252.

it pointed. The needle of course set itself along the line of greatest pull, just as if it had threads attached to each end and pulled opposite ways. Being pivoted, it turned more easily than the iron filings of Fig. 243 and the field is traced farther out, past the Neutral Points XX, into regions where the earth's influence preponderates. The earth's lines when undisturbed are of course straight lines running magnetic N. and S.

This map of the combined or resultant magnetic field which we are now calling a force diagram is equally the map of the stream lines of a fluid exuding from a supply at N. and being removed equally fast at S., the experiment being done in a brook

gently flowing up the page. Where the flow is rapid (force strong enough to move filings) the slow current of the brook (weak field of the earth) does not appreciably distort it, and it is only the feebler outer lines that show signs of its interference.

If there were no force other than the lengthwise tension in the lines they would all pull up into one shortest straight line joining the two pieces of iron. Evidently there is also a sidewise pressure among them, spacing them out over the whole field. This can be used in explaining the repulsion between similar poles as in Fig. 245, the mutually cramped-up streams of lines tending to swell to their natural width.

§ 521: But *Iron* lying on a curved line is pulled tangential *and also pulled sideways* into the hollow of the curve. For the forces on its ends, which lie at separated points on the curve, are not exactly opposite and have a small inward resultant. In this way the filings in the densest part of Fig. 247 have been raked in, as the card was tapped, from the weaker parts of the field.

Thus *iron always tends to move into the stronger parts of the magnetic field.* Work is done in moving it, hence the energy left in the field is less when iron is present. The energy per cubic centimetre in iron is less than in air ; it is inferred that the pull of a magnetic line is less in iron than in air ; this we have no experimental means of measuring.

Chapter LIII

MAGNETIC FIELDS

It will now serve our purpose best to lay aside the idea of stream lines and to merely regard magnetic action as direct attraction or repulsion between point-poles at a distance, much in the same way as we resorted to geometry in optics. This is done solely because stream-line calculations are much more difficult; it is justified because in all worked-out cases the two methods have given the same results.

§ 522. The point-poles of a magnet can be regarded as the 'centres of gravity' of two magnetic charges. To find them, bring up one end of the magnet to within an inch of a charm compass in such a way that the needle is not deflected at all, ink on the magnet the line of pointing of the needle. Slew the magnet round to some other position, still so as not to deflect the needle, again ink in its line of pointing. The ink lines cross at the 'pole.'

In a bar magnet the poles are usually ·85 its length apart (one-fourteenth from either end). The line joining them is the magnet's **magnetic axis,** it is this line of course which sets in the magnetic meridian when the magnet is free to turn in the earth's field.

§ 523: In **measuring the strengths of poles** it must be conceded that a pole possessing m units of strength produces everywhere m times the magnetic effect of a unit pole placed in its position.

I. **The unit (N.) pole repels with a force of 1 dyne another unit (N.) pole placed 1 cm. away from it.**

II. A pole of m units repels unit pole with m dynes, and further, it repels pole of strength m' with their product mm' dynes. [South poles are given a — sign, a — force means attraction.]

This can be demonstrated by a 'Magnetic Balance' as follows:—

Several steel knitting needles AA′, BB′, etc., are magnetized;

let their north-pole strengths be A, B, C, etc. AA′ is laid on the pan of a delicate balance and counterpoised. Above it is fixed BB′ with its N. pole B vertically above A at a distance d, Fig. 253, and repelling it down. The weight necessary in the other pan to restore equilibrium may be called AB, the repulsion between poles A and B. It should be in dynes, but as the experimental accuracy is vitiated by cross attractions with the distant S. poles A′ and B′, milligram weights serve well enough as units of force.

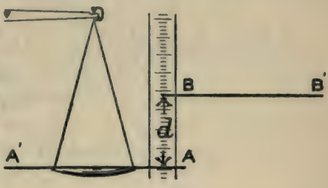


Fig. 253.

BB′ is now exactly replaced by CC′, DD′, etc.: weights AC, AD, etc., restore equilibrium. Since the repelled pole has remained the same, the

ratio of pole B to pole C to pole D = force AB : AC : AD, etc.

This gives a relative measure of the poles: now removing AA′ and taking any pairs, placing one on the pan and the other at d above it, the *repulsion between them will be found proportional to their product.*

III. The force varies inversely as the square of the distance.

Put magnet BB′ at other distances, $1{\cdot}2d$, d, $\cdot 8d$, etc., and the repulsions will be found to be AB/1·44, AB, AB/·64, etc. [Caution.—Like poles put too close together partially demagnetize each other.]

Putting wood, etc., between the poles makes no difference, but iron plates of course upset the experiment.

Hence the complete law of magnetic action can be put:—

The repulsion, measured in dynes, between two point-poles, is equal to the product of their strengths divided by the square of their distance apart in centimetres.

$$f = \frac{mm'}{d^2}$$

We can calculate the absolute values of the knitting needles' poles. For taking B and C, their repulsions on A give

$$\text{Pole ratio } \frac{\text{B}}{\text{C}} = \text{repulsive force ratio } \frac{\text{AB}}{\text{AC}}$$

Then discarding AA′, their mutual repulsion at distance d cm.

$$\frac{B\times C}{d^2}=BC \text{ dynes.}$$

Multiplying the two equations together

$$\frac{B}{C}\times\frac{B\times C}{d^2}=\frac{AB}{AC}\times BC$$

$$\therefore \text{ pole } B=d\times\sqrt{\frac{AB\times BC}{AC}} \text{ units,}$$

$$\text{or dividing, pole } C=d\times\sqrt{\frac{AC\times BC}{AB}} \text{ units.}$$

§ 524 : **Strength of field.**

The strength of the magnetic field (often briefly referred to as '**the field**') at a place, is defined as being equal to the force in dynes that would be exerted on a unit N. pole placed there.

It is sometimes (mis)called the **magnetic force**, but it should be carefully distinguished from actual Mechanical Force, to obtain which it must evidently be multiplied by pole strength.

The mechanical force on pole m' distant d from pole m

$$=\frac{mm'}{d^2}=\frac{m}{d^2}\times m'=\text{field strength}\times\text{pole strength. [or 'field'}\times\text{'pole']}$$

$$\frac{m}{d^2}\times 1=\text{field}\times\text{unit pole}=\text{dynes force on unit test pole.}$$

Thus **strength of field** at distance d due to a single pole $=\pm$**pole**$\div d^2$, directly away from a N. pole or towards a S. pole.

§ 525 : **Magnetic lines in relation to field strength, pole strength, etc.**

Here we may bring together the geometrical theory and the magnetic stream lines. Taking a square centimetre at right angles to the lines, *let a field of unit strength be represented by one 'unit magnetic line' passing perpendicularly through that square centimetre : a field strength H by H unit lines per sq. cm.*

Actually of course the stream pervades the whole square centimetre, there is no striated structure in the field—as the card is tapped some of the lines of filings will probably move sideways and settle down where blank spaces were—but it is convenient to think of unit lines, each the axis of a tube of flow, as one might count wicks in a box of candles that had accidentally softened

into a solid lump. For instance, the earth's 'total field' 0·33 is represented by 1 unit line to each 3 sq. cm. of an area perpendicular to it (or a line of $\frac{1}{3}$ unit strength per sq. cm., or a $\frac{1}{300}$ line per sq. mm., etc., but the first is most convenient).

Since a unit pole produces a field of unit strength ($1 \div 1^2$) at 1 cm. from itself in all directions, and since the sphere of radius 1 cm. has an area of 4π sq. cm., each of which must have its unit line, therefore *Unit N. pole must be regarded as emitting* 4π *unit magnetic lines; and pole m emits* $4\pi m$ (*about* $12\frac{1}{2}m$) *unit lines*, all of which have travelled to it through the metal.

§ 526: The strength of pole per square centimetre surface of the metal is called its **Intensity of Magnetization (I)**.

If this magnetization is entirely induced by a magnetizing field strength H, the ratio I/H is the specimen's **Susceptibility** to magnetization.

We have already (§ 514) defined its permeability, and can put it now = lines per sq. cm./H. To these lines the field itself contributes H, the induced pole contributes 4πI, hence

$$\text{Permeability} = \frac{\text{H} + 4\pi\text{I}}{\text{H}} = 1 + 4\pi \times \text{Susceptibility}.$$

Permeability is the stream-line way, susceptibility the external geometrical way, of reckoning magnetizability.

§ 527: The 'distribution of free magnetism in a magnet,' or in other words, the comparative numbers of magnetic lines leaving its surface per sq. cm. at various parts, can be studied by finding at each, with a weak spring balance, the force necessary to pull off a small piece of soft iron ($\frac{1}{2}$ in. of french nail) clinging to the magnet, Fig. 254. The iron gets magnetized, by the lines that enter it, proportionally to their number per square centimetre, i.e. its induced pole is proportional to the strength of the polar surface it is clinging to. **The force observed is proportional** to the product mm' of these 'poles,' i.e. **to the square of the quantity in question.**

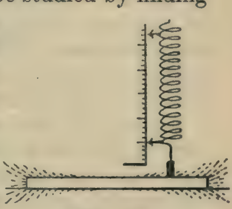


FIG. 254.

It can be shown that the **force of magnetic adhesion** in dynes per sq. cm. is $\frac{1}{25}$ (lines per sq. cm.).[2] Thus a tram brake-block magnetized to 20,000 line-density adheres to the rail sixteen

times as hard as if magnetized only to the 5000 customary in electrical machinery.

Bits of iron always prefer clinging to the edges rather than to the flat face of a magnet pole. For the lines crowd into the corners (Fig. 254) so as to have as great a proportion of their course in the easily permeable metal as they can, hence the attractive force is much greater there.

§ 528 : **Measurement of the strength of a magnetic field.**

Theoretically one puts a unit N. pole at the place and measures the force on it in dynes. Doubtless this might be done satisfactorily were it not for the persistent presence of the corresponding S. pole on the same piece of steel. The problem has to be attacked indirectly.

We have at disposal two distinct ways of *comparing* the strengths of fields in which this south pole causes no difficulty. With the second of these we can combine the experiment of § 523 and get absolute numerical results, but only within 2 or 3 % (see end of § 523). Fortunately, however, a little calculation will enable us so to combine the two comparative methods as to do away altogether with the necessity of using § 523, and to give field strengths and magnet strengths with great accuracy.

§ 529 : **Method I. Comparison of magnetic fields by Deflection.**

The earth's field cannot be got rid of, *ergo* the best thing to do is to make use of it, as a standard. [For its daily variations (§ 546) are too small to affect any but the finest work (in which they are allowed for), and though its value may be altered by the proximity of girders, gas-pipes, etc., it will keep sufficiently constant at any one place, provided one guards against movable iron anywhere near—steel tools, fire-irons, gas-burners, stray magnets ; pocket-knives, keys, or any other steel in one's attire. One soon recognizes the transient signs of passing trams and trains.]

Arrange the field under test so as to act at right angles to the earth's field on a compass needle. [Sometimes mere symmetry facilitates this, but in any case it can be effected thus : The whole apparatus is arranged on a square board in such fashion that the compass is quite undeviated, i.e. the test field is in line with the earth's. Now turn the compass box till 90° is under the needle, and then turn the board through a right angle. 0° should be under the needle, but instead there is a deflected reading D°.]

Then (Fig. 255) the N. pole m of the compass needle is being pulled magnetic northwards by the earth's field H with a force

$m \times$ H dynes and magnetic east (or west) by the test field F with force $m \times$ F dynes. The needle turns and settles down so that the resultant pull acts along the line joining pole to pivot, when of course it has no further tendency to turn either way. Meanwhile the S. pole is undergoing exactly similar but opposite actions.

Then by the rectangular parallelogram of forces mABC

$$\frac{m \times \mathrm{F}}{m \times \mathrm{H}} = \frac{\mathrm{AC}}{\mathrm{A}m} \textit{ or } \frac{\mathrm{F}}{\mathrm{H}} = \tan \mathrm{D}.$$

Where D is the angle the needle is deflected from its natural position.

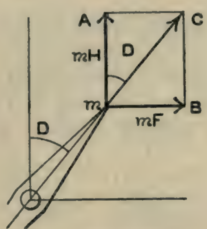


Fig. 255.

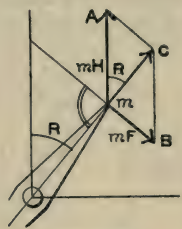


Fig. 256.

[Note.—With some forms of apparatus it happens to be more convenient to keep the test field always perpendicular to the needle, as in Fig. 256. This is effected (after the preliminary setting as before) by turning further till the scale-zero comes under the needle again (meanwhile its deflection increases somewhat). Measuring this rotation R of the turntable—

$$\frac{m \times \mathrm{F}}{m \times \mathrm{H}} = \frac{\mathrm{AC}}{\mathrm{A}m} \textit{ or } \frac{\mathrm{F}}{\mathrm{H}} = \text{sine R.]}$$

Other test fields can then replace the first and produce deflections D_2, D_3, etc., when evidently

$$\mathrm{F}_1 : \mathrm{F}_2 : \mathrm{F}_3 \ . \ . \ . : \mathrm{H} = \tan \mathrm{D}_1 : \tan \mathrm{D}_2 : \tan \mathrm{D}_3 \ . \ . \ . : 1$$

[or in the second modification $=$ sine R_1 : sine R_2 : sine $\mathrm{R}_3 : 1$.]

For a particular case of field due to magnet see § 532, and for fields due to electric currents see Galvanometers, § 609.

The compass, called in this connection a Deflection **Magnetometer,** may be a small surveying compass, or a Kelvin card, or simply an inch of magnetized knitting needle with a long light pointer attached, suspended by a silk fibre over the centre of a graduated circle in an iron-less draught-proof box with a glass

lid. Eliminate error of centring by reading both ends and taking mean.

A more delicate magnetometer has a gridiron of half a dozen horizontal 1-cm. strips of magnetized 'hair-spring' stuck on a flattened vertical rod (magnesium ribbon), which also carries a small mirror and is suspended by 2 or 3 in. of very thin silk fibre in a small shallow upright draught-proof box (cf. Fig. 309). Its motion is observed by telescope or lamp and scale as described in § 362.

§ 530: **Moment of a magnet.**

Suppose a magnet held at right angles to the lines of a field H as in Fig. 257. A force Hm acts at right angles on the N. pole and an equal force in the opposite direction on the S. pole, both tending to turn it one way, with *a turning moment* about any point O

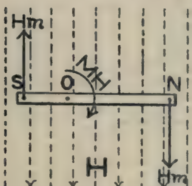


Fig. 257.

$=\mathrm{H}m\times \mathrm{NO}+\mathrm{H}m\times \mathrm{SO}=\mathrm{H}m\times \mathrm{NS}$

$=$field$\times$[pole$\times$straight length of magnet between poles].

Thus if the magnet lies with its axis at right angles to a field of unit strength, the couple or turning moment acting on it

=strength of one pole×length between its poles

and this product is called the **Magnetic Moment, M, of the magnet.**

Neither the strength nor the position of a pole can be quite satisfactorily ascertained, but this product can be measured (as below) with very great accuracy, and it is always used to express the magnetic value of the magnet.

§ 531: **Method II. Comparison of magnetic fields by Vibration.**

Let now a magnet, pivoted or suspended by a torsionless fibre and perfectly balanced, be turned through a small angle from its position of rest (straight down the field) and then let go. It oscillates under the steady uni-directional pull of the field just exactly as a compound pendulum does under the steady vertical pull of gravity, and according to § 55 its time of vibration T sec.$=2\pi\times\sqrt{}$(moment of inertia, I$\div$turning moment acting on it when held out at right angles to the force). By the last

paragraph this turning moment = field H × moment M, of magnet.

$$\therefore\ T=2\pi\sqrt{\frac{I}{HM}}$$

provided that the arc of swing is small, § 38,

$$\text{or}\quad MH=\frac{4\pi^2 I}{T^2}$$

I is calculable from the mass and size, § 54; T is obtained as usual by dividing the time occupied by a number of successive vibrations by their number. We still have M to find (unless we accept an experiment as in § 523), but without that, here is a valuable method for comparing fields.

Time the swings of the magnet first in one field, then in the other; M and I do not alter and hence $H \propto 1/T^2$, the strengths of the fields are inversely as the squares of the periods of vibration. A swing twice as fast means a field 4 times the strength. If n is the number of swings per minute, say, $n \propto 1/T$, and hence $H \propto n^2$; **the field strength is proportional to the square of the number of swings per minute.**

Fields compared in this way are of course the resultant fields of magnet (or coil) and the earth, whereas Method I distinguished between these.

The vibration method is used in magnetic surveys, on all scales of magnitude. Thus in Fig. 252 the values of field strength marked would be obtained from the squares of the numbers of vibrations per minute of a magnetometer (a fibre-suspended brass bob with ½ in. of knitting needle stuck through; vibrates longer and steadier than the charm compass). In the absence of the magnet the earth's field $H=\cdot 185$ gives a standard N^2, hence the actual values of the fields = squares of vibrations $\div N^2 \times \cdot 185$ c.g.s. units.

[As a matter of fact the pole strengths and field values marked on Fig. 252 were obtained three years after it was drawn by graphic calculation from the shape and size of the field.]

[NOTE.—If the magnet is of complex shape and I cannot be calculated, a brass bar of calculable I′ is attached to it and a second time T′ of swing observed, $T'=2\pi\sqrt{\frac{I+I'}{HM}}$

Squaring both T′ and T and subtracting, $T'^2-T^2=4\pi^2 I'/HM$.

Now removing the magnet, if the silk suspension has any appreciable torsional stiffness it will cause the brass bar to oscillate in the long time $T''=2\pi\sqrt{I'/S}$ where S is the additional control due to stiffness. Then $T'^2-T^2=4\pi^2 I'/(MH+S)$.

$$\therefore MH=4\pi^2 I'\left(\frac{1}{T'^2-T^2}-\frac{1}{T''^2}\right)$$

§ 532: **Calculation of field due to a magnet at a point on its axis produced—"End-on."**

Let magnet have poles $\pm m$ separated length l; to find field d from its centre along axis produced. Fig. 258.

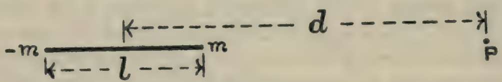


FIG. 258.

Field strength at P due to N. pole $=\dfrac{m}{(d-\frac{1}{2}l)^2}$ to right.

" " " S. " $=\dfrac{m}{(d+\frac{1}{2}l)^2}$ to left.

$\therefore$ Resultant field strength at P $=\dfrac{m}{(d-\frac{1}{2}l)^2}-\dfrac{m}{(d+\frac{1}{2}l)^2}$ to right

$$=\frac{m(d+\frac{1}{2}l)^2-m(d-\frac{1}{2}l)^2}{(d^2-\frac{1}{4}l^2)^2}=\frac{2mld}{(d^2-\frac{1}{4}l^2)^2}$$

$$=\frac{2Md}{(d^2-\frac{1}{4}l^2)^2}\text{ to right along axis}$$

and if d is much greater than l (say 5 times) this very nearly $=\mathbf{2M}/d^3$, so that the field strength of a small magnet falls off very rapidly, being inversely as the *cube* of the distance.

Example 1. Calculate field on axis at 25 cm. and at 85 cm. beyond the N. pole of a 10-cm. bar magnet of pole strength 250.

Here $d=30$ or 90 cm., $l=8{\cdot}5$ cm. (§ 522). $\therefore M=250\times 8{\cdot}5$.

Hence at 30 cm., accurately ·164, approx. ·157 (too close)

" 90 " " ·00583 " ·00582

§ 533 : Calculation of field due to a magnet at a point in its equatorial plane—" Broadside-on," Fig. 259.

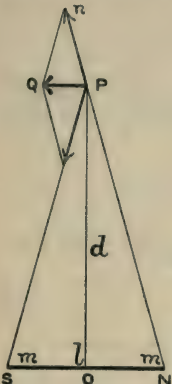


FIG. 259.

Field strength at P due to $+m$

$$=\frac{m}{\mathrm{NP}^2}=\frac{m}{d^2+\frac{1}{4}l^2}\text{ up along NP.}$$

Field strength at P due to $-m$

$$=\frac{m}{\mathrm{SP}^2}=\frac{m}{d^2+\frac{1}{4}l^2}\text{ down along PS.}$$

To get their resultant, resolve each along PO and PQ ; the components along OP are equal and opposite, and cancel; along PQ the fraction Po/Pn of each acts towards the left, where o is the middle point of PQ. Triangle Pon being similar to NOP, this fraction$=\frac{1}{2}l\div\sqrt{d^2+\frac{1}{4}l^2}$,

$$\therefore \text{ Field strength is towards left and}=2\times\frac{m}{d^2+\frac{1}{4}l^2}\times\frac{\frac{1}{2}l}{\sqrt{d^2+\frac{1}{4}l^2}}$$

$$=\frac{\mathrm{M}}{(d^2+\frac{1}{4}l^2)^{\frac{3}{2}}}$$

and if d is much greater than l this very nearly$=\mathbf{M}/d^3$, *which is just half the field strength at the same distance along the axis.*

Ex. 2. For the same magnet,
$d=30$ cm., accurately ·0765, approx. ·0785 (too close)
90 ,, ,, ·00291 ,, ·00291

§ 534 : Gauss's proof of the Inverse-Square Law.

Now if the reader will rewrite these two calculations but using the inverse pth power of the distance instead of the inverse square —thus $m/(d-\frac{1}{2}l)^p$, etc.—he will find that approximately the field on the axis$=p\mathrm{M}/d^{p+1}$ and on the equator$=\mathrm{M}/d^{p+1}$, the ratio being about p. Only if the inverse-square law holds is the ratio of fields 2 (compare the last figures in the two Examples).

Gauss, experimenting as in the next paragraph, with a magnet 'end-on' and also 'broadside-on' to a magnetometer, proved the inverse-square law in this way to 1 part in 10,000.

§ 535 : **Now to calculate both M and H—**

Vibration experiment. Suspend the magnet by a silk fibre and find its time T of small oscillation

$$MH=\frac{4\pi^2 I}{T^2}$$

Deflection experiment. Place the magnet E. or W. (magnetic)* of a compass and pointing towards it, so that the field F of § 532 $=2Md/(d^2-\frac{1}{4}l^2)^2$, then :—

$$\frac{M}{H}=\frac{(d^2-\frac{1}{4}l^2)^2 \tan D}{2d} \text{ [or sine R throughout]}$$

Multiply these two equations together and take the square root.

$$\therefore M=\frac{2\pi(d^2-\frac{1}{4}l^2)}{T}\sqrt{\frac{I \tan D}{2d}}$$

Divide first equation by second and take the square root.

$$\therefore H=\frac{2\pi}{T(d^2-\frac{1}{4}l^2)}\sqrt{\frac{2Id}{\tan D}}$$

* For accuracy, place 4 ways at d, centre to centre,

(1)	to E.	N. pole	pointing	E.,	read both ends	of needle.
(2)	,, E.	,,	,,	W.	,,	,,
(3)	,, W.	,,	,,	W.	,,	,,
(4)	,, W.	,,	,,	E.	,,	,,

The mean D of the 8 readings is free from numerous possible errors.

§ 536 : **Geometrical construction of magnet's field.**

To find the relative strength and direction of the field due to a magnet with poles NS at a point P, Fig. 260. Join PN, PS, on PN produced mark off away from N in any convenient units a length proportional to $1/(PN)^2$ and on PS mark off towards S a length proportional to $1/(PS)^2$ (using tables of reciprocals). Complete the parallelogram, the diagonal PP′ represents the resultant field strength and direction at P.

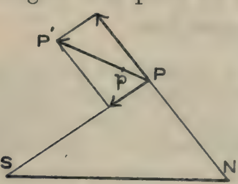


FIG. 260.

Starting again at p another vector can be worked out, and so on, approximating to the complete curve by a succession of straight lines.

EXAMPLES.—CHAPTER LIII

3. Explain how to magnetize a knitting needle to have a North pole in the middle and a South pole at each end. If then suspended horizontally from its middle point, which end would point North ? [L.]

4. What occurs when a small piece of soft iron is suspended in various typical positions near a bar magnet ? [L.]

5. Represent approximately the magnetic lines in and near a bar of iron 15 cm. long and 2 cm. diam., placed in an originally uniform field, parallel to axis. [L.]

6. Show in a figure the lines of force inside and outside (1) a magnetized steel tube, (2) a solenoid carrying a current. [L.]

7. Explain how to uniformly magnetize a wire. [Ab.]

8. What is meant by magnetic induction, and how do you account for it on the molecular theory of magnetism ? [D.]

9. Define magnetic moment, intensity of magnetization, and magnetic susceptibility. An iron wire 40 cm. long, and of cross-section 0·005 sq. cm., is placed along a field of strength 0·5, its magnetic moment becomes 2 ; calculate the intensity of magnetization, and the susceptibility. [L.]

10. Explain magnetic ' permeability ' and ' induction.' Show how the behaviour of different specimens of iron may be conveniently represented by curves. How does temperature affect permeability ?

11. Describe the magnetization curve. [M.]

12. Show how it is possible to have a rod of soft iron magnetized to different intensities at different times, although under the influence of the same magnetizing force. [L.]

13. How would you prove experimentally that the two poles of a bar magnet are of exactly the same strength ? Describe an experiment which shows that the bar is magnetized along its whole length. [L.]

14. What is meant by the magnetic axis of a magnet ? How would you determine the magnetic axis of a magnetized steel sphere ? [L.]

15. Explain '*strength of a magnetic field*.' Magnet and soft-iron bar of same size are placed parallel ; draw lines (a) when well apart, (b) when nearly touching. [M.]

16. Draw the lines of force for two magnets crossed at right angles. [M.]

17. Define *magnetic pole of unit strength*, *magnetic force at a point*. How would you investigate experimentally the relation between the magnetic force due to a pole and the distance ? [L.]

18. Define the magnetic moment of a magnet. Describe an experiment by which the moments of two magnets could be compared. [L.]

19. Obtain an expression for the force due to a long straight magnet at any point on its axis produced. How can the moment of a magnet be determined in terms of the strength of the earth's field ? [L.]

20. Calculate strength and direction of field at point distant d from centre along axis of magnet length l, poles $\pm m$, and find effect on the field of placing an equal magnet on the first but at right angles to it. [M.]

21. The maximum intensity of permanent magnetization in a steel bar 10 cm. long by 1 cm. square has been found to be 225 c.g.s. units. Find tangent of greatest deflection of a magnetometer which this magnet could cause if centre of needle were 30 cm. east of centre of magnet. [H=0·18 c.g.s.] [L.]

22. Two 1 cm. long magnets are placed in line, centres 10 cm. apart. What is force between them if poles are 4 and 3? [M.]

23. Two magnets, each of effective length 8 cm. and moment 80 units, lie in the same straight line, with their N. poles 6 cm. apart. Calculate the repulsive force between them. [L.]

24. A long thin bar magnet weighing 22 grm. appears to weigh 20 grm. when an equal and similar magnet is placed parallel to it 2 cm. below. Find the strength of each pole of each magnet. What would be the effect of placing the lower magnet so as to attract the upper? [L.]

25. Two thin magnets 30 cm. long are parallel along two sides of a square, moment of each = 300. Calculate resultant force on either.

Chapter LIV

THE EARTH'S MAGNETISM

§ 537 : We have seen that a magnet freely poised, under the influence of magnetic force alone, sets itself along a magnetic line with its N. pole 'down stream.' Such a magnet controlled by the Earth's Magnetic Field sets itself (in this country) with its N. pole pointing between N.W. and N. and dipping steeply downwards, i.e. the field runs in this direction, cf. Fig. 261, T.

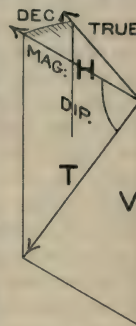


Fig. 261.

To get an accurate description of the direction, the vertical plane in which the magnet's axis lies is first drawn. This is called the plane of the **Magnetic Meridian**. The horizontal angle ('bearing') between this plane and the **Geographical (astronomical) Meridian** is called the **Declination** [**'Dec.'**] or, nautically, the **Variation of the Compass,** from true N. The angle in the magnetic meridian plane at which the magnetic axis dips down below the horizontal is the **Dip.**

These angles are actually studied separately, the Dec. in a compass whose S. end is overweighted to prevent the N. dipping, the Dip with a dip needle which is placed in the meridian and allowed to roll on a horizontal cross-axis.

§ 538 : **Declination.** The variation of the compass from true N. in Europe is said to have been noticed in 1269, and Columbus in 1492 doubtless observed that the angle changed on his voyage across the Atlantic. But it is remarkable how persistently the belief that the compass points true N. is held among the educated and unobservant public at the present time, even among those so far advanced in astronomical lore as to point out the north star correctly and to condemn the sundial's timekeeping.

The Declination can be measured on an ordinary mariner's compass by taking a bearing of the pole star by the following *azimuth instrument* (a home-made one) shown on the binnacle glass in Fig. 268. Stuck on the glass cover of the compass box is a brass centre having in it a small hole exactly over the pivot of the compass card. In this hole fits one of the three feet of a little stool; the other two stand on the glass, thus the stool rotates concentrically with the card. A narrow slit rises in a vertical plane from the outer end of the stool, and a vertical wire from its inner end. Looking through the slit, one turns the stool till the wire is seen 'cutting' the object whose bearing is being taken, then carrying the eye down the wire its lower end is seen cutting the graduated edge of the compass card in the magnetic bearing required. A horizontal mirror lying on the stool enables bearings of high stars to be obtained by reflection.

In observing the pole star in this way a correction is necessary, because the star is not quite at the pole. This is easily made; trace a line from the pole star to the middle star of the Great Bear's tail (the handle of the Plough), the pole lies $1\frac{1}{5}^\circ$, say two moon's diameters, down this line; sight that point. The use of any other star involves one in astronomical calculations.

The Declination of the compass W. of true N. is then of course equal to the compass bearing of the celestial pole E. of N.

The accuracy of such observations of course depends on the accuracy with which the compass card is made and attached to its magnets, and on the true centring and verticality of the azimuth instrument. Land magnetic-survey instruments have elaborate means of combating errors, most important perhaps being the invertibility of their compass needles (which are directly observed). For the magnetic axis may not quite coincide with the geometrical, say therefore the latter lies a little E. of magnetic N. Turning the needle top for bottom now changes this error to W. of N., and the mean is true.

§ 539: **Dip** was discovered by Norman, a London instrument maker, in 1576. "Always finishing and ending his needles before touching them with the stone, he continually found himself constrained to put some small piece of ware on the S. point to make them level again. And having once spoiled a large needle by cutting too much off the north end, in some choler he applied himself to seek further into this effect" and he straightaway made the first Dip Circle and found a dip of 71° 50′.

The Dip needle is a pointed magnet 4 or 5 in. long, originally balanced as perfectly as practicable on an axle passing transversely through its centre. The perfecting of the cylindrical shape of this axle makes exceptional demands on the skill of the mechanic. It rolls with the minimum of friction on horizontal edges of agate.

The plane of motion of the needle in front of its concentric vertical graduated circle must first be set into the magnetic meridian. This is done by compass, or else by finding the azimuth in which the dip needle sets vertical, and then turning at right

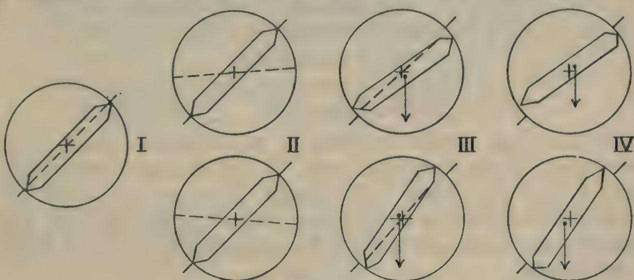


FIG. 262.

angles. [For if the horizontal drift of the field pulls the needle neither way, it must be at right angles to the plane of motion, which was therefore E. and W.]

This done, a perfect needle would give the Dip in one reading, but in practice it is the mean of 16. For :—

I. Both ends of the needle must always be read to get rid of error due to eccentricity, as in Fig. 262 (i), where, reading continuously round, one end is evidently as much short of the truth as the other is beyond it (+180°).

II. The zero line of the circle may be not quite horizontal—eliminated by readings with the whole circle facing E., and facing W., Fig. 262 (ii).

III. The magnetic axis may not coincide with the line joining the needle's points, and also the centre of gravity may lie to one side of the points of support—both eliminated by reversing the needle in its bearings, when the bias both faults (say) gave it to point too high is reversed to make it point as much low, Fig. (iii).

IV. The c.g. of the needle may lie some distance *along* it from

the centre of the axle. This must be remedied by remagnetizing it equally and oppositely and repeating all the $2 \times 2 \times 2 = 8$ readings foregoing. The heavy end (say) now dips instead of the light, and as much deeper as that was too high, Fig. (iv).

§ 540: The strength of the earth's field, called its **Total Intensity** or **Total Field,** can be measured by counting oscillations of the dip needle and then using it to deflect another dip needle according to the principles of § 535.

Usually, however, one supposes the Total Intensity T replaced by two components, the **Horizontal Component Intensity H** and the **Vertical Component Intensity V.** Then (Fig. 261) :—

$$H = T \textit{ cosine } (\textit{dip}).$$
$$V = T \textit{ sine } (\textit{dip}).$$
$$V = H \textit{ tan } (\textit{dip}).$$

The compass needle rotating in a horizontal plane is not influenced by the 'vertical force' V but is controlled entirely by the 'horizontal field' H. Hence the more usual and convenient experiment is to measure the important H with horizontally moving magnets as in § 535, and to deduce V and T from H and the Dip.

The Magnetic State of the Earth

In considering maps of the magnetic state of the earth as a whole, avoid 'Mercator's projection.' The better you understand that deceptive contrivance the more you will appreciate the advice.

§ 541: **Dip.** The needle sets nearly horizontal in equatorial regions, the N. end dips more and more as it is carried N., while in the southern hemisphere the S. end dips increasingly. That is, the lines of force run out of the southern hemisphere of the earth into space and return into the northern hemisphere, very much in the same way as they run out of and into the circle in Fig. 244.

The great circle of the earth on which there is no dip is the **Magnetic Equator. N.** and **S.** of it are the successive 'small circles' of **magnetic latitude** (called also **Isoclinals**) on which the dip is 1°, 2° . . . 89°. Those for every 15° are shown in Fig. 263. The two places at which the needle stands always vertical are called the **Magnetic Poles.**

This system of circles is inclined to the geographical system, the magnetic equator rising to 10° N. latitude in the Indian

Ocean and sinking to 16° S. latitude in Brazil. The circles are more or less distorted.

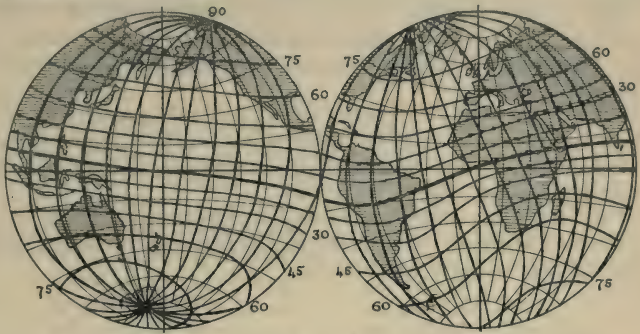


FIG. 263.

§ 542: **Declination.**

Dividing the magnetic equator into 360°, starting off from each and following the direction of the compass needle N. or S., there are traced out **meridians of magnetic longitude** all converging towards the magnetic poles, Fig. 263. These are all great circles, slightly distorted; one of them practically coincides with the geographical meridian 90° E.W. (the outside rim of the figure), the others are all more or less inclined, this of course reaching a maximum 90° away from the foregoing, or practically on the 0° 180° meridian of Greenwich (centre lines of figure).

The indispensable arrows, indicating magnetic N.S., drawn at various parts of a chart, are really short pieces of these lines.

It must be noted that these are not lines of force, for the latter enter and leave the earth's surface steeply. They may, however, be regarded as the lines of the H component force.

[May I here recommend the holiday tourist with a predilection for a half-inch map and a pocket compass, to get a compass with a moving card, to unfasten the needle and slew it round and re-attach it at the average variation of the district in prospect? He will avoid the constant bother of allowing for the variation, especially troublesome when looking southward.]

ALTERNATIVELY, Statistical Lines called **Isogonals** may be drawn; along each the Declination has some fixed value. They run much more irregular courses. A straggling **agonic** great 'circle,' everywhere on which the needle points true N., separates

a smaller Atlantic ‘ hemisphere ’ of westerly declination from a larger Pacific one of easterly declination. In Eastern Siberia there is another agonic ‘ oval ’ within which the needle goes west again.

§ 543 : Besides these, statistical lines may be laid down to join places of equal H, of equal V ; or of equal Total Force, Fig. 264.

H decreases from a maximum ·36 in equatorial regions (Borneo, etc. ; but not *on* the magnetic equator) to 0 at the magnetic poles.

V increases from 0 on the magnetic equator to about ·66 in polar regions (not just *at* the poles).

Total force increases from ·36 in equatorial regions to maxima at the four Magnetic Foci shown in Fig. 264 as plain dots. [The Poles are marked as ringed dots.]

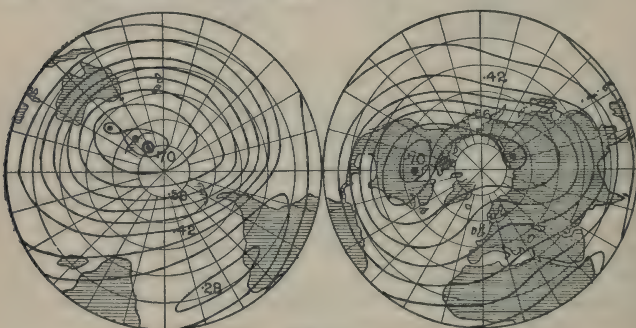


Fig. 264.

That these Foci show a stronger field than the poles themselves need not cause surprise, for the poles are defined so differently to those of a bar magnet—as places of vertical dip, instead of centres of distribution of magnetism. It is almost as if there were *two* magnets buried in a sphere, each stretching from an arctic to an antarctic focus ; such a sphere rolled in iron filings would gather them thickest at the four points nearest the buried poles, but at these places they would lean away from the twin repelling poles and it would be at some intermediate place that a thinner crop stood upright.

Wilde constructed a terrestrial globe inside which two concentric spheres were each wrapped in wire carrying an electric current. The axes of the windings were somewhat inclined and so produced the effect of two magnets crossing each other. Having

in addition overlaid his oceans with thin sheet iron he did obtain something of an imitation of the actual distribution of the earth's magnetism, and further, by rotating one sphere inside the other, he could reproduce the secular change of § 545. But of the true origin of the earth's magnetism we are profoundly ignorant.

§ 544 : Before leaving the Magnetic Poles there may be recalled an academic controversy the echoes of which have hardly died out. It dates from the days of the geometrical action of pole on pole, before the more valuable concept of a magnet 'setting' down the magnetic stream lines had become general. Certain people, mindful that 'like attracts unlike' and solicitous as to the magnetic charge of that inaccessible northern land, insisted on calling the northern end of a compass needle a *true south pole.* Others, less bold, compromised on *north-pointing* or *north-seeking pole,* hardly an apt description of an instrument which placed true N. and S. and let go immediately swings round to somewhere N.N.W.

Sailors speak of red magnetism in the northerly end of the needle and blue in the southerly, but probably most of us obtained our early notions of magnetism from the products of the toy-maker, and he blues the northerly end of the charm-compass and paints red the south end of his bar magnets.

Nowadays the end of the magnet that would point northerly anywhere on the inhabited earth is called its N. pole.

It follows that the magnetism of the earth's northern hemisphere is S. and that of the southern hemisphere, N. The possible confusion as to which is which of its Magnetic Poles—like the question as to which is the south border in the garden—is best avoided by calling them the Arctic and Antarctic magnetic poles, or, if you like, the Boreal and Austral.

§ 545 : The earth's magnetism continuously alters, the magnetic elements — Dec., Dip, and Force—at any place undergoing a slow but considerable **Secular Change,** and even the poles wandering.

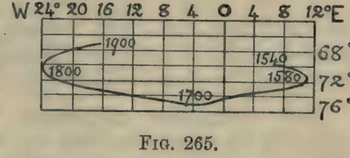


Fig. 265.

For instance, Boroughs at Limehouse in 1580 recorded a declination 11° 18′ E. ; his reputation suffered when in 1622 Gellibrand and Gunter found 6° 15′, but was re-established when in 1634 the same two observers found 4° 5′ E.

The recorded motion of the N. pole of the free needle at Greenwich, as seen from its centre, is shown in Fig. 265.

§ 546: There is also a **diurnal variation** which is a sort of miniature of the secular, the pole of the needle travelling round in a cycle of, roughly, 8′ angular radius, every day.

Breaking in on this quiet daily march come **Magnetic Storms** which may fling the compass needle to and fro more than a degree from its mean position.

It is known that these **daily variations** are due to some cause exterior to the earth's surface, and magnetic storms are often accompanied by notable displays of the *Aurora Borealis*, manifesting electrical disturbance of the upper atmosphere. That both these are connected with that variation of solar activity evidenced by the prevalence and extent of **sunspots,** is deducible from the occasional violent terrestrial disturbance accompanying a particularly large and active sunspot, and with more certainty from the occurrence of an eleven-year period in the frequency of all three, their maxima coinciding within a year or two.

At Kew in 1911 the Declination was 16° 0′ W., it is decreasing $5\frac{1}{2}'$ per annum and usually swings during the day from 5′ W. to 3′ E. of its mean value. The Dip was 67° 0′, and decreases 1′ per annum. $H = \cdot 185$, very slowly increasing.

Ross in 1831 obtained a dip of practically 90° in Boothia at 70° N. lat., 97° W. The Antarctic pole has been located thrice much more recently, it seems to be wandering about somewhere between 72° and 73° S. lat., 155° and 156° E.

The Error of the Compass in Iron Ships

The compensation of the Error or Deviation of the compass in iron ships involves a collection of illustrations of the magnetic principles worked out above.

Part of a ship's magnetism is due to the earth's induction, part is permanent. The induced part varies with course and position, and can only be compensated by an equal induction in soft iron; the permanent part can be corrected by steel magnets.

§ 547: **Temporary magnetism.** To a first approximation the ship may be represented by the soft-iron bar, Fig. 266. On a N. or S. course this becomes magnetized, but the compass needle lies along its axial line of force and is undeviated. On an E. or W. course the bar is not magnetized. On other courses, however, the bar becomes more or less magnetized, and being in an oblique

position its attraction or repulsion deviates the needle. As drawn the deviation would be easterly on a N.E. or S.W. course, westerly on a S.E. or N.W. course: this is the **Quadrantal Error.**

The breadth of the ship partly compensates for its length, the

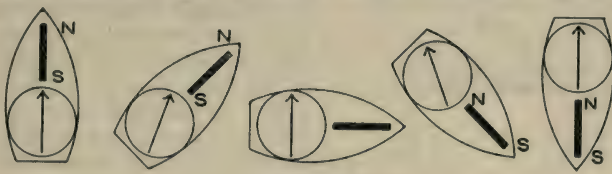


FIG. 266.

remaining soft iron to annul the error is disposed athwartship in the form of two large iron spheres at an adjustable distance (perhaps 9 in.) on either side the card.

Vertical magnetism of the ship causes no error so long as she is on an even keel, but when heeled over the lines of force no longer come up quite perpendicularly to the card, and have a horizontal component which causes a deviation perhaps twice as great as the angle of heel. The temporary part of this **Heeling Error** is corrected by the Flinders bar, a very stout soft-iron bar 2 ft. or more long fixed upright in front of the binnacle with its upper end just above it. The temporary heeling error of course vanishes at the magnetic equator, where the vertical force$=0$, and thereafter changes sign.

§ 548: **Permanent magnetism.** Representing the ship by the steel magnet Fig. 267, there is again no error on a N. or S. course, but on an E. or W. course there is a maximum deviation. This **semicircular error** is corrected by steel bar magnets fastened on adjustable supports in the pedestal

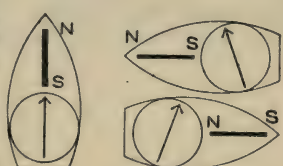


FIG. 267.

beneath the binnacle, and pointing fore-and-aft the opposite way to the ship's magnetism.

The magnetization of the cross-beams is compensated by a similar magnet placed athwartship, and the ship's permanent vertical magnetization by vertical magnets slid up or down beneath the centre of the card. The binnacle with the various correctors and an azimuth instrument is sketched in Fig. 268.

§ 549: Without going into the usual method of adjustment it may be pointed out that, when correct, the effect of the ship on the compass must be the same on whatever course she is laid, or the joint controlling force of (earth+ship+compensators) is invariable. A deflecting magnet, on a geometrical carriage, can be centred on the glass of the binnacle, and is so adjusted in strength that when lying over the E. and W. of the card the latter is deflected 85°. The deflector is removed and the ship put on another course: the deflector replaced and kept on the E. and W. of the card must again turn it through 85°, and this on five different courses; if it does, the compass is correct. Heeling trials can be made with a special dip needle held in the place of the compass card.

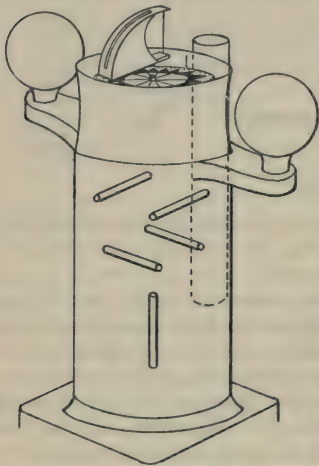


FIG. 268.

Thus, thanks chiefly to the genius of that great Sea Father, the late Lord Kelvin, it is possible to float a serviceable compass on board a 20,000-ton steel fort, and to check its correctness even in a fog.

EXAMPLES.—CHAPTER LIV

1. Indicate the effect on the resultant magnetic field at a place, of a vertical iron column, (a) near its foot, (b) near its head; and of steam-boilers fixed N. and S. in the basement on the field in the rooms above.

2. What measurements would be necessary to ascertain the direction of the magnetic field of the earth at any place? If a soft-iron rod were placed (1) horizontally N. and S., (2) horizontally E. and W., (3) vertically, what would be the general effect on the earth's field in the neighbourhood of the rod? [L.]

3. What are declination, dip, and total force of the earth's magnetic field? Outline a method of determining the last experimentally. [L.]

4. A dip needle oscillating in the meridian makes 35 oscillations per minute in a locality where dip is 60°. At another locality, where dip is 45°, the needle makes 40 oscillations per minute. Find (a) ratio of earth's total intensities, and (b) ratio of horizontal components of earth's field at the two places. [L.]

5. How would you determine experimentally the horizontal component of the earth's magnetic field? [L.]

6. Describe and explain how a magnetic needle, free to rotate about a horizontal axis through its centre of gravity, will move as the direction of this axis changes. [L.]

7. Describe how you would measure the Dip. [D.]

8. Explain Declination and Dip. Would you expect latter to be greater in Scotland or in Italy, and why? How find resultant intensity given dip and H? [Ab.]

9. What is an isogonal, and what an isoclinal? Describe briefly the character of the isoclinals in the northern hemisphere.

10. A dip needle removed from its bearings and hung by a fibre so as to swing horizontally makes 10 complete vibrations in 102 secs. Its moment of inertia is 726. ·1 grm. wt. 6·7 cm. from centre causes it to set horizontal when in its bearings. Calculate Dip (tan 67° 2·356, tan 68° 2·475).

ELECTRICITY

Chapter LV

FRICTIONAL ELECTRICITY

§ 550. The crackle and sparkle in dry hair and fur and its lifting towards the hand that stroked it must have been at least as well known to the hunters of the cave bear and the mammoth as to their descendants of the present day. And to the Egyptians of the later dynasties, with their employment of resins, their veneration for cats, and their torrid climate, the active attraction and adhesion of dust and light stuff was doubtless an occurrence too familiar to be placed on record.

It is to Thales the Ionian, student in Egypt, one of the 'seven wise men' of Greece, whose teachings were not confined to traditional lore, that the first discourses on the attractions of magnets, and of the fossil resin ἤλεκτρον (amber) are ascribed.

From that word is derived the modern 'Electricity,' and lately it has been adopted bodily as the name of the 'atom of electricity'—the uncut unit—the Electron.

The first important record of advance was the book in which the Elizabethan physician, William Gilbert of Colchester, discoursed "*De magnete . . . etc., plurimis experimentis demonstrata.*"

He observed that without Friction few bodies gave out their natural 'emanation and effluvium' and he made up lists of things in which friction excited this attractive effluvium and gave them the name of **Electrics.** Such are amber, resin, lac, wax, sulphur, paper, dry wood, silk, glass, etc.

Substances from which friction drew no effluvium—metals, stone, etc.—were **Non-electrics.**

"Electrics attract all things save flame and objects aflame and thinnest air, the effluvia are consumed by flame and igneous heat, yet they draw to themselves the smoke of an extinguished candle."

§ 551. Gilbert found the necessity of getting rid of the damp usually adherent to everything. "Moisture suppresses the effluvium, but olive oil does not." and as a practical point in frictional electrical experiments this is all-important. Everything should be dry and warm (though there is no need to risk melting wax or cracking glass); and so should the atmosphere. The room should have been warmed for some time, and even then the presence of a number of people may moisten the air too much for really successful work; frictional electrical experiments are among those things that will go wrong in public.

Amber is always reliable, but unobtainable in any size. Sticks and plates of Ebonite (black hard-vulcanized rubber, the stuff your fountain-pen is made of) are expensive, but most generally useful: it should be tough and of good quality, cheap brittle varieties are not much use. Preferably its polish should be removed with fine glass-paper, and thereafter it should be kept in the dark, otherwise its surface is apt to oxidize and spoil. Sulphur, Shellac, and Sealing-wax are good, but brittle; common 'bottle-wax' is useless. Brown paper is excellent when dry, but is hygroscopic and must be scorched before the fire every minute or two. Celluloid electrifies easily, but leaks. All these are electrified by rubbing with dry fur or flannel, or the coat-sleeve. Glass is apt to collect a surface film of moist dirt, it should be washed in hot soap and water, rinsed in hot water and wiped dry, and is then freely electrified by warm silk.

The property of becoming electrified by friction is, however, not confined to 'electrics,' as Gilbert supposed. No amount of drying can make metals 'electrics,' but if a tube or plate of metal be mounted on a handle made of an 'electric,' and be whacked with dry fur or silk, it will be found electrified.

Tests of electrification. The picking up of light stuff—paper, feathers, hair, dust, etc., is a rough test. A curious woolly tickling is felt on the nose and face when an electrified plate is held close to it; perhaps this is due to the lifting of the fine hair of the skin and to slight electric discharges to it. Much more delicate is the attraction of a little pill of elder-pith suspended by a fine thread. [And still more sensitive is the gold-leaf electroscope, to be described later.]

§ 552. There seem to be two opposite kinds of electricity obtained by rubbing different substances; much as there are two opposite polarities of magnetism. The mutual repulsion of

bodies charged with the same kind (same 'sign') of electricity is shown by rubbing two sticks of sealing-wax, placing one in a stirrup of wire or card suspended by a plaited thread or very narrow ribbon, and bringing the other near it. The same repulsion occurs with glass rods, but glass and sealing-wax attract each other. The repulsion is very easily shown by stripping a doubled silk ribbon through the fingers, the two halves straddle wide apart. And on occasion one's own hair, dried after a too-thorough wash, becomes electrified and quite unmanageable.

Indirectly, the repulsion can be shown by use of a pith ball hung by a thread or fibre of silk. It comes up to touch the electrified glass and then flies away. That this repulsion is due to its having picked up electrification from the rod is proved by the now increased strength of its attraction towards rubbed sealing-wax: the ball jumps rapidly to and fro between the two opposite rods.

The electrification developed on glass was called 'vitreous' and now **positive** (+); that upon resin, sealing-wax, sulphur, ebonite, etc., 'resinous,' now **negative** (—).

§ 553. If the pith ball is suspended by cotton or thin wire from a glass or sealing-wax holder it will be found to gain an electrical charge from the electrified rod drawn across the thread. The electricity has travelled, or has been *conducted*, along the 'non-electric' material.

This immediately explains why non-electrics do not ordinarily show electrification after friction; they conduct the developed electricity away to the hand, and it passes through the experimenter's body down to the earth, the great receptacle for all stray electric charges.

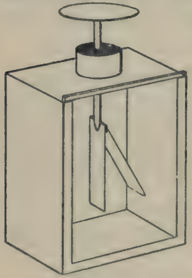


FIG. 269.

Consequently 'non-electrics' are nowadays called **Conductors**; while 'electrics' are non-conductors or **Insulators,** for on them the electricity cannot travel about, but remains isolated in patches, often difficult to remove.*

§ 554. The **Gold-leaf Electroscope** takes advantage of conduction. There is a metal stem bearing at the top a knob or plate to which the various charged bodies to be tested

* Wiping only makes more. Clasp in a damp hand, or best, pass over a flame.

are presented. The stem passes down through an insulating plug of wax or ebonite into a draught-proof box of metal and glass, to its flattened lower end are gummed a pair of strips of gold-leaf * [or for all ordinary use, Dutch-metal]. Charge given to the top of the stem travels down and is shared by the leaves, which thereupon open out by mutual repulsion.

§ 555. The attraction exerted by electrified on unelectrified objects leads by the argument of § 512 to the idea of **Electric Induction.** The mere proximity of an electric charge induces a separation of + and — electricity in the uncharged body, whichever charge is of opposite sign is drawn nearest to the inducing charge, and the attraction between these overbalances the repulsion between the inducing charge and the more distant residuum of the same sign.

That this separation does occur is shown by an experiment like Fig. 270. A ' conductor ' is made up of two separable halves, e.g. two apples hung by silk threads and touching each other, a charged rod is brought near one, and they are separated. Both will now affect an electroscope, but oppositely, and the effect of the one that was nearest the rod is opposite to the rod's own effect.

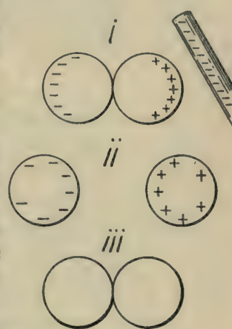


Fig. 270.

Thus *as* in Magnetism opposite charges have been induced to separate, but *quite unlike* Magnetism they can be isolated on separated halves of the conductor. [They reunite if the two halves are touched together again in the absence of neighbouring charges (iii).]

On a non-conductor these charges cannot move apart, and if this explanation of attraction of uncharged bodies is true, a non-conductor ought not to be appreciably attracted towards a charged rod. A very simple and striking experiment shows this, cotton and silk threads hang side by side over the finger, a rubbed rod of glass or sealing-wax is brought near, the conducting cotton rises high to meet it, the non-conducting silk hangs indifferent.

The third law of motion, that action and reaction are equal and

* For measuring purposes it is better to make one ' leaf ' a vertical strip of stiff metal as in Fig. 269.

opposite, applies of course to forces of electrical origin just as to any others.

That there is attraction between two bodies leaves it an open question as to which carries the 'inducing charge.' Other circumstances sometimes tell us, e.g. if a pith ball spontaneously moves up to meet the hand we know that it was the ball that was electrified.

Mutual repulsion necessarily means electrification of both.

§ 556. Since the two opposite induced charges sprang into being without the conductor being touched in any way (and without any conduction through the air by *spark*), and subsequently neutralized each other without leaving any residue, they must have been exactly equal. There was a temporary separation of electricities, but no creation.

There never is a creation. Mount on sealing-wax handles a disc of ebonite as big as a penny and a similar disc of card covered with cloth. Rub them together, the handles prevent either of them losing any of the charge developed on it by the friction. Hold them together near a pith ball—no effect—separate them and the ball dances from one to the other, showing that they are oppositely charged.

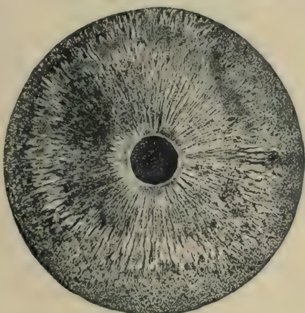
Ordinarily the cloth, etc., used as rubber is held in the hand, and as it is not a very good insulator its electrification soon travels down through the experimenter to earth and is lost sight of, and there appears to have been a production of one sort of electricity only [and rubbing a metal plate both charges travel away, leaving no signs of electrification at all].

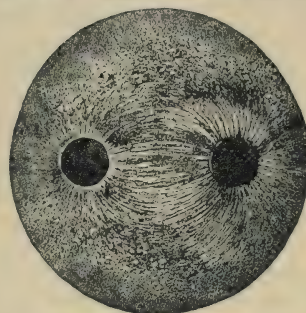
Why friction should cause this separation of positive and negative electricities we do not know. Conceivably, however, it produces local heating and increases the natural tendency to oxidation of the sulphur, resin, insulated metal, etc., and this may be the obscure beginning of an electro-chemical process. This makes it depend on the presence of an atmosphere, but of that the merest clinging traces would suffice, and the question is not to be tested by merely pumping out 'a vacuum' over the surfaces.

§ 557. Just as with Magnetism, the most graphic way of explaining electrical actions is by filling the field with Lines of Electric Force.

Each line links together a + and a — charge; it is said to originate on the + charge and run from it till it ends on the —

charge. The shapes of the lines in a few cases are shown in the photographs. Fig. 271 represents their uniform spreading from a + charged body to end on an equal — charge induced on a surrounding wall; Fig. 272 their path from a positively to an equal negatively charged body; Fig. 273 shows two equal charges of the same sign, and Fig. 274 shows the action of a gold-leaf





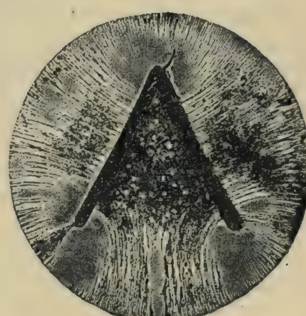


Fig. 271.
Fig. 273.

Fig. 272.
Fig. 274.

electroscope. Notice how the 'repulsion of similar charges' appears rather as a pulling apart, by the lines joining them to opposite charges which they have induced on surrounding conductors, e.g. the metal case of the electroscope.

Notice that these electric lines are not supposed to be continued through the substance of the conductor as magnetic lines were through the magnetized iron. The existence of a line presupposes

a + and — charge at its ends, and if these are situated in the same conductor the line joining them immediately pulls them together, and line and charges disappear. Conductors are blanks on an electric-line diagram.

§ 558. Nor is there any need for the conductor to be solid throughout. For suppose there were + and — charges on a hollow conducting shell, Fig. 275, and a line joined them; under its pull the charges run round the shell to meet each other and coalesce and the line has disappeared.

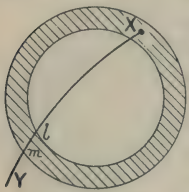


Fig. 275.

But might there not be a line such as XY crossing the hollow on its way from a + charge on the conductor to some remote — charge elsewhere? This would have to cut through the conductor at *lm*, the piece *lm* disappears since it is in a conducting material, therefore *l* is the end of a line X*l* and must be a — charge, X*l* shrinks up as before, and all that is left is the beginning of a line, i.e. a + charge, at *m*. In other words, X has travelled round to *m* under the pull of the line to Y.

Hence *a charge on a conductor produces no lines inside it, i.e. no electric force inside it, whether it is solid or hollow.*

Faraday thoroughly tested this remarkable result. He built a large box, suspended it by silken ropes, and connected it with an electrical machine so that sparks several inches long could be obtained from all over it. Meanwhile he was inside with delicate electroscopes, trying, and failing, to detect any sign of electric force there.

§ 559. It has an important practical application in **Electric Shielding.** Any instrument entirely enclosed in a conducting envelope is perfectly shielded from all external electrical disturbance. All that the latter can do is to induce various charges *on the sheath.* This is very strikingly shown by an experiment in which a pith ball hangs inside a soap bubble, an electrified rod is brought near and the soap bubble bulges out to meet it, but the pith ball hangs quite unaffected. Bringing the rod too near, the bubble bursts and instantly the ball flies up towards the rod. Coarse wire gauze makes an efficient shield. Recent experiments have shown indeed that very carefully paraffin-waxed paper is the only substance sufficiently perfectly non-conducting to have no screening action.

§ 560. Since the charge on a hollow conductor is unable to produce lines inside it, **no part of the charge is on the inner surface.** For if it were, lines would arise from it and must pass across the cavity.

And none of the charge remains at rest in the body of the metal by the argument of § 558.

All is on the outer surface, brought there by the pull of the lines joining it to the equal and opposite charges on other conductors elsewhere. And the lines it emits leave the surface perpendicularly, otherwise their 'resolved component' parallel to the conducting surface would tow the charges along it until the pull became entirely at right angles to it.

This absence of Charge inside a closed hollow conductor is easily demonstrated. The hollow conductor may be a tin can, with a $1\frac{1}{2}$-in. hole cut in its lid, insulated by standing on wax or ebonite, and charged. A small insulated conductor called a 'Proof Plane,' say a halfpenny on the end of a stick of sealing-wax, is lowered into the can and touched on its inside, then taken out and touched on a gold-leaf electroscope. No effect. But if touched on the outside of the can and then tested, the leaves of course diverge.

[Notice particularly that if a wire attached to the electroscope and twisted round a sealing-wax handle is lowered in to touch the inside of the can, the leaves do diverge just as much as if the wire touched the outside. For now can, wire, and gold leaves combine to form one conductor, and this is not a hollow or nearly closed one.]

Faraday also showed that an insulated charged butterfly-net gave up no charge from its inside to a proof plane, and that when pulled inside out by a cord attached to its bottom the charge travelled through so as still to be on the outside only.

§ 561. Nothing that has been said precludes the existence of lines inside a conductor provided that they emanate from separate charged bodies inside and insulated from it, and these lines then do induce opposite charges on the inner surface of the cavity. For instance, an electrified rod inside a room.

But it does follow that **if any of these charged bodies is touched on the wall it gives up the whole of its charge to the hollow conductor,** instead of merely sharing it. Thus we can transfer the whole of the charge on anything, a proof plane, for instance, to an electroscope, by standing a deep narrow can on

the plate of the electroscope and lowering the proof plane to touch the can inside near the bottom. Nearly enough it is then inside a 'closed' conductor.

As the charged proof plane is lowered into the deep cavity the leaves spread out, and it will be noticed that the final touching has no sudden effect. This leads on to the whole question of Charging by Induction and its explanation in terms of electric lines.

Charging by Induction

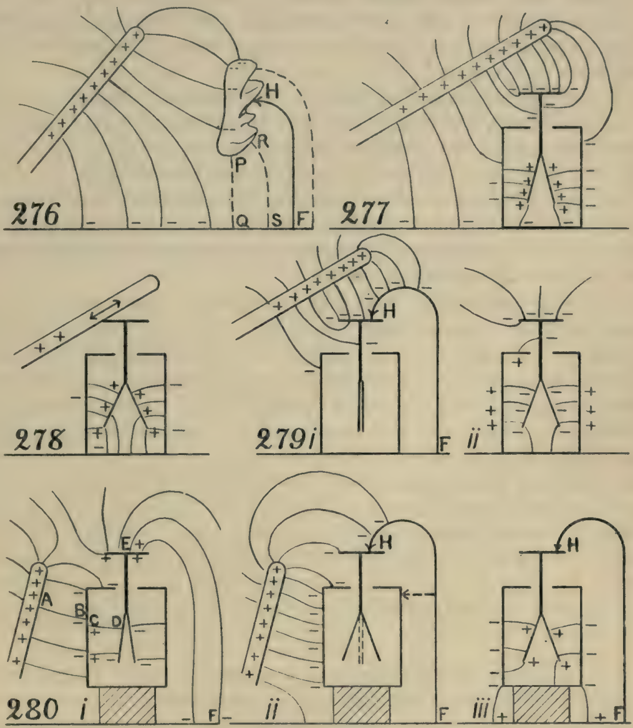
§ 562. We have seen that when a charge is produced by friction there is an equal and opposite charge on the rubber. As the two things are separated the quasi-elastic lines of electric force draw out and spread out so as to fill the surrounding space, but each trying to remain as short as it can consistently with the sideways pressure of its neighbours.

If a magnetic line ran near iron it bent so as to run as much of its course as possible in the iron. When an electric line comes near a conductor it bends towards it and may break in halves, the broken ends on the conductor—meaning that equal and opposite charges are induced on it—and these broken ends (charges) separate without difficulty. The weakening of the magnetic line in iron is superseded by the total obliteration of the electric line in the conductor, and the pieces left at the ends are together shorter than the original line.

Thus the lines of the field very soon resemble Fig. 276, and now if, as usual, the rubber is not a good insulator and is connected to earth by the experimenter's body (represented by the earth wire HF), the lines (shown dotted) joining it to the table have both ends on the same conductor (table-earth-body-rubber) and therefore pull together out of existence. There is left the charged rod radiating lines to a collection of charges on table, experimenter, walls, etc., totalling the same value of opposite sign.*

* The line PQ in the figure is as short as it can be, why should the charges PQ travel a long way round to meet and neutralize each other? Recollect that this line is not alone. There are some lines running near HF, these shrink up along it as described, leaving a vacancy in the field which is filled up by the sideways expansion of the tubes of force of which PQ, RS, etc., are the axial lines (cf. § 525). RS, etc., therefore continually approach HF and presently disappear as they pull R and S together in it (*a current flows in* HF). The tube of which PQ is axis swells continuously and its strength is continuously sapped by HF; axis PQ quickly loses all significance even if it does not move.

§ 563. Now bring the charged rod near an insulated conductor, conveniently the cap stem and leaves of a gold-leaf electroscope. Near the cap, for of course the lower end and the leaves are screened by the metal box. This box now represents part of



Figs. 276–280.

the earth, and lines will shorten on to it, but some will find a still easier course to earth via the conducting stem, breaking into a piece from rod to cap and a piece from leaves to box ; these latter pieces pull the leaves open. Fig. 277.

In other words, an opposite charge is induced on the parts of the conductor nearer to the rod, and as it is insulated, this leaves an

equal charge of original sign in the farther parts. This latter charge then induces opposite charge in the conductor near those parts, i.e. in the enclosing box. The box's residue of original sign has the whole earth to spread over.

Thus the leaves begin to open out with charge of the same sign as on the electrified rod. If the rod is removed they close again, but we can give them a permanent set in two ways:—

I. By wiping the rod on the cap, or if instead of a glass rod it is a charged *conductor*, merely touching it on the cap. This makes a conducting path along which the lines joining rod and cap shrink up and disappear, the charges neutralizing one another, Fig. 278. The 'field' between rod and cap has been destroyed and the rod can be taken away, but that between leaves and box remains as it was, the leaves remain permanently apart, charged with electricity of the same sign as on the rod.

II. '**By Induction,**' Fig. 279. While the inducing rod is in position touch the electroscope cap with the finger and so 'earth' it, HF (i), thus putting it in conducting connection with the case. The field between leaves and case disappears and the leaves therefore collapse. But the field between rod and cap remains unchanged, and there is a considerable negative charge on the cap with many lines from the rod ending on it. Now removing the rod these lines spread about and as there is a conductor (table +electroscope case) in the neighbourhood many of them break into two pieces, one from rod to a near part of table, and one from case to a near part of (cap+stem+leaves) i.e. to leaves. Thus, Fig. 279 (ii), the leaves now open with a charge *opposite* in sign to that on the rod. This is commonly called 'charging by induction.'

§ 564. If the electroscope case is insulated, as by standing on ebonite or wax, more puzzling actions can occur. The approach of a charged rod cannot now cause so large a divergence of the leaves. Each line that breaks and jumps from rod to leaves via the table has now to make also an intermediate jump between table and case. The shortening obtainable along this course is not much, accordingly fewer lines are induced to take it and the available forces are diminished.

It is now practicable to charge the leaves 'by induction' with the *same* sign as the rod. Hitherto stem and leaves have been the insulated conductor and case was earthed. Now use the

case as insulated conductor and connect the leaves to earth by *keeping* the fingers on the cap. Bringing the rod near *the case*, lines will run into it and thence to earth, via the leaves, pulling them open, Fig. 280 (ii). [Note that this does not contradict § 558, for the case has an aperture through which another conductor, the stem, etc., has been inserted.] Touch the case momentarily (dotted line), case and leaves communicate, field inside vanishes and leaves collapse. That between rod and case remains and the case has a charge opposite to the rod. Remove the rod, the usual spreading of lines occurs, some break and go from case to table via the leaves and cap, and the leaves are pulled open with the *same* sign as on the rod, Fig. 280 (iii).

Fig. 280 (i) suggests why if the leaves are not earthed they scarcely open when a rod is brought near the insulated case, but open widely when the cap is touched, (ii). Such paths as AB +CD+EF are long from rod to earth, as in the last paragraph, and will be traversed by but few lines (forced into that course by the sideways pressure). When the necessity for EF has been done away with by a conducting connection, D becomes ' earth,' such courses as AB+CD are short and many lines crowd along them, pulling the leaves wide open. Ex. 5 is left to the reader to puzzle out.

§ 565. The **Electrophorus** (electricity carrier) is an important instance of charging by induction. It is the simplest sort of ' electrical machine ' by means of which considerable quantities of electricity may be obtained without continual waste of labour in friction.

On the table lies a plate of ebonite, glass, etc., or a sheet of scorched brown paper, rubbed or brushed to electrify it as usual. Upon this is laid a smaller plate of thin metal, usually a disc of tin or brass 3 to 6 in. diameter, to which is attached an insulating handle. Brown paper, and the lid of a tin which has been stuck while hot on to half a stick of sealing-wax, is a homely combination but works as well as anything.

Any sharp corners and edges on the metal plate should be smoothed off or the charge would readily leak from them into the air (§ 666). The handle should be held by its upper part only or the charge may leak to the fingers. The handle should occasionally be cleaned by a rag moistened with spirit.

The electrical condition is now represented by Fig. 281 (i). The — charge on the ebonite receives lines from a + charge induced up

towards it on the table top, a few of these arch over via the metal plate which has a small + charge induced on its lower surface and a — left on top. The metal is by no means in that close contact which is necessary to actually pick off charge from the electrified

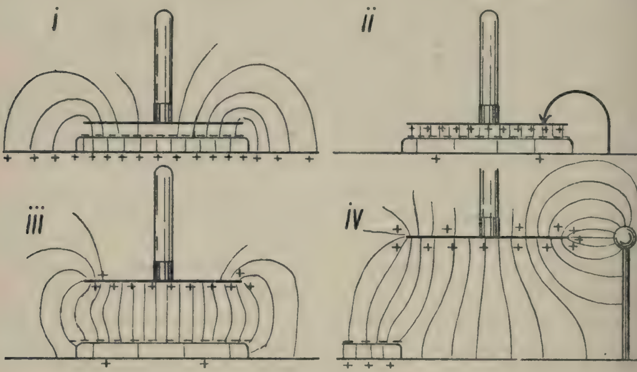


FIG. 281.

surface (§ 553), probably this occurs at only a few small patches, *therefore it is shown in the diagram as distinctly separated.* Now touch the plate, i.e. earth it, Fig. 281 (ii) ; the shortest way to an opposite + charge is now across the narrow air gap, the vast majority of lines take that course, the rest of the field practically disappears, any pith balls, electroscopes, etc., standing near, which had hitherto been perturbed, now hang dead.

Remove the finger. Lift the plate by its handle ; the lines are drawn out (Fig. iii) just as were those between rod and rubber in § 562 and presently break off on to neighbouring conductors, hand, table, etc. The slab is left with all its original — charge and the + charged electrophorus radiates lines in all directions (Fig. iv), ready to concentrate on any near object, e.g. the knuckle, and to such an intensity as often to break down the electric strength of the air and pull the opposing charges together in a Spark.

The reader will have seen that the Electrophorus is just a convenient improvement on the two-apple experiment of Fig. 270. It corresponds to the nearer sphere. In one of its many varied forms—a plate with insulating handle, a knob hung by a silk thread, a tray supported on three or four tumblers, a patch of

tinfoil on a glass plate, etc. etc.—it is the most convenient ‘ charged body ’ for experiments.

Work has to be done in pulling out the electric lines, a light plate feels perceptibly heavier to lift, the charged brown paper will often lift with it and have to be torn off. As the system (slab +electrophorus), or equally the system (rod+insulated rubber), was electrically inert before separation, it is evidently this Work done in pulling apart the oppositely charged bodies [and stored as a whole regionful of contractile lines of electric force] that provides the store of electrical energy which can move light stuff, produce the heat, light, and sound of electric sparks, etc.; (cf. § 513).

Electrical Machines

§ 566. **Frictional Machines.** The early machines for producing electricity consisted of cylinders or large circular plates of glass which were rotated against leather-covered pads smeared with tin amalgam. [This substance rendered them much more efficient ; it adheres to glass, as in mirrors, and is torn off by the motion, probably it is this separation that is the effective action.] The electrified glass then came opposite to the sharp points of a metal comb, attached to a brass cylinder or ball insulated on glass legs and called the ‘ prime conductor ’ of the machine. Negative electricity streams off the sharp points to neutralize the + glass, leaving a corresponding + charge on the conductor. Sometimes the rubber also was insulated so that its negative charge could be accumulated instead of going to earth, but the rubbing pressure required is so great that the rubber's glass legs soon gave way. The machine had to be thoroughly warmed and dried before use, during use the driver's exertions kept it and him quite warm enough.

These ‘ Friction Machines ’ have been superseded by ‘ Induction Machines ’—continuous acting improvements on the electrophorus—free from this wasteful heavy friction. Two very different patterns of these will be described, the Kelvin water-dropper as an illustration of principle, and the machine invented by Mr. Wimshurst as the most successful machine in practice, standing in something the relation to the electrophorus that the rotary newspaper press does to the old hand platen.

§ 567. **The Kelvin Water-Dropper**, Fig. 282, is a machine any tinman can make. Water drips from a couple of taps on a water-pipe which of course is ‘ earthed.’ Surrounding the nozzles are

insulated metal jackets J J. The drops fall into insulated leaky cans K K whence the water drips to waste. Jackets and cans are cross connected by separate wires as shown, J_1 to K_2, J_2 to K_1.

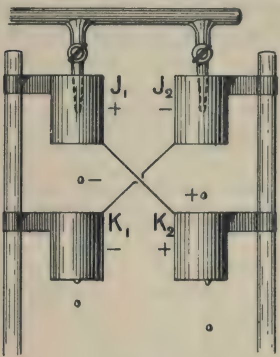


Fig. 282.

Give J_1 a + charge. It induces up from the earth a — charge on to the nozzle and hanging drop inside it, the drop falls off carrying a —charge with it and gives it up almost completely to the can K_1, which at once shares it with J_2. Drops falling through J_2 therefore acquire + charges by induction and give them up to K_2 which shares them with J_1, increasing its activity, and so on.

Thus the charges on all four go on increasing, the energy of fall of the water being converted into electrical energy, until the drops begin to fall wide of the repelling K K and probably spoil the insulating quality of the glass supports.

§ 568. **The Wimshurst machine**. In this there are two glass discs a foot or more diameter and $\frac{1}{4}$ in. apart, rotated rapidly opposite ways by a hand-wheel and open and crossed driving cords. In Fig. 283 they have been represented as concentric drums, in which form in fact they are occasionally made. Sixteen or more short strips of tinfoil are gummed on the outer sides of the plates (inside and outside of cylinders). At opposite ends of the horizontal diameter are double ' combs ' attached to insulated prime conductors. There are also two stiff wires fixed across the machine at 45° and carrying tinsel brushes which just sweep the tinfoil ' sectors ' as they pass.

Give the sector at 1 a + charge. This is done by induction with an ebonite rod, but unless the machine is too damp the trifling friction of the brushes will electrify it enough for a start. 1 moves to position 2, here it induces a — charge along the 45° rod up to the inner sector at I, while J gets the residual + charge. When 1 arrives at 3 it gives up nearly all its charge to the points of the surrounding comb and the attached prime conductor gets a + charge. Meanwhile I moves on to II and here induces a + charge on an outside sector at X while the opposite outside sector at Y gets a residual — charge. Thus—

(a) There is kept up a succession of + charges going over the outer ring from X and accumulating on the right-hand prime conductor.

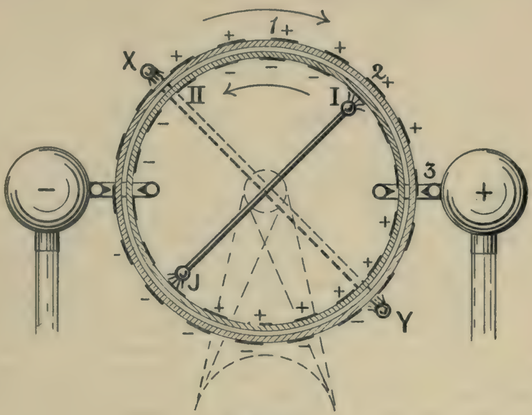


Fig. 283.

(b) A succession of — charges travel from I and pile up a — charge on the left-hand prime conductor.

(c) Meanwhile the + residuals discharged on to the inner sectors at J are travelling to the right, helping in the induction along YX and then joining the accumulation on the conductor.

(d) And the outer sectors carry — charges from Y to the left, helping induction along JI as they pass.

(c) and (d) are replicas of (a) and (b).

The reader will see that these processes go on in a mutually intensifying fashion and in a very few seconds the machine is prepared to give off long sparks from either prime conductor. If there is nothing near enough to spark to, brush and glow leakage (§ 666) takes place and the whole machine fizzes, shines in the dark, and ozonizes the air, producing a strong characteristic smell.

When actively working the machine is perceptibly harder to turn than when rotating idly without charge, work is being done in pulling apart attracting charges.

Wimshurst machines are occasionally made with several pairs of plates. Small machines have also been worked in compressed air, which reduces the abundant leakage from all parts and considerably increases the efficiency.

EXAMPLES.—CHAPTER LV

1. Describe the construction and action of the gold-leaf electroscope. How could the instrument be used to show the absence of charge inside a hollow conductor ? [L]m.

2. How would you test whether a conductor is charged positively or negatively by means of a gold-leaf electroscope ? [L]m.

3. How show that electricity has greater density on corners and points ? Describe two practical applications. [St. A]m.

4. An uncharged metal disc is lowered nearly into contact with the flat top of a charged electroscope. What effect is produced in the leaves if the disc is (a) insulated, (b) earth connected ?

5. The case of an electroscope is insulated, and the stem and gold leaves are given a charge. Explain, in terms of potential or of lines of force, what happens when one connects to earth first the case, then the stem, then the case again, and so on alternately. [L.]

6. How and under what conditions can one conductor be made to give up its charge entirely to another ? How could you ascertain which of two charged conductors had the greater charge ? [L.]

7. A metal can is insulated inside another, which stands on a gold-leaf electroscope. Account for behaviour of electroscope when (1) a + charged ball is lowered into inner can, (2) outer can is earthed momentarily, (3) ball allowed to touch inner can, *or* (4) ball removed without touching.

8. Describe the construction and action of the Wimshurst influence machine. What would be the effect of a small piece of metal held between the rotating plates (a) opposite a neutralizing brush, (b) opposite a collector ? [L]m.

Chapter LVI

ELECTRIC FIELD AND POTENTIAL

§ 569 : The forces acting between electrical charges at a distance can be investigated in a way resembling that of § 523, or by a delicate but troublesome instrument called a torsion-balance, or by the following contrivance :—

Blocks A B slide on a graduated wooden bar, Fig. 284. From them hang by silk fibres L a leaden bullet and P a pith ball, at the same level, both in front of a scale marked on mirror glass. L and P are both charged by an electrophorus. L is heavy enough to hang as a plumb line under all circumstances, by its aid the mirror scale is made to tally with the beam scale.

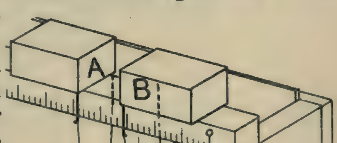


Fig. 284.

Pushing B near A, P is repelled by L, and LP remains greater than AB. The difference LP—AB=deflection of BP from vertical, and so long as this is small it is directly proportional to the repulsion acting on P. [$r/w=$ PN/BN=PN/BP if P not far from N; w and BP are constant. $\therefore r \propto$ PN.]

Putting B at different distances PN is determined at each and it is found that PN$\times$(LP)^2 is constant, i.e. repulsion$\times$distance^2 is constant, or *force* $\propto 1/d^2$.

Now bring up to L an equal uncharged ball on a silk thread, and touch them together, holding the new bullet beside L so that both are at the same distance from P. L's charge is halved

between them,* removing the ball, NP [or strictly NP/LP^2] will be found halved. Repeating, NP is halved again. Or if P is touched with an equal pith ball, NP is halved. Hence one concludes that *force $\propto$ either charge, i.e. $\propto$ product of charges.*

Hence Force $\propto ee'/d^2$.

Defining the Unit Electric Charge as that which repels equal charge 1 cm. away, in air, with the unit force of 1 dyne, this becomes [cf. § 523]

Force of electrical repulsion in dynes
= product of charges ÷ square of cm. distance apart in air.

§ 570: The best Proof of the Inverse-Square Law is that which Cavendish (ca. 1772) based on the absence of electric force inside a hollow closed charged conductor.

Suppose, Fig. 285, the conductor a sphere charged uniformly with e units per sq. cm. of its surface. Place at any point P inside a small test charge. P may be chosen as the vertex of a pair of slender cones; the axis APB of these meets the sphere at the same inclination at both ends, and hence the areas the cones cut out on the surface are proportional to AP^2 and BP^2, and bear charges $\propto e \,.\, AP^2$ and $e \,.\, BP^2$. These are distant AP and BP from P and together produce no resultant force along APB on the test charge at P.

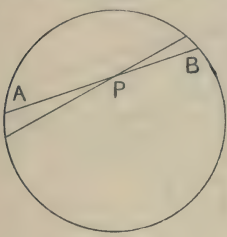


Fig. 285.

This condition is fulfilled by the equation, force $e \,.\, AP^2/AP^2 =$ force $e \,.\, BP^2/BP^2$, or the force is proportional to the charge and inversely as the square of the distance.

Since the whole sphere can be filled with similar pairs of cones with vertices at P, and every pair must fulfil the condition independently, this is the only possible solution.

§ 571: *The strength of the Electric Field at a point is defined as equal to the force in dynes which would act on a unit of positive charge placed at the point.*

Since at 1 cm. from a unit charge in air the test unit would experience a force of 1 dyne there must be unit field-intensity there,

* The examinee who glibly desires one to 'give the ball 2, 3, etc., times the charge' is proposing a task of no ordinary difficulty, requiring elaborate appliances.

i.e. all over the 4π sq. cm. surface of a 1-cm. radius sphere surrounding the unit charge. Representing unit field strength by 1 Unit Electric Line per sq. cm. (in air), we must say that unit charge emits 4π lines.

[$4\pi = 12{\cdot}56$. . . . It may be helpful to think of a blackberry of 12 plump drupels and a smaller one, each representing a unit ' tube ' and the contained seeds the axial lines.

Customarily unit electric charge is looked upon as emitting only 1 unit line, which then is 4π times the strength of our lines. But this merely obscures the similarity of the magnetic and electric formulæ, without making any difference in the end.]

The force on charge E placed in field F is therefore EF dynes.

§ 572 : If E is pushed 1 cm. forward against F, EF ergs of work must have been expended on it, and to push forward *unit charge* s cm. against field F, Fs ergs of work are demanded.

This will be obtainable again by letting the charge move back the s cm. under the force F. It has been stored as potential energy, or as we say the **Electrical Potential** of the charge has been increased.

Having expended 100 ft.-lb. of work on a pound weight by carrying it uphill we have increased its gravitational potential energy by 100 units, we have carried this *unit weight* to a place of 100 units higher (gravitational) potential, simply another way of saying 100 ft. vertically higher. Measuring the work done on this 1 lb. is thus a method of measuring difference of level. It is frequently useful to think of ' charge ' as electrical weight and ' difference of electrical potential ' as difference of electrical level through which it is lifted, the work done in the process being the product of the two.

§ 573 : We can do the same amount of work on a unit charge and therefore rise through the same difference of potential either by working against an intense force for a short distance or a weaker force for a longer distance, just as we can reach the same height by scrambling a few yards up the face of the hill or by walking a few rods on the sloping back. We can speak therefore of a steeper or easier ' potential gradient ' with obvious meaning, and we can draw **equipotential surfaces** analogous to the ' contours of equal altitude ' on a map.

Contours are crowded together where they run across the steep slope ; so equipotential surfaces are close together where they cross parts of the electric field of high intensity.

In the alternative more picturesque method of marking hills, the 'hill-shading' lines are packed closest together where they run down the steepest slopes; just in the same way the unit lines of force are closest in the strongest parts of the field.

Fig. 286 represents roughly the equipotential surfaces and the lines of force between a +charged egg at potential 8 and a

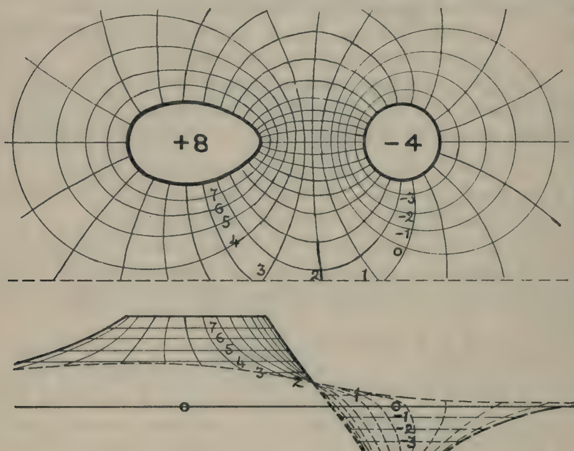


FIG. 286.

—charged ball at potential —4. It might equally represent the contour lines and hill-shading of a flat-topped hill 800 ft. high and a flat-bottomed pit 400 ft. deep, as in the sectional elevation beneath, which gives a side view of the whole of the upper figure. The contours or equipotentials are marked with their ± heights above the zero level.

Difference of potential between two places is of course evidence that electric force would be acting on a charge placed between them, and if there is a conducting path the electricity will be driven along it from the place of higher to that of lower potential. Hence Potential Difference is commonly, in dealing with electric currents, referred to as **electro-motive force,** E.M.F.

It follows that *if electricity is at rest on a conductor* the whol conductor is at one potential, at the same electrical level through out, *the surface of the conductor is an equipotential surface.*

And since there is no force inside a charged hollow conductor

due to any charge upon or outside it, i.e. no work would be done in moving a test charge about anywhere inside it, therefore the inside of the conductor is throughout at the same potential as its surface.

Hence the flat top and bottom in the diagram : a high and a low lake, joined by many a hill-side ' force '—stream lines of water, or of electric force.

The *equipotential surfaces cut the lines of force at right angles ;* for if not the force would have a component parallel to the surface which would cause a potential difference as one moved along the surface, and that is contradictory. Similarly canals dug along contour lines would be full of stagnant water and would cross all the hill-side streamlets at right angles.

Parallel lines of force indicate a field of uniform strength and hence are cut at right angles by equipotential surfaces spaced at equal distances.

§ 574 : **Potential due to a charge on a very small conductor.**

Let the conductor have charge $+e$. At r from it this produces a field strength (i.e. a repulsive force on a unit test charge) $=e/r^2$.

Push the test charge nearer by the very small distance d, the work done against the electrical repulsion $=d\times e/r^2$ and it has arrived at distance $r-d$ from e.

Now $\dfrac{e}{r-d}-\dfrac{e}{r}=\dfrac{ed}{r^2-rd}$ and if d is very small (and it can be as small as ever we like to make it) rd can be neglected compared with r^2, and the expression becomes our ed/r^2. Hence the work done, which is the increase in potential, has been expressed as the difference of two similar quantities, each being (charge $\div$ distance from it). Hence :—

The potential at a point due to a small charged conductor in the neighbourhood (in air) = charge ÷ distance from it, e/r.

Making r infinitely great, $e/r=0$, and the actual potential is theoretically the work done in bringing unit $+$charge from an infinite distance up to the point. Practically, the potential of the earth is arbitrarily chosen as zero, and potentials are reckoned above or below it just as heights and depths are reckoned from sea-level.

§ 575 : If there are several charged bodies the potential at the point is $\pm e_1/r_1 \pm e_2/r_2 \pm e_3/r_3$, etc., $-$ being used for a negative charge. This may be seen from a topographical analogy ; a house

30 ft. high is perched on the side of a small hill 100 ft. above the stream at its foot, but the whole district is on the long slope of a distant ridge and the foot of the little hill is 1000 ft. above sea-level. Very naturally one thinks of the altitude of the house-top in three independent steps, 30+100+1000 or, 1130 'units of potential' (measured by carrying 1 lb. up to it).

§ 576: **Potential of sphere, radius *r*, due to charge *e* on itself.** By symmetry the charge will spread itself uniformly over the sphere and will have the same effect at external points as if concentrated at its centre [much as the mass of the earth attracts gravitationally as if situated at its centre of mass]. For if it were eccentric the potential of the nearer side would be higher, and electricity would be driven round to equalize it. Every point on the sphere's surface is distant r from its centre and hence its potential, in air, is e/r.

§ 577: Evidently the crowding of a large charge on to an isolated conducting sphere would raise it to a high potential, but it by no means follows that every heavily charged surface is at a high potential, for there may be large negative charges on neighbouring conductors, lowering the potential all around them, just like lumps of ice cooling their whole neighbourhood. In Fig. 281 (i) there is shown a large + charge on the earth's surface, yet the earth is at zero potential, it is the proximity of the — on the ebonite plate that keeps it so.

There can be large quantities of electricity at low potential and small charges, or uncharged conductors, at high potential, just as there are large populations in the plains and few or no inhabitants of the hill-tops. Or there can be very different **surface densities** of electricity at different places on one conductor, which is of course at the same potential throughout. If a wire held by a sealing-wax handle be brought from an electroscope and touched on the egg-shaped conductor of Fig. 286 the leaves will open to the same extent wherever it touches, for the conductor (egg+wire +electroscope) is throughout at one potential. But if a proof plane be touched on the pointed end and then carried away to another electroscope there would result a wider opening of its leaves than if touched on the round end. The closer packing of lines shows that there is more charge per square centimetre—a greater surface density of electrification—on the little end, and this spreads to the proof plane. Now when the latter gets away

and is free from the equalizing influence of the conducting surface it will have a higher potential.

NOTE.—The lift of the gold-leaf is of course a measure of the work done on it, i.e. of the *difference of Potential* between it and the case.

EXAMPLES.—CHAPTER LVI

1. Describe experiments to verify the law of inverse squares (a) in light, (b) in electro-static action. [St.A]m.

2. Describe how the inverse-square law may be accurately established for electro-static forces. [L.]

3. Explain the meaning of electric potential. Is a positively charged conductor necessarily at a positive potential ? Can an uncharged conductor be at a high potential ? Show how the approach of an earth-connected metal plate alters the potential and capacity of a charged parallel plate. [L.]

4. How would you find experimentally the field direction near a charged conductor, and how prove no field inside ? [L]m.

5. A + electrified sphere is in the open air 6 ft. above the ground. Draw the lines of force, and show how they would be altered if a large earth-connected metal plate were held horizontally 3 ft. above the sphere.

6. Sketch the lines of electric force in the field of (a) a charged conductor with an uncharged one near it, (b) a charged gold-leaf electroscope, and give an account of the phenomena which the lines of force represent in each case. [L.]

7. Show by a diagram the general arrangement of the lines of force *or* equipotential surfaces when an insulated uncharged conductor is placed in the neighbourhood of a positive charge. Is it possible for a conductor to be all at one potential while it has positive electrification on one part of its surface and negative on another ? [L.]

8. A small positively charged conductor is half-way between an insulated sphere and the walls of a room in the middle of which the sphere is placed. Indicate approximately the lines of force and equipotential surfaces (a) when the sphere is insulated, (b) after the sphere has been earthed. [L.]

Chapter LVII

ELECTRIC CAPACITY AND ENERGY

§ 578: Only very small quantities of electricity can be stored on isolated conductors of ordinary size, for leakage through the air inevitably begins if charged to more than about 200 units of potential.

The charge or quantity of electricity that raises the potential of a conductor by 1 *unit is the measure of the* **Electrical Capacity** *of the conductor.**

Charge e given to an isolated sphere of radius r, in air, raises it to potential e/r, § 576. For this to be equal to 1, $e=r$ and now $e=$its capacity.

Hence the capacity of an isolated sphere in air is equal to its radius in centimetres. [Not proportional to its surface, in spite of the electricity being spread there.]

Then Total Charge=Capacity×total rise in potential.

Thus a football 9 cm. radius could at most hold only about $9\times200=1800$ units of charge.

[*The capacity of an isolated disc in air=diameter÷*π.]

Take advantage, however, of § 577, keep the potential of the charge down, and so obviate its leaking off, by providing another charge of opposite sign close to it. It will now be possible to crowd on much more electricity, and arrangements of this nature are called '**Condensers.**'

§ 579: In the **Concentric Sphere Condenser** used by Faraday a hollow sphere, radius b, encloses the ball, radius a, which is given its charge e by way of an insulated wire passing through a small hole in the outer shell. This causes a potential e/a all over the ball's surface and e/b all over the inside of the shell. Connecting the shell to earth lowers its potential to 0, and that of the inner

* Comparable to measuring the capacity of a tank by finding how much water would fill it a foot deep. Then suppose that all tanks begin to leak under the pressure of 200 ft. height of water.

sphere to $e/a-e/b$, the fixed difference between them. Putting this potential difference equal to 1, e becomes equal to the capacity of the inner sphere, called *the capacity* C *of the whole condenser*

$$\frac{e}{a}-\frac{e}{b}=1 \quad \therefore\ e=C=\frac{ab}{b-a}$$

§ 580. A Condenser much easier to construct consists of a pair of large flat **Parallel Plates,** sheets of tinfoil, for instance, gummed on the inner faces of two pieces of plate glass, spaced apart by bits of glass rod. Giving one plate a + charge by an electrophorus or machine, and keeping the other earthed by touching it, or by a wire to the nearest gaspipe, practically all the electric lines run straight across from one plate to the other, that course being so much the shortest. [Thus one plate catches all the lines from the other, just as the outer necessarily caught all the lines from the inner sphere, i.e. the charges on the plates of a condenser are equal and opposite, hence only *one* is considered in stating the 'charge of the condenser,' or its capacity.]

If e is the charge per square centimetre and each unit emits 4π lines there are $4\pi e$ lines per square centimetre, or the field between the plates has strength $4\pi e$. If d is the distance between plates the work done in carrying unit charge from one to the other$=4\pi e\times d$. This is their potential difference; putting this equal to 1, e becomes the capacity per square centimetre of plate and $=1/4\pi d$, and the *Capacity of an air condenser with parallel plates each of* S *sq. cm. area and d cm. apart*$=\dfrac{S}{4\pi d}$

[This could have been obtained from the sphere condenser, put $b=a+$small thickness d, then C very nearly$=a^2/d=4\pi a^2/4\pi d$ $=$area of sphere $S/4\pi d$. From this expression the radius has disappeared and there is no obligation to keep to the spherical form so long as the plates are close together.]

§ 581. Now the earliest attempt to collect electricity from a machine (a large ball of sulphur rotated in a lathe against a man's hands) was the very natural one of holding a glass of water so that a chain hanging from the 'prime conductor' dipped into it, with the idea that the electric fluid might run down the chain and dissolve in the water. The attempt succeeded, for on going to lift out the chain with his other hand Cunæus of Leyden suffered a shock that scared him horribly.

See now the resemblance between this arrangement and the condensers we have been describing. The observer's hand grasping the glass is an earthed conductor which closely surrounds the charged water inside, being insulated therefrom by the *glass*, and into it a large opposing charge is induced up from the earth. Touching the chain of course connected the opposite charges and they flowed together through the observer's arms and chest. It was soon discovered that a tinfoil coating pasted on outside and inside the glass did better than the hand and the water, and the form of electrical condenser called the **Leyden Jar** was evolved. It is still the most common and convenient pattern for high-potential purposes, Fig. 287. There is an open-mouthed jar of glass, preferably 'flint,' and fairly thin. Tinfoil is pasted on inside and out about two-thirds way up; the glass margin is cleaned and varnished with shellac and well baked. [The shellac surface retains its insulating power better than a glass one.] From a thick wooden disc lying in the bottom of the jar rises a brass stem and knob; disc and lower end of stem are wrapped in tinfoil to secure good conducting connection. Small jars can be made by chemically silvering 'boiling tubes' inside and out, a wire twisted into an open spiral is stuck in for a stem. 'Franklin's Pane' is a sheet of glass with tinfoil pasted on both sides, leaving a 2-in. margin all round, it is not very handy.

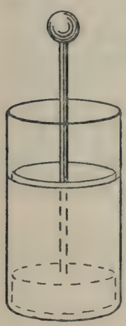


FIG. 287.

§ 582. In all these practical forms of Condenser there is *glass* between the opposing conductors instead of air. Does this make any difference?

Faraday filled in the air space of one of his spherical condensers with *shellac* and found that the capacity was much increased. It took 3 or 4 times as many sparks from the electrophorus before refusing more charge; when it was made to share its charge with a similar air condenser it lost only a quarter or less instead of half, as judged by the spreading it could still produce in an electroscope. And experiments with plate condensers show that glass is more effective still, the capacity with glass between the plates is 6 or 7 times as much as with air.

The insulating material gets the name of the **Dielectric,** as the inductive action takes place through (*dia*) it, and

the ratio of the capacity of a condenser made with the dielectric to that of an equal-sized one with air only, is called the **Specific Inductive Capacity** *of the Dielectric* (*S.I.C. or in formulæ*, k).

The Capacity of a **Parallel Plate Condenser,** area of plate S, dielectric of s.i.c. $=k$ and thickness d, is therefore $\dfrac{Sk}{4\pi d}$

Even an *isolated sphere* is of course a condenser, for the lines from it end on walls, etc., somewhere. Hence the capacity of a sphere immersed in a large block or tank of dielectric is k times its radius.

[The capacity of a pair of **long concentric cylinders** such as a submarine cable, radii a and b, separated by thickness $b-a$ of dielectric k, is $1{\cdot}17k \div (\log b - \log a)$ per cm. length.]

Some **Specific Inductive Capacities** are :—

Paraffin wax or oil, carbon disulphide, india-rubber	2·0 to 2·2
Resin, vulcanized rubber, ebonite . . .	about 2·5
Shellac, sperm oil	3
Gutta-percha	4
Mica, castor-oil	5
Glass	6 to 8

§ 583: We can see now why it was necessary to specify 'in air' in defining Unit Charge, etc. For a k times increase in capacity of a condenser means that k times the charge must be put on one plate to produce the same difference of potential between them. This difference $=$ force $\times$ distance, distance is unaltered, therefore force is unaltered although there are k times as many lines crossing (for we still credit unit charge with 4π lines). In a dielectric therefore an electric line represents only $1/k$ of the strength of field it did in air, just as in a magnetic material a magnetic line represented only ($1 \div$ permeability) of the strength of magnetic field it did in air. *The force between two unit charges* 1 *cm. apart in a dielectric is only* $1/k$ *dyne, and the field strength is only* $1/k$ *the number of lines per sq. cm.*

In addition to the gain of capacity by using glass between the plates there is the advantage that much greater potential difference may be applied without spark discharge ensuing. A $\frac{1}{8}$-in. air gap will stand only about 40 units of p.d., even if it can be kept free of threads of dust, a $\frac{1}{8}$-in. glass plate should easily withstand 500: beyond this there is a risk of the glass puncturing as if by pressure of a sharp punch. There is also the mechanical advantage that the attraction between the oppositely charged plates cannot possibly pull them into contact.

§ 584. **Coupling condensers 'in parallel.'** Any number of jars, etc., are coupled in parallel by joining, by wires or strips of tinfoil, all the right-hand (or inner) plates together in one, and all the left-hand (or outer) plates together in another bunch. The total capacity of this 'leyden-jar battery' is just the sum of the individual capacities added together, and the P.D. to which it can be charged is that at which the weakest dielectric breaks down.

Compact condensers of very large capacity for low P.D.'s (used for cable telegraphy, induction coils, etc., § 605) are made of alternate tinfoils and larger leaves of thin mica, or more cheaply india paper baked dry and steeped in melted paraffin wax, the whole pile being subsequently consolidated between rolls while hot. The odd foils all project at the left-hand end and are soldered together, the even foils project and are soldered at the right. In effect it is a heap of 'Franklin panes' joined in parallel. A pocketable condenser of this construction may equal in capacity a thousand 'half-gallon' leyden jars, though probably it cannot endure a thousandth the electrical pressure without damage [and hence, see § 586, could contain no more energy than a single jar.]

§ 585. **Coupling 'in series' or 'in cascade.'** The left-hand plate of the first condenser is connected to the machine. Its

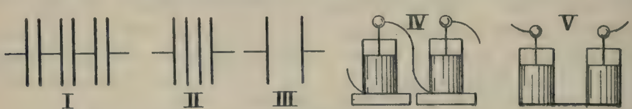


FIG. 288.

right-hand plate is connected to the left-hand plate of the second jar, and so on, as in Fig. 288 (i). The reader will see that this might almost as well be (ii), and now the intermediate plates are doing nothing and might be left out, as in (iii). So that assuming the n condensers all equal in size, this arrangement produces merely one of the same area but with dielectric n times as thick. The joint capacity $Sk/4\pi(nd)$ is only $1/n$th of *one* of them, but the combination is n times stronger to resist excessive charging pressures.

In practice leyden jars can be connected up as in (iv), each well insulated on a glass plate; or as in (v), the common way of

connecting pairs on to Wimshurst machines. [Don't put your knuckle to a machine with jars attached.]

§ 586 : **Energy of charged condenser, and force between plates.** It has been pointed out in § 572 that the work done in carrying a charge from a place of low to one of high potential = charge × difference of potential, and that this work is stored as potential energy in the electric field. The energy stored by a conductor which has been raised from zero potential to V by giving it a charge of E units is not, however, the full product EV. For at first the conductor was uncharged, and the first small fraction of the charge could be brought up to it on a little electrophorus without any repulsion having to be overcome, i.e. without doing any work ; just as the first brick of a wall could be pushed along the ground into position without lifting it, and possesses no available gravitational energy because it cannot fall. The next $1/n$th fraction of the charge has to be brought up against the repulsion of the fraction already in possession, this having raised the potential of the conductor to $1/n$th its final value. The third $1/n$th has to be lifted to a place of $2/n$ths the final potential, and so on, just as successive bricks have to be lifted higher and higher. And precisely as the total gravitational energy stored in the wall (and set free if it falls) is found by considering the height to which the centre of mass has been raised, and is *half* the product of its mass and full height, so the electrical energy of a body which has been raised from potential 0 to V by giving it charge E is $\frac{1}{2}$**EV.**

Thus the energy of the football of § 578 is $\frac{1}{2} \times 1800 \times 200 = 180{,}000$ ergs $= \cdot013$ ft.-lb., just enough to produce a slender thread of light, a little heat, a tiny crack, and a brief tingling in the knuckle brought up to receive the spark which discharges the ball.

§ 587 : **Force between plates of charged condenser.**

As already mentioned in § 565, it is the work done in lifting an Electrophorus that gives it its electrical energy. Regarded as a parallel plate condenser its capacity is about inversely proportional to the distance apart of the plates. Its charge E = capacity × potential difference, and the latter is therefore about proportional to height lifted (i.e. to the length of the parallel lines of force), and so is the energy, $\frac{1}{2}$E × potential difference. Since this energy is equal to the work done in sepa-

rating the plates against their mutual attraction, i.e. to distance $\times$ force of attraction, we can now calculate the force of attraction per square centimetre between parallel plates, or, putting it in a more convenient way, the *Force per square centimetre on a charged plate in a field of given strength.*

In Fig. 289 (left), the plates being supposed large compared with their distance d apart, the electric lines run straight across between them, and number everywhere $F=4\pi e$ per sq. cm., e being charge per sq. cm. of plate. Potential difference $=4\pi ed$. Energy per sq. cm. $=\frac{1}{2}EV=\frac{1}{2}e\times 4\pi ed=2\pi e^2d$. Separating the plates another cm. to $d+1$ increases this, to $2\pi e^2(d+1)$, by $2\pi e^2$ ergs $=\frac{1}{2}Fe$ ergs $=\frac{1}{2}Fe$ dynes $\times$ the 1 cm. lifted.

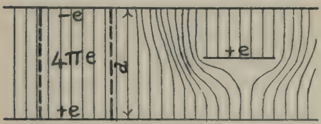


FIG. 289.

Hence the force in dynes acting on each sq. cm. of a charged plate in a field F = *half the product of charge per sq. cm. and field*, $\frac{1}{2}eF$ [or $2\pi e^2$, or $F^2/8\pi$, by substitution from $F=4\pi e$].

How is this to be reconciled with the definition of § 571 that the force on e in field $F=eF$? Consider a square centimetre plate charged $+e$ in the uniform field F between large parallel plates, Fig. 289 (right). F lines connect it with the $-$plate and pull it with force $\frac{1}{2}eF$. F units of charge on the large $+$plate, which originally sent lines straight across to the $-$plate, now send them round the edges of the intruding disc, and their bent parts exert a sideways pressure on it, squeezing it away, and this supplies the remaining half of the total force eF.

The gold-leaf electroscope depends on this attraction between plates—the gold leaf and the wall of the case. In the **Attracted Disc Electrometer** the force pulling a light 'earthed' disc down towards the charged plate is directly measured by springs or counterweights. The expression $F^2/8\pi$ for the attraction shows that the force between plates at a constant distance apart is proportional to the square, $(Fd)^2$, of their difference of potential, and this is what the instrument is employed to measure. In the **Quadrant Electrometer** there is a horizontal figure-of-eight shaped plate of thin aluminium hung by its middle from a torsion-fibre of bronze or silica. The ends of this 'needle' are drawn by an oblique attraction between plates into quadrantal boxes, the rotation measuring the (p.d.)^2.

§ 588 : The fact that the energy of the 'system' increases with the distance between the plates, i.e. with the volume of dielectric traversed by the electric lines, rather suggests that **the Energy is** really contained **in the Dielectric,** which is *strained* by the electrical *stress* and should therefore contain energy (§ 100).

That this strain is very real is shown by the puncturing of the glass of an overcharged leyden jar, or by the following experiment. Two parallel metal rods lie $\frac{1}{4}$ in. apart in a liquid dielectric, carbon disulphide. A beam of polarized light is sent from end to end of the narrow space between them and is stopped by a 'crossed Nicol.' When the rods are connected to the opposite conductors of an electrical machine, however, light begins to get through the analysing Nicol, showing that the liquid has become doubly refracting, and it has much the same depolarizing effect as a bit of glass squeezed between pincers.

The experiment of dissecting a charged leyden jar shows too that the energy is stored in the dielectric. A large tumbler is fitted with removable inner and outer coatings of tin, it is charged, the inside tin is hoisted out by a loop of silk, the outside is pulled off by hand, and the two are laid together on the table. Yet when the inner casing is dropped back and the outer shell put on again the jar will give the usual strong spark to the discharging tongs, or to the experimenter's knuckle, whichever he prefers. Or instead of putting back the tins, some strong sulphuric acid can be poured into the jar, which is then stood in a deep dish of the same conducting liquid ; a wire carried round from the outer will spark to the inner acid just before it touches.

Evidently the function of the conductor has been merely to distribute the charge over, or to collect it quickly from, the insulating surface of the Dielectric. The tinfoils on a Wimshurst (§ 568) act mostly in the same way, with good brushes the machine can work without any sectors at all.

§ 589 : We can say then that the little centimetre square column of Fig. 289 which is responsible for $2\pi e^2 d$ ergs in all, *contains* $2\pi e^2$ ergs per c.c. $=\frac{1}{2}e\mathrm{F}$ or $\mathrm{F}^2/8\pi$ ergs per c.c., in air, or $k\mathrm{F}^2/8\pi$ in a dielectric.

The energy in ergs per c.c. in a Dielectric in which is an electric field F (*represented by* kF *unit lines threading through each sq. cm.*) *is* $k\mathrm{F}^2/8\pi$.

When the dielectric breaks down in a conducting spark the electric lines close in on the spark just as on to a wire, bringing their energy with them to provide its light, heat, etc.

EXAMPLES.—CHAPTER LVII

1. Describe any means of charging a leyden jar. How is it possible to touch first the outside and then the inside without discharging ? [Ab.]

2. The inner coating of a leyden jar is connected to an electroscope and a charge given to it. How would the deflection of the gold-leaves differ according as the jar stood on glass or on the table ?

3. Two equal leyden jars are charged with their inner coatings connected by a wire and their outer coatings connected to earth. The wire is then removed and one jar is placed upon a glass plate, and its outer coating is connected to the inner coating of the second jar. What effect does this produce upon the potential of the inner coating of the first jar ? [L.]

4. Define electrical capacity. How would you determine which of two condensers had the greater capacity ? [L.]

5. Describe the construction of the quadrant electrometer, and explain how it can be used for comparing charges on conductors. [L.]

6. A 24-cm. diam. sphere is charged with 15 units of electricity and deflects an electrometer 48 divisions. On connecting it by a long wire to a 12-cm. sphere deflection is reduced to 36 divisions. Calculate capacity of the electrometer.

[Here $15 \div (12 \text{ cm.} + x) = 48d$
$15 \div (12 + 6 + x) = 36d$, solve simultaneously.]

7. Show that the energy required to charge a condenser of capacity C to a difference of potential V is $\frac{1}{2}CV^2$. What becomes of the energy when the condenser is discharged ? [L.]

8. What is the unit of capacity in the electro-static system of units ? Two hollow conductors have capacities 180 and 40, and they are charged to potentials 30 and 20. Find the change in energy when the second is placed inside the first and in contact with it. [L.]

9. Define dielectric constant or specific inductive capacity. Give the principle of some method of determining it. [L.]

10. Define the electro-static capacity of a system.
Explain the effect on the capacity of two parallel planes, one of which is insulated, of (i) moving them farther apart, (ii) inserting a plate of ebonite between them. [L.]

11. Discuss the changes of charge, potential and energy, that occur when a sheet of glass is inserted between the plates of a condenser (1) when the plates of the condenser are joined to the poles of a battery, (2) when the condenser is charged and disconnected from the battery. [L.]

12. Show that capacity of a sphere = its radius multiplied by specific inductive capacity of dielectric surrounding it. [L.]

13. What is capacity of condenser of sheet glass 2 mm. thick with tinfoils 30 cm. square if s.i.c. of glass = 7·5 ? [L]m.

14. A sheet of gutta-percha 0·15 cm. thick has a tinfoil 30×40 cm. pasted on each side. The s.i.c. of g.p. is 5. This condenser is charged

to 10 electro-static units of potential. It is allowed to share its charge with a similarly coated sheet 0·45 cm. thick. Find the resulting loss of energy of the system in ergs.

15. An air condenser with plates 10 cm. square and ·5 cm. apart is charged with 100 units. Find loss of electric energy when it is plunged under oil of s.i.c. 2. [L.]

16. Show that stress on a conductor due to electro-static action is $2\pi \times (\text{surface density})^2$.

17. Calculate the electrical attraction between two parallel plates immersed in a liquid whose specific inductive capacity is k. [L.]

18. An attracted disc electrometer is immersed in oil s.i.c. 2. Disc is 50 sq. cm. and ·5 cm. from fixed plate, pull is 500 dynes, find p.d.

19. Two plates 2 cm. apart are connected to the terminals of a battery of 60 volts = ·2 unit of p.d. Express the electric field in the air space and find dynes per square centimetre tending to draw the plates together. State how these forces are affected when the space is filled with a liquid of specific inductive capacity K. [L.]

MAGNETISM AND ELECTRICITY

Chapter LVIII

MAGNETIC FIELDS AND ELECTRIC CURRENTS ELECTRO-MAGNETIC INDUCTION

We have now to endeavour to find some connection between Magnetism and Electricity.

§ 590. Experiments made in any of the ways suggested in these last two sections of the book would disclose none. A magnet has no more effect on an electrified body than the unmagnetized steel would have; like most things it is a conductor of electricity, but nothing more. Steel and brass balls can be suspended and electrified, both attract a pith ball but only one moves towards a magnet. A suspended electrified lath makes no attempt to set N. and S. So far there is no connection.

But set the electricity into motion. In the middle of a 2-ft. length of electric-light wire twist a little helix of three or four turns, lay a sewing needle in the coils, and bend the long ends of the wire to touch the outer coating and come near the knob of a charged leyden jar. A spark jumps, the electrical charges travel along the wire, and the needle will be found able to pick up iron filings or to set N. and S.; it has become magnetized by the passage of a 'current' of electricity in the wire encircling it.

In experiments made by Rowland and others a charged disc was quite prevented from exerting any electric attraction on a delicate magnetometer needle by the interposition of an earthed metal plate. But when the disc was spun rapidly the moving charge produced a magnetic effect which was felt through the metal plate, for the needle was deflected.

Lightning has frequently been observed to cause magnetization or demagnetization.

Hence electric charges in motion can affect a magnet; in other words, **an Electric Current gives rise to a Magnetic Field.**

§ 591. Several devices for separating electrical + and − charges have already been described; the flowing together again of these charges constitutes an electric current. But although these devices yield high electric pressures (differences of potential) capable of forcing current through an inch or two of air perhaps, yet the currents they supply are usually too intermittent and always too scanty in total quantity to be of much practical value. The abundant and continuous currents from Voltaic Batteries (Chap. LXIII), in which electric charges are being separated by chemical action, are most commonly used in magneto-electric experiments. The chemical action produces only a very small electric potential difference, only a thousandth or less of that required to produce a *very* small spark in air, consequently a current path of **good conducting** copper, brass, solder, etc., must be provided all the way, and the current is quite unable to pass out of this into the air. And on wire wound in close coils a thin wrapping of cotton, silk, paper, etc., forms ample **insulation**, just to prevent metallic contact of adjacent turns, through which current might 'short-circuit' without travelling the whole length of the coils.

The electric current obtainable from the public mains, and produced by the electro-magnetic machinery of § 601, is of 100 to 200 times higher pressure, and not to be recommended to beginners for laboratory experiments, but it is not till the 'extra high-pressures' of the electrical engineer, 200 to 500 times those of domestic supply, that we again reach the long sparks and the imperative necessity for glass and ebonite insulators that we found in electro-static experiments. And considering that all the unpleasantness arising from a leyden-jar shock is caused by the passage of a current for a few millionths of a second, the reader will understand the extreme precautions taken by an engineer who is supplying current at these pressures constantly.

Suppose then we have a suitable voltaic battery—one or more cells of a 'bichromate' or some large 'dry cells'—by which we can send a strong current through two or three yards of thin copper wire, cotton covered. *The current is defined to travel through the wire from the carbon +plate to the zinc −plate of the battery.*

§ 592. Stretching the wire ¼ in. above a card sprinkled with iron filings, the latter will arrange themselves, when the card is tapped, in short straight lines at right angles to the wire, giving the clearest proof of the existence of a magnetic field near the wire (and cutting the plane of the card in directions at right angles to the current).

As in § 520 a small compass needle is more sensitive than the filings and also tells which way the lines are running. Stretching the wire E. and W. and bringing it just above or below the compass will not tell us much, for we have just seen that the field due to current is perpendicular to it, and being thus N. and S. is merely added to or subtracted from the earth's controlling field, without altering its direction. But holding the wire more or less N. and S., parallel to the needle, and bringing it above or below, the needle will be seen to deflect opposite ways in the two cases, and ultimately set practically perpendicular to the wire when very close. And its movement will be found to agree with the Rule—Swimming in and with the current, facing the magnet, the north pole moves towards your left hand. As the N. pole sets 'down stream,' this **Ampère's Rule** may be more generally stated thus—**Swimming in and with the current, the field in front of you runs towards your left hand.**

If the wire is stretched on a level with the compass the needle is not deflected E. or W.—there is no field straight towards or away from the wire—but one or other pole ducks down, and as one would have to swim on one side to face the needle, it is evidently obeying the Rule.

The photograph Fig. 290 shows the lines of filings* round a wire which ran vertically up or down through the paper.

Taking the results of these experiments altogether it will be evident that the magnetic lines are circles surrounding the current, these circles lie in planes perpendicular to a straight current, and the direction in which the lines travel round (i.e. N. poles move) is given by Ampère's rule or by a rule easily derivable from it—that the directions of the current and of the magnetic lines are related like the forward motion and the rotation of an ordinary screw. Notice that there is no tendency to attract a pole directly towards the wire, nor any tendency to drag it along the wire, the magnetic field is strictly perpendicular to both these directions.

* That these lines look coarser than those in **Figs. 243–8** is merely due to a larger scale of reproduction.

§ 593. Fig. 291 is a photograph of the filing lines due to the combined action of a magnet's N. pole and current flowing up through the paper. Recollecting that the lines are on the

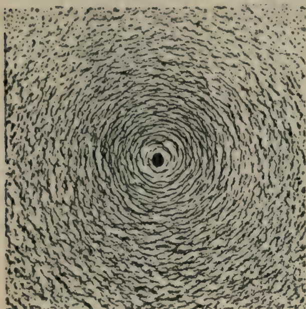


Fig. 290.

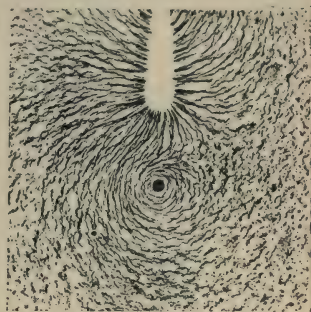


Fig. 291.

stretch this figure plainly shows that the N. pole will be pulled to the left of a swimmer up the wire and facing it ; *or conversely*, if the magnet is more securely fixed than the wire, *that the wire is forced bodily towards the right*, ' action and reaction being equal and opposite.'

A few instances of this mutual action between magnet pole and current-carrying conductor * may be given here :—

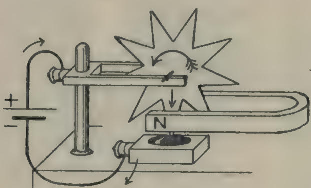
Barlow's wheel. The battery, represented at the left, Fig. 292, sends a strong current through the axles, vertically down the prong of the copper star-wheel which happens to be lowest, into the mercury in which it is dipping, and back to battery. A horseshoe magnet sends magnetic lines horizontally through the wheel from front to back. According to the Rule this magnet's N. pole would be pulled to the left, therefore the wheel gets a push horizontally to the right and rotates in the direction of the arrow, usually in a feeble fashion that hardly suggests this machine as the great ancestor of the electro-motor.

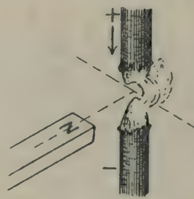
A light compass needle brought near the arc-lamp carbons of Fig. 293 would have its N. pole driven to the left ; a larger magnet

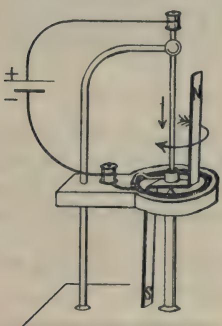
* Of course it is the conductor that experiences the force. A magnet held over a sheet of tinfoil in which current is flowing does not distort the lines of current flow at all, but tends to push the whole tinfoil sideways.

drives off the current-carrying flame of the arc itself towards the right (at right angles to the direction of approach of the pole) and may stretch it so much as to extinguish it.

Fig. 294 shows an apparatus in which the N. pole of a pivoted crooked magnet is feebly driven round and round a strong current.







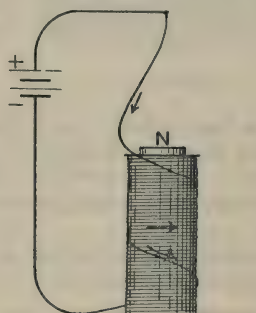


Fig. 292. Fig. 293.
Fig. 294. Fig. 295.

Fig. 295 shows how a slack current-carrying wire will wind itself round the leg of a great magnet, or will unwind and wind on the opposite way when the current is reversed.

In Fig. 296 a heavy magnetic needle is pivoted in the middle of a 6-in. coil of wire hung by two long thin wires through which current is supplied. The coil is at first suspended in the magnetic meridian so that magnet and coil lie together much as in Fig. 297. The current circulating perhaps 100 times round the coil is equivalent to a 100 times greater current passing once down and up: observe how the lines of force are wrenching magnet and coil round opposite ways. In the experiment of Fig. 296 the magnet swings out one way but comes to rest at a deflection

such that the couple exerted on it by the earth is equal and opposite to that due to the coil. And the coil swings round the

FIG. 296.

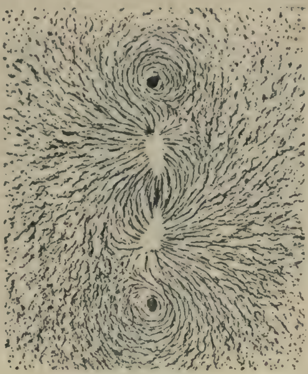


FIG. 297.

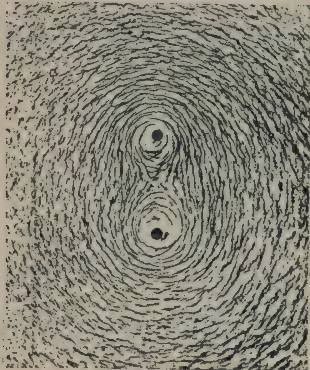
other way under the reaction of the magnet till the twist on the suspending wires checks it.

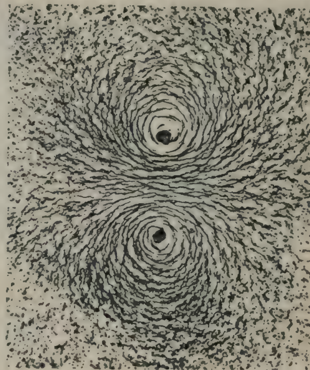
§ 594. Now look at Fig. 298, a photograph of the iron-filing lines surrounding two parallel wires both carrying a current in the same direction. The circles have flowed into one another, and the oval lines, like elastic bands, are evidently pulling the wires together.

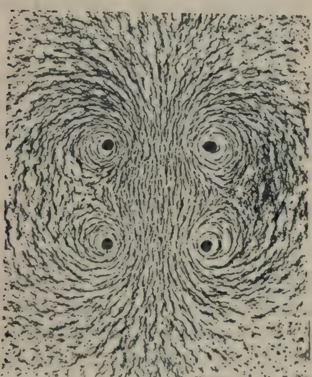
Contrast Fig. 299, where one current is going down through the paper and the other coming up. The circles running opposite ways round get squeezed up and are no longer concentric with the wires; recollecting the sidewise pressure between the lines (§ 520), it is evident that the wires are being pushed apart.

This action, the direct attraction between parallel conductors carrying currents the same way, and the direct repulsion between conductors carrying currents opposite ways, was discovered by Ampère. It can be easily observed in two thin wires hung from a picture nail and almost touching along their whole lengths. A current from two or three large cells, sent up one and returning by the other, causes them to bulge apart an inch or more; if sent up to the nail by a third separate wire and returning by both wires they cling together closely. Fig. 300 shows both attractions and repulsions.

§ 595. Fig. 302 (A) shows two currents, one down at D, the other up at U. The magnetic lines due to *one* current only are shown, and this one current may be regarded as *entirely replaced*







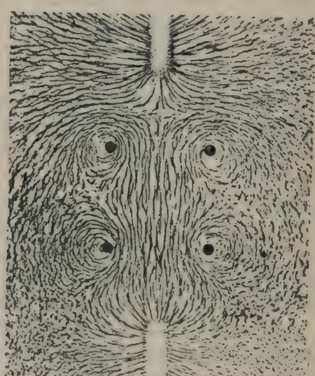


FIG. 298. FIG. 299.
FIG. 300. FIG. 301.

by its magnetic field. Fig. B shows the lines running from a magnet and at right angles to them the current-carrying conductor U.

In A, we have just seen that the action is a motion of conductor U directly away from D. In B, it is a motion of U to the right

(Fig. 291). We see that both these can be described as the same action, under one general rule :—

A conductor carrying a current moves so as to cut across magnetic lines.

The Direction of the Motion is always obtainable by careful application of Ampère's rule ; *swimming in the current and facing the place the lines come from, that place must move off to the left, i.e. the conductor is pushed to the right.* Or a mnemonic device perhaps quicker of application in many instances is this :—

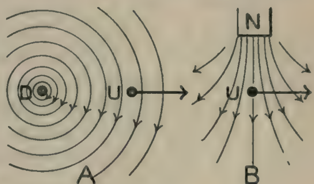


FIG. 302.

Hold up the Left Hand, thumb and index finger outstretched, middle and other fingers naturally partly bent; then a current flowing out along the middle finger, across magnetic lines running out parallel to the index finger, is acted on by a force out along the thumb.

The conductor will always endeavour to move so as to cut most lines, i.e. at right angles to itself and at right angles to the magnetic lines.

This way of always reducing the experimental conditions to a current flowing across a magnetic field may seem a one-sided way of looking at the problem, but it is the way along which the electrical engineer has made all his progress.

Looking at Figs. 292–6 in this new way, it will easily be seen how the moving wire is 'mowing down' as many magnetic lines as it can.

§ 596. Now if the current flowing across a magnetic field causes a body-moving force on the conductor, what will happen when an empty conductor is bodily moved across a magnetic field ? Will there arise an electricity-moving (electro-motive) force tending to drive a current along the conductor ? This is by no means the only thing that might happen, but experiment shows that it is what actually does happen.

For instance, instead of attaching a battery and watching Barlow's wheel go round, Faraday attached a galvanometer and spun the wheel by hand ; an electro-motive force was induced in each of the succession of spokes as it crossed the magnet's field, and a current was driven through the galvanometer.

Hence the **fundamental statements of Electro-magnetic Induction** can be put as follows :—

A conductor carrying a current tends to move across a magnetic field so as to cut the lines.

Forcibly moving a conductor across a magnetic field so as to cut the lines tends to make a current pass along it.

Which way will the induced current flow ? Suppose it went the same way as before, the way which would assist the very motion that produced the current. The motion would go on faster, causing a greater current, which would help more, and so on, always faster and stronger without any help from without. This would be the Perpetual Motion, ever vainly sought for through the centuries. Therefore

The current is always in such a direction as to oppose the motion inducing it. Its direction is the reverse of that found in § 595.

This is **Lenz's Law,** it is another fundamental statement of electro-magnetic induction, it is the appropriate form of the principle of the Conservation of Energy.

§ 597 : Now I. *How great is this force that acts on the conductor carrying a current in the magnetic field ?*

And II. *How great is the electro-motive force that tends to drive a current along a conductor moving across a magnetic field ?*

The answers to these two questions constitute the actual Definitions of the Units of Current and Electro-motive force fundamental in Current Electricity [or Electro-magnetism, or Electro-dynamics].

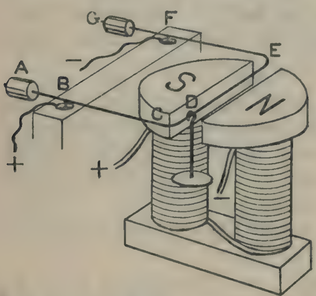


Fig. 303.

Fig. 303 represents an apparatus which, though incapable of accurate results, serves very well to suggest how both questions are to be dealt with. ABCDEFG is a frame of wire pivoted at B and F in mercury cups scooped out in a fixed wooden bar. Through the mercury in these it makes good conducting connection with the remainder of a circuit. A scale-pan hangs at D on CE and the whole frame is exactly balanced by counter-weights at AG.

The straight wire CE moves up and down, parallel to itself and at right angles to the lines of the magnetic field in the narrow gap between the pole-pieces of a magnet NS.

For simplicity, suppose this field uniform in the gap and negligible outside it. It can be measured by magnetic methods and we can therefore tell to start with how many lines would be cut if the horizontal wire CE moved 1 cm. vertically; it = lines per sq. cm. × no. of sq. cm. the wire sweeps over in its motion = *field strength* × *length of wire in field* × 1 *cm.* Let this total number = n.

Now **I.** Load the scale-pan at D with n dynes. Send a current along CE so as to lift it, and adjust the current till there is equilibrium again, i.e. the upward force acting on CE is equal to n dynes. This is then the theoretical **Unit Current,** the **Decampère** [= **10 Ampères**].

The Force on the conductor (*in dynes*) = *current* (*in decampères, hereby defined*) × *lines cut when conductor moves* 1 *cm. in direction of force.**

Or—If a wire crosses at right angles a magnetic field of unit strength, and Unit Current flows in the wire, there will be a force of one dyne exerted on each centimetre of it.

The Ampère is one-tenth of this. It is the practical unit.

To get a great force a large current must cross a broad and strong magnetic field.

II. Move the wire CE at the steady speed of 1 cm. per sec., so that it ' mows down ' magnetic lines at the rate of n per sec. The electro-motive force† caused in CE = n units of e.m.f.

The Electro-motive Force in a conductor is equal to the number of unit magnetic lines it cuts per second.‡ This is the modern form of **Faraday's Law** of Electro-magnetic Induction.

Or—If a conductor is moved so as to cut one unit magnetic line per second, Unit Electro-motive Force arises in it.

The **Volt** = 100 million times this unit [and even then proves to be only $\frac{1}{300}$ the electro-static unit of potential difference].

To get a high e.m.f. a great length of wire must be moved rapidly across a strong magnetic field.

* Supposing field uniform. It is the ' space rate of cutting ' for those who comprehend that expression.

† Or ' the electro-magnetic measure of the potential difference ' caused between C and E.

‡ Supposing speed uniform. It is the ' time rate of cutting.'

What current the electro-motive force succeeds in setting going depends on how good-conducting is the circuit of which the moving conductor forms part.

Example 1. Calculate the total force in dynes acting on a 30-cm. length of wire which is carrying 20 amp. at right angles to a magnetic field of 5000 unit lines per sq. cm.

$$\text{Force} = 2 \text{ decamp.} \times (5000 \times 30) = \underline{300{,}000 \text{ dynes.}}$$

Ex. 2. A telegraph wire carries a current of ·1 amp. magnetic east in the earth's horizontal field (·18). The wire weighs 1 grm. per cm. By how much will its weight be apparently increased or diminished by the electro-magnetic action ?

$$\text{Diminished by } \cdot 01 \times \cdot 18 \div 981 = \underline{\cdot 0000018 \text{ grm. per cm.}}$$

Ex. 3. The wire in Ex. 1 is moved at right angles to itself and to the field at a speed of 15 cm. per sec. What difference of potential is induced between its ends [e.m.f. in wire] ?

$$(5000 \times 30) \times 15 = 2{,}250{,}000 \text{ lines cut per sec.}$$
$$= \underline{\cdot 0225 \text{ volt.}}$$

§ 598. Fig. 299 illustrates also the action of a coil of wire of one turn (or of several hundred bunched into one) round which the current circulates. Notice that in the middle the lines are all going one way. and just in the centre are perpendicular to the plane of the coil, uniformly spaced, and shortly parallel, i.e. the field is approximately uniform for a small space hereabouts.

In Fig. 300 the current is going down the two wires on the right and coming up the two on the left ; this is a coil of two turns, the small beginning of the long helical coils or **Solenoids** (Σωλεν, an eel) familiar in electrical apparatus. Notice that the lines run along the axis of the coil, where they tend to keep uniform and parallel. Consequently a pair of ring coils such as these, or a long coil, is of great use when a uniform magnetic field is required, e.g. for measurement, or for magnetizing steel magnets uniformly.

The running of lines out from one end and into the other end, shown to perfection in the photograph Fig. 301 where two magnets have been placed with their N. poles near the ends of the 'solenoid,' indicates that the coil acts like a magnet (with the distinction that now the return of the stream through the interior is traceable). A few dozen turns of wire wound on a paper tube and connected to a 'dry cell' make a coil whose opposite ends attract and repel a compass needle just like rather feeble magnet poles. It does not matter whether the coils are in one or more long layers (solenoid) or bunched into a ring.

If the inside of the long coil is filled with iron, many score times more lines will flow through, because the iron is so very permeable, and we obtain a strong **electro-magnet.**

Thus an Electro-magnet is easily made by winding several turns of insulated wire round a wrought-iron bolt and connecting the ends of the wire to a battery. The turns must all go the same way round, but whether they run up or down the iron, or in how many layers, makes no difference. If only a weak current is available there must be many hundred turns: the total flow of current round each cm. length of iron must be kept large.

In winding a 'horseshoe' the wire must cross over between the legs and wind on them opposite ways, Fig. 303, to produce the opposite poles required. (Straightening out the horseshoe, this would form a continuous coil.)

The N. pole of the iron, from which lines run out, is towards the swimmer's left as he faces the iron, by Ampère's Rule. The current enters the magnet in Fig. 303 by the wire marked +.

Soft-iron electro-magnets are much stronger than permanent steel ones; they let go when the current is cut off and are immensely useful in all sorts of electrical machinery.

§ 599. **The Electro-motor.** Suppose a rectangular loop of wire ABCDEF, Fig. 304, free to rotate on axis XY, in a cylindrical space between the pole-pieces N S of a magnet, where magnetic lines are running across as dotted. A current is sent from A round to F, there will be a force on BC lifting it upward and on DE pressing it downward, and the loop will turn till it stands vertical, when the vertical forces can turn it no farther.

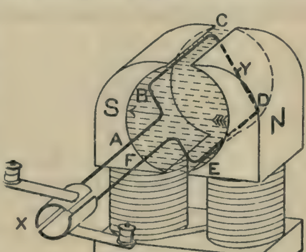


FIG. 304.

Suppose, however, that its inertia carries it on, and also that as it passes this vertical dead-point the current is reversed so as to flow from C to B and from E to D: BC, now on the right, is driven down and DE is driven up on the left, i.e. the loop continues to rotate in the direction SCN.

The usual way of making the machine itself effect the reversal of current is shown at X. The wire ends are attached to two

half-cylinders of copper enclosing the axle, quite separated from each other by insulating material (mica, etc.). Against these press two fixed 'brushes' of copper or block graphite to which a continuous current is supplied. When the loop is vertical the insulating gap has come under the brushes, a moment later the copper segment that had just escaped from the left-hand brush slips under the right, and vice versa, so that the current is now being sent into the loop the other way round.

The actual electro-motor suited to work with continuous (or 'direct') current is this machine modified in detail :—

(1) There are many similar loops of wire arranged at equal angles to fill the whole circumference. The half-cylinders of the 'Commutator' * are slit up into narrow strips so that each loop gets its pair of segments. The loops are also all connected to one another end to end (in series), and consideration will show that the effect is merely to get a stronger and more continuous rotation, the principle being quite unaltered.

(2) The cylindrical space is nearly filled with a mass of soft iron. This enormously increases the number of magnetic lines, and therefore the forces acting. Whether this iron core stands still or rotates makes little magnetic difference, consequently for mechanical reasons the wire is wound on the iron and this whole massive 'Armature' revolves.

§ 600: Now let us turn to further instances of the production of electric current by moving a conductor across a magnetic field.

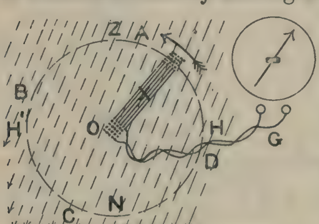


Fig. 305.

Consider first the **Earth Inductor**, shown, in section by the N.S. plane, in Fig. 305.

Taking a rectangular loop of a few score turns of wire, with its ends connected to a sensitive galvanometer G, hold one horizontal side magnetic E. and W. and steadily rotate the loop on this as axis in the earth's magnetic field. This side does not move, two sides move in planes parallel to the lines and cut none; attention can therefore be confined to the fourth side only. As this moves near A it is cutting lines fast, and the electro-motive force in-

* If alternating current is supplied to a motor the commutator can be dispensed with.

duced in it drives a current which deflects the galvanometer needle strongly to the right. Approaching B it is cutting across lines much slower and the galvanometer needle creeps back towards zero. At B there is momentarily no cutting, towards C it begins to cut lines the other way and the galvanometer swings to the left, reaching maximum at C, zero at D, and so on.

Thus an alternating current is being produced.

There is no obligation to use one side as axis. For suppose the axis at X, the fourth side moves only half as fast, but the first side, in which the wire runs *back*, is now also cutting lines *the other way* ; $\frac{1}{2}-(-\frac{1}{2})=1$; i.e. rotating the coil on a central or any other parallel axis has the same effect.

And further, the shape of the coil does not matter, so long as its area remains the same. At OB the number of lines passing through = field strength $\times$ area of coil ; arrived at OD all these pass through the reverse way. The total change = total lines cut = twice field strength $\times$ area of coil, i.e. if the area is the same the induced e.m.f. is the same whatever parts of the wire happen to do the actual cutting.

The electro-motive force = rate at which lines are being cut.

So long as no additional obstruction is placed in a wire circuit the current moved in it is proportional to the electro-motive force [Ohm's Law, § 615].

Hence current is proportional to rate of cutting lines, e.g. in this apparatus to speed of rotation.

Multiplying both sides by the Time spent in the process

Current $\times$ time of flow $\propto$ rate of cutting lines $\times$ time spent.

The left-hand side is the total *Quantity of Electricity* induced to move past any particular point in the circuit.

$\therefore$ *Quantity $\propto$ total number of lines cut.*

This is a general and important result. The rush of electricity is often too rapid for the moving parts of a galvanometer to keep pace with, but a heavy slow-moving 'ballistic' galvanometer will give a scale-swing proportional to the total Quantity that passed in the rush, just as a heavy pendulum swings out proportionally to the whole momentum of a bullet shot into it.

In the Earth Inductor, for instance, rotation from B to D gives a swing proportional to whole area of coil $\times$ earth's total field (§ 540). Turning over from H to H′ gives a less throw, it misses lines at the start and cuts some backwards at the end. But resolving the field into the Horizontal and Vertical Com-

ponents, this turning over flat on the table gives a throw proportional to V, and turning over from N to Z or in any manner from facing north to facing south, gives a throw proportional to H. Hence the apparatus can be used to find the Dip, etc.

Moving the coil parallel to itself produces no current, for the following half, in which the wire is coming back, cuts as many lines as the leading half, and neutralizes the induced e.m.f.

§ 601. **The 'Dynamo.'**

Replace the earth's field by the field of a magnet as in § 599. Taking Fig. 304, instead of supplying current, turn the loop round by hand. The two sides of the loop co-operate to produce an Alternating Current, and this can either be led out as it is, or be passed through the commutator, where the brushes gather rushes of current, always passing out at the same brush [left-hand, §§ 595, 596].

Then by multiplying loops of wire and using soft iron, as before, one gets a more uniform current from a much more compact machine, the **electric generator, dynamo-electric machine,** or **'dynamo'** of commerce.

It is precisely the same machine as before, with a new name and function.

As Motor it is supplied with current and does work.

As Dynamo it is supplied with mechanical energy and produces current.

The 'magneto' of petrol engines is a small dynamo in which the magnetic field, is more conveniently supplied by steel permanent magnets instead of the usual electro-magnets. It has very many turns of fine wire on its armature and therefore produces a very high electro-motive force.

The crank one turns on a telephone drives a magneto which sends alternating current 'to line' and round electro-magnets at the far end. These alternately pull and push a magnetized steel rocker, thus oscillating a hammer between two bells.

The medical magneto-electric machine is an ancient pattern in which a soft-iron yoke is driven round, by hand-wheel and gearing, so that it alternately bridges the polar gap of a steel magnet and stands at right angles to this position. Thus a thick stream of lines alternately pours through the iron and is wrenched out of it, thereby cutting the wire wound in two bobbins on the iron and producing an alternating current, which is led out to the handles.

§ 602. So long as there is a mutual cutting of magnetic lines and conducting circuits it does not matter in the least whether the circuits move and the lines stand still, or the lines move and the circuits stand still, or perhaps both move. There follow some instances of Moving Lines.—

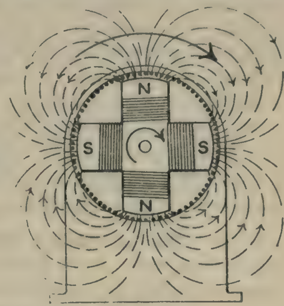


Fig. 306.

In some dynamos it is more convenient to move the magnets and keep the loops of wire fixed, e.g. in the 7000-h.p. alternators which supply the underground railways of London a huge cross-shaped electro-magnet is driven at 1000 revs. per min., and its magnetic lines sweep across the strands of wire shown in section as dots in the fixed cage, Fig. 306, inducing in them an e.m.f. averaging 11,000 volts but reversed four times per revolution.

Returning to laboratory dimensions, pushing a magnet's pole towards a coil of 50 yards or so of wire will cause a deflection in a low-resistance galvanometer connected to it, showing that a current is circulating in the coil as the lines, moving with the magnet, cut through the wire.

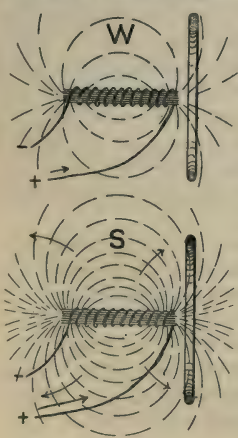


Fig. 307.

The direction of the current is most easily found by recollecting that it will always oppose the motion [Lenz]. Thus facing the coil and pushing N. pole towards it the current circulates against the clock, giving the face of the coil N. polarity so as to oppose the oncoming pole. The current continues till the magnet is half-way through, when the lines, now parallel to the magnet, cease to be cut. Then an equal reverse current flows as the S. pole passes through.

Now this might have been an electro-magnet, and then instead of moving it, it could be magnetized in position by sending a current round it. The effect of this is a rapid spreading of magnetic lines as

the magnetism strengthens, the weak Fig. 307, W changing to Fig. 307, S. These spreading lines cutting the coil induce in it a current resisting the magnetizing process, just as previously it resisted the coming of the magnet. When the magnetizing current is stopped the lines collapse again on the failing electromagnet, and cutting the coil as they move, now induce in it an equal direct current tending to hinder the demagnetization.

And the effect will be the same, though weaker, if there is no iron present at all, § 598. So that sending or stopping a current in a coil of wire induces in a neighbouring coil transient currents tending to oppose its starting or to prolong its running.

§ 603. **The Alternating-current Transformer** is developed from this pair of coils. In a typical transformer there is a long core of 'laminated'* iron, on which is wound a coil of thick insulated wire. Around this coil and insulated from it is wound another coil containing a many times greater length of thinner wire. Sometimes the core is straight but more generally it forms a closed ring of iron; Fig. 308 will serve as an illustration of the straight pattern.

When a current is sent into the inner coil the magnetic lines, starting as rings round the individual wires, speedily fuse into elongated loops like those of Fig. 300. The inner straight sides of these magnetic loops pack together by thousands in the very permeable iron core, the outer sides bulge out rapidly, cutting through the wires of the second coil as they spread. When the current is stopped all these lines shrink back on to the wire, and now if a reverse current is sent the system spreads out again, with each magnetic line reversed in direction; this *continues* the current in the second coil induced by the stoppage of the direct current in the first.

Thus, when an alternating current, i.e. a current which is reversed 50 to 100 times a second, is sent into the **primary** coil, another alternating current flowing nearly in opposition to the first can be drawn from the **secondary** coil. As the secondary coil contains very many turns of wire, the rate of cutting of lines and wires, and therefore the electro-motive force in the circuit, is high, and the transformer enables us to 'step up' a large low-pressure alternating current to a small high-pressure alter-

* i.e. composed of strips of thin soft iron separated by insulating paper. For in solid conducting iron currents would be induced to circulate; these would oppose the driving current and nearly neutralize its effect, besides greatly heating the iron.

nating current much more suitable for economical transmission to a distance. A miniature transformer of this sort converts the 3-volt-pressure current in a telephone into a high-pressure current capable of negotiating several miles of line without much loss, while transformers weighing many tons are in use in distant-power-transmission systems.

Per contra when the high-pressure current is supplied to the secondary coil, a large low-pressure alternating current can be drawn from the comparatively few turns of the primary coil. Transformers are therefore used to ' step down ' from the dangerous voltage of the transmission line to the 100 volts or so safe for domestic use.

§ 604. The electro-motive force produced in the secondary of the ' step-up ' transformer is proportional to the rate at which the magnetic lines cut the wires. Suppose therefore we could instantaneously stop the primary current ; the lines would travel in at enormous speed (the speed of light), and at first sight it seems that a practically unlimited voltage would result. But on trying the experiment, say by snatching away the supply wire from the binding screw of the primary coil, the secondary voltage, though high, will seldom be found able to drive a spark through half an inch of air (45,000 volts). The reason is seen in the primary break, a flash of light $\frac{1}{8}$ in. long or more follows the snatched-away wire, through this flash the current continues to flow, and its stoppage is by no means the utterly abrupt one intended.

Whence this flash ?

As the current dies away in a coil and the wide magnetic lines shrink down into little rings round the individual wires, each has had to cut a number of neighbouring wires, i.e. a large amount of cutting of lines and wires has gone on in the coil *itself*. Therefore a current has been induced in the coil itself and this current tends to oppose what is being done, it is a direct current delaying the dying away. Moreover, the quicker we attempt to do away with the current the quicker is this cutting and the higher the electro-motive force, which becomes quite able to drive this ' extra current at break ' across a short air gap after the retreating wire.

Thus not only is there ' Mutual Inductance ' between two coils, but every coil possesses **' Self-inductance '** of its own. If there is an iron core, enabling very many magnetic lines to be formed,

this self-inductance may be very large; a regular flame appears on breaking the circuit of a large electro-magnet. Compare this action of the current in a self-inductive circuit with that of the stream of water in the hydraulic ram, § 65.

§ 605. **The Induction Coil** is a step-up transformer in which the production of exceptionally high electro-motive forces is specially aimed at. A coil is shown in section in Fig. 308. It possesses—

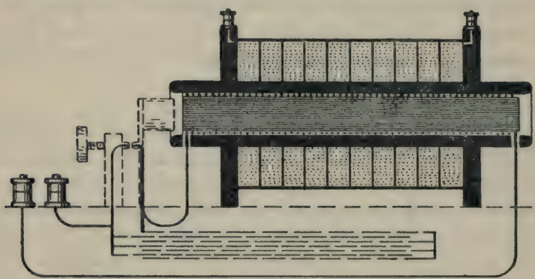


FIG. 308.

(1) A long stout *core of laminated soft iron* well magnetized by a large battery current in the '*Primary*' *winding*, of one or two layers of thick copper wire. This primary coil is connected, through a *break*, with the terminals on the left.

(2) A *secondary Coil containing an enormous number of turns* of (necessarily) thin wire. This coil must be extremely well insulated; usually a tube of ebonite $\frac{1}{2}$ in. thick (black in figure) separates it from the primary, and it is built up of a score or more flat ring-shaped coils strung on this tube and separated from one another by ebonite discs. The ends of the secondary coil are led out to the little terminals on the top.

(3) *Some contrivance for breaking the primary current with great rapidity.*

The commonest contrivance is a Spring Hammer Break. A vertical spring stands up from the base-board and holds a soft-iron hammer-head just opposite the end of the iron core. When the core is magnetized it attracts this hammer and in so doing draws the spring away from contact with a platinum-tipped screw carried by a second upright on the base-board. As this screw and spring form part of the primary circuit, shown as a

continuous black line, this breaks the current, the core loses its magnetism, the spring and iron spring back into contact with the screw again, the primary current restarts, and so on. The arrangement is the same as that to be seen inside an ordinary electric bell.

But the moving apart of spring and screw is not *very* quick, and sparking at the break (due to self-induction) would rob the contrivance of almost all its value were not an additional device employed to extinguish it. This is a Condenser, such as described in § 581, which is placed as a shunt across the break. The 'extra current at break,' instead of rising to the high pressure necessary to drive a lengthening spark across the gap, flows into this condenser, charges it, and comes to a stop, not instantaneously indeed, but comparatively quickly—for a blazing path through air, once established, offers very little resistance to the continuance of even a low-pressure current. And the condenser charge is ready to assist in the re-establishment of the primary current on 'make.'

This pattern automatic break is handy and self-contained, but noisy and irregular, for the sparking cannot be entirely quenched and the platinum contacts soon burn rough. However, it serves on small coils. Break and condenser (below) are shown in broken lines in Fig. 308.

A much better contrivance is the modern Mercury Break, in which a jet of mercury impinging on a fixed plate and carrying the primary current is broken perhaps 20 times a second by the broad teeth of a crown wheel driven by the motor which is also pumping the mercury. The break takes place under paraffin oil or in an atmosphere of coal gas and is very abrupt. Motor breaks of this description are now very largely employed for working Röntgen-ray coils.

A curious sort of interrupter was invented by Wehnelt. The current from the 100-volt mains passes between two lead plates in a tank of dilute sulphuric acid. One plate is inside a wide glass test-tube and the current has to pass through a small hole in this tube. Bubbles of electrolytic gas and steam forming at the hole rapidly interrupt the continuity of the liquid and therefore of the current.

Induction coils of all sizes are stock articles of commerce, from the little medical shocking coil worked by a dry cell and capable merely of inflicting harmless torment on the patient, and from the small sparking coils of petrol motors, to great coils giving

sparks a foot or two in length and used in Röntgen-ray work and in wireless telegraphy.

In the induction coils used for physiological experiments the iron core is removable, and also the secondary is mounted on a 'sledge' and can be slid away from the primary coil. This very much weakens the induced current and permits adjustment of the electric stimulus given to the nerve under observation.

There is also a current induced in the secondary at the 'make' of the primary current, but the back electro-motive force self-induced in the primary coil prevents the current increasing rapidly, i.e. the rate of cutting of lines and secondary wire is comparatively small and the e.m.f. is seldom sufficient to cause a reverse spark. The discharge from the Induction Coil therefore consists of a succession of rushes of small quantities of electricity at high pressure (30,000 volts per cm. spark) all in the same direction. A moderately large coil averages perhaps ·02 amp. at half a million volts.

EXAMPLES.—CHAPTER LVIII

NOTE.—Strength of field due to *long* straight wire $= 2 \times$ ampères $\div$ cm. distance from wire.

4. What would happen if a current-carrying wire were dipped in iron filings ? [L.]m.

5. How would you investigate the magnetic field at different distances from a long vertical wire carrying a current ? Show in a figure the position of the apparatus used. [L.]

6. A long vertical wire carries a descending current, and a small compass needle is placed successively N., S., E., and W., at equal distances from it. How will the position of rest of the needle and its time of oscillation vary at these different points ? [L.]

7. Parallel wires carrying currents opposite ways are observed to repel each other : clearly connect this action with Ampère's 'swimming' rule. [L.]

8. What are direction and magnitude of force acting per centimetre on a current-carrying conductor in a magnetic field ? Illustrate by some current-measuring instrument. [M.]

9. Make a diagram showing the magnetic lines near a straight wire carrying 10 amp. in a field of strength 2 perpendicular to the wire. In what direction would the wire tend to move ? [L.]

10. Describe the nature of the magnetic forces in the neighbourhood of a long straight wire through which a current is passing.

Calculate attraction between two parallel wires of a metre length and 2 cm. apart when 1 amp. is sent through each. [L.]

11. A single turn of wire in the form of a rectangle 15 cm. $\times$ 8 cm. is suspended in a horizontal field of strength 2·5. Indicate the forces acting on each side of the rectangle, and calculate the couple acting upon it when its plane makes an angle of 45° with the field and 200 amp. flows in it. [L.]

12. Describe the use of Barlow's wheel as a dynamo and as a motor.

13. Sketch the distribution of currents in a large copper plate drawn between the poles of a horseshoe magnet.

14. State Lenz's Law of induced currents and say exactly how you would prove it experimentally. [L.]

15. Give an account of the phenomena of electro-magnetic induction, describing experiments to illustrate them. State Faraday's Law and Lenz's Law, and point out how they apply to the experiments you have described. [L.]

16. A telegraph wire running magnetic east is blown down. Calculate the mean voltage induced in the wire per metre length, supposing it to fall freely from a height of 5 m., and state in which direction the current will flow. [Take g 1000, H ·18.] [L.]

17. A ship of 39·4-ft. beam sails at 14·4 knots at a place where H = 0·2 and tan (dip) = 2. Find the difference of potential between her sides. Will it make any difference if she is of steel or of wood, copper sheathed ? How will the voltage vary (a) if she suddenly changes her course, (b) on a voyage from London to the Cape ?

18. A coil carrying a current is mounted so that it can turn freely about a horizontal axis. How will it set itself in the Earth's field when the axis is (1) E. and W., (2) N. and S., (3) in any other horizontal direction ? [L.]

19. Show that when a current flows in a wire wound in the form of a long straight solenoid the magnetic force inside the solenoid is uniform. [L.]

20. A circular coil of wire is rotated about a diameter on a vertical axis in a horizontal field. Describe and explain the changes in current during one complete revolution. [L.]

21. How could you use a small coil of wire to explore the field of a tangent galvanometer coil ?

22. How would you find the Dip by using a coil of wire and a galvanometer ?

23. Prove experimentally that the flow of electricity developed by induction in a circuit depends upon the change in the number of lines of magnetic force passing through the circuit. A vertical hoop falls sideways to a horizontal position on the ground. If it was originally (a) in the meridian, (b) perpendicular to the meridian, in which case would the induced current be greater ? [L.]

24. Describe any arrangement by which a continuous electric current may be produced by electro-magnetic induction. [L.]

25. A simple coil of wire with a cylindrical iron core is rotated about a cross-axle between the poles of a permanent magnet. Describe fully how the electric pressure in the coil changes during 1 revolution.

26. A N. pole is brought down to the middle of a coil lying on the table. Which way is the induced current, and how would you prove this ? [L]m.

27. What effect is shown by a galvanometer connected to a horizontal coil in these two cases: (a) vertical magnet placed half-way through coil is dropped, (b) horizontal magnet at middle of coil is dropped. [L]m.

28. Explain different ways in which a current can be induced in a wire, stating its direction. [M.]

29. Explain the cause of the flash on breaking the circuit of the field magnet of a dynamo. [M.]

30. Describe experiments to show the storage of energy in the field of a current through a coil.

31. Describe an induction coil capable of giving long sparks, carefully pointing out the use of each part. [L]m.

32. Describe the action of an induction coil, and explain the effect of having a condenser in the primary circuit in parallel with the interrupter.

What effect is produced on the discharge of the secondary when the opposite plates of a condenser are connected to the terminals ? [L.]

33. Explain why a coin cannot be spun in a strong magnetic field.

34. A copper ring is suspended so as to encircle the pole of a horizontal electro-magnet. Show that it will jump off when the magnet current is 'made,' will *slowly* return to its place, and will jump on when the magnet current is broken.

35. Show that a copper ring is repelled from the pole of an alternating-current magnet.

Chapter LIX

THE MEASUREMENT OF ELECTRIC CURRENTS

The principle of the electro-magnetic method of measuring electric currents has been explained in § 597. It consists in measuring the force on a conductor carrying the current across a known magnetic field, or conversely, of measuring the reaction on the magnet producing the field. In the first case, the force can be measured in grammes weight, i.e. against gravity, or against the known strength of a spring; and in the second case also against the known magnetic pull of the earth.

The gravity instrument which is in use for determining the Ampère absolutely will be briefly described in § 617; the instruments now about to be described are suitable for measuring the relative values of currents and are called **Galvanometers.** If their scales are graduated so as to read directly in Ampères, they are called ampère-meters or **Ammeters.**

§ 606. In Fig. 304 the force tending to rotate the coil is of course proportional to the current flowing in it. Instead of letting the coil go on rotating, suppose that a spring like the hair-spring of a watch is fastened round the axle. Now as the coil turns it must wind up the spring, and it does this until the increasing elastic resistance just balances the turning effort due to the current. We have put an electro-motor to push against a spring balance, so to speak, and we have produced a **Moving-coil Galvanometer**, wherein the rotation of the coil, as shown by a pointer moving over a graduated scale (40° or 50° long) is nearly proportional to the current flowing in it.

Moving-coil instruments have coils, perhaps an inch square, of 100 or 200 turns of very thin copper wire. The coil moves on pivots in jewelled bearings, between the hollowed-out poles of a strong steel magnet; current is led in and out by extremely thin and flexible silver strips, and a spring of phosphor-bronze controls the motion. Inside the coil is a lump of soft iron, corresponding

in function to the iron of the motor armature, but now more conveniently fixed to the instrument, so as to relieve the pivots of unnecessary weight. Instruments of this description form the highest class of commercial ammeters (and voltmeters) for the measurement of direct current. They are usually very sensitive instruments and take but a small fraction of the current, the great part of which passes by them in an appropriate 'shunt,' § 623.

In the still more delicate galvanometers required in the laboratory the control is often given not by a spring, but by the twisting of the exceedingly slender strip of phosphor-bronze (the finest wire rolled out flat) on which the coil hangs, pivots also being dispensed with. Provided with mirror, lamp and scale (see below) they usually give about 1 mm. deflection for one one-thousand-millionth of an ampère.

The great advantage of moving-coil instruments is their freedom from external magnetic interference, for the field in the gap in which the coil moves is many thousand times stronger than the earth's.

§ 607: A permanent-magnet instrument is evidently useless for **Alternating Current,** which changes direction so rapidly that the coil never has time to move either way. An instrument with a laminated soft-iron electro-magnet has recently been introduced, the alternating current flows also in this magnet and reverses its polarity in step with the reversals in the coil, so that now the pointer is pushed always the same way and reads as usual. More commonly the magnet iron is dispensed with altogether, leaving the empty solenoid to produce the necessary alternating field.

§ 608: In the remarkable Oscillograph the moving coil has been reduced to a single narrow loop of tightly stretched slender phosphor-bronze strip, with a very small mirror attached across its middle, and all immersed in oil. This 'coil' has of course an exceedingly small inertia and it can therefore respond very rapidly to the electro-magnetic forces and the considerable tension; it is able to follow oscillations of current up to a frequency of 10,000 per second, the reflected spot of light tracing a wavy current curve on a rapidly moving photographic plate.

A recent introduction is the 'String Galvanometer.' It is Fig. 303 with the long narrow polar gap vertical, and instead of the copper wire an exceedingly fine thread of silvered quartz carries the minute current and therefore sways across the magnetic

field. Its motions are observed through a microscope arrangement, magnifying 800 diameters, which pierces the pole-pieces of the electro-magnet. While not so quick it detects much smaller currents than the oscillograph, and is of great value in electro-physiological experiments.

§ 609. Now we must turn to the older patterns of Galvanometer, those in which the coils stand still and the magnet moves. The controlling restraint is here occasionally provided by gravity or by a spring, but far more usually by the (horizontal component of the) earth's magnetic field. As in § 529 there is a compass needle pulled N. and S. by the earth and pushed E. and W. (by the action of the current in the coil); then the tangent of its deviation is the ratio between the magnetic field due to the current and the earth's field.

Looking at Fig. 297 we can see how a magnet placed in the plane of an encircling coil of wire will be twisted out of that plane when a current flows. Looking also at Fig. 299 we see that the magnetic field due to a coil is, *just in the very middle*, uniform in strength and at right angles to the plane of the coil. Accordingly if we place round a *small* compass needle a large vertical coil of wires with its plane magnetic N. and S. we shall have the magnetic forces at right angles contemplated in § 529, and this arrangement forms a **Tangent Galvanometer.**

Let m be the strength of the needle's pole, practically at the centre of the ring of radius r, composed of n turns of wire carrying a current C. All over the sphere of radius r surrounding the pole m, the magnetic field is radial, and of strength m/r^2 (§ 524), i.e. this number of unit magnetic lines passes out of each square centimetre of it. If the encircling belt of n turns of wire, each $2\pi r$ cm. long, were to move 1 cm. at right angles to itself (as if slipping off the imagined sphere) there would be $m/r^2 \times 2\pi rn$ cuttings of lines and wire. Therefore, by § 597 the coil is acted on by a force in this direction (say, West) $= \mathrm{C} \times m/r^2 \times 2\pi rn = 2\pi n\mathrm{C}m/r$ dynes, and of course an equal reduction on the pole drives it E. The horizontal component H pulls the pole N. with force Hm dynes, therefore we have as in Fig. 255, § 529, a deflection D from the N. such that

$$\tan \mathrm{D} = \frac{2\pi n \mathrm{C} m/r}{\mathrm{H}m} = \frac{2\pi n\mathrm{C}}{r\mathrm{H}}$$

or C *decampères* $= \dfrac{r}{2\pi n}$ H tan D, or A *ampères* $= \dfrac{10r}{2\pi n}$ H tan D.

Example 1. What current in ampères would cause a deflection of 35° (tan 35° = ·575, from the tables of natural tangents) in a tangent galvanometer having 7 turns of wire on a ring 16 cm. diameter, at a place where H = ·20 ?

$$A = 10 \times 8 \text{ cm.} \times \cdot 20 \times \cdot 575 \div (2 \times 3\cdot 14 \times 7)$$
$$= \underline{\cdot 21 \text{ ampère.}}$$

With coils of different shape and position $2\pi n/r$ ceases to be correct, but there is always a 'galvanometer constant' G depending on the size and dimensions, and either calculable or found by experiment (sending a current which is simultaneously measured on a standard tangent galvanometer near by), so that in general

$$C = (H/G) \tan D.$$

And when always used at one place H/G may conveniently be calculated out as the 'reduction factor, K,' by which one multiplies the tangent of the observed deflection and obtains the current at once, C = K tan D.

§ 610. It does not follow, however, that because there stands on the bench an instrument called a Tangent Galvanometer, having a large vertical hoop of wire with a small magnet in its middle, and because the instrument has been turned round and levelled until the ends of the pointer, stuck crossways on the needle, both read zero on the graduated arcs over which they swing, that therefore a single observation of a deflection D will suffice to determine a current accurately through the above relation. For the pointer may be neither straight nor at right angles to the needle, the needle may not be in the centre of coil and graduated circle, and the silk fibre* by which it hangs is very likely twisted, and these mean that the two forces are not satisfactorily at right angles to start with. If, however, a galvanometer be freed as far as possible from these visible defects and both ends of the pointer be read with a direct and also with a reverse current, then the mean of these four readings may reasonably be taken as D.

Unless the needle is quite small the above argument begins to fail : the needle swings out into the curved weaker parts of the field in Fig. 299. Some tangent galvanometers have a pair of large coils, rather closer together than in Fig. 300 : these produce a very uniform field near the middle, and a longer compass needle is allowable.

When the earth's H at the place of observation has been determined as in § 535 and the average radius r of the coil can be

* A pivot is preferable, provided it is of proper 60° conical shape and is occasionally sharpened up with a stone.

accurately measured (e.g. a few turns of thin wire on a marble ring), then $C = rH \tan D/2\pi n$ gives the current absolutely in Decampères; but various experimental difficulties stand in the way of extreme accuracy and the tangent galvanometer is seldom used nowadays for the absolute measure of current (or indeed for any other than educational purposes).

§ 611. To evolve the more **Sensitive Galvanometers** required in many electrical methods—

(1) We must increase the mutual action of current and magnet as much as possible;

(2) We must enfeeble the controlling action of the earth; and

(3) We must devise means of measuring the very smallest deflections.

(1) This is done by carrying the current very closely around a small magnet in a very large number of turns of (inevitably) fine wire. At once the accurate application of the tangent principle is interfered with, but that cannot be helped.

(2) There are three ways of doing this.

Firstly, a large 'control magnet' is fixed near the galvanometer in such a position as to almost neutralize the earth's field (needle near neutral point, Fig. 252). This is easy, but has the disadvantage of leaving the galvanometer very unprotected against little accidental external magnetic disturbances.

Secondly, the galvanometer may be provided with an Astatic pair of needles. Two magnetized needles, as exactly equal as may be, are fixed to the wire stem which carries the pointer, one above the other and pointing opposite ways. The earth's actions on them are equal and opposite and the combination should therefore have no tendency to stand N. and S., hence its name. But the coil surrounds only one of the needles, or sometimes there are two coils wound opposite ways round the two needles, and the combination feels the full force of the current. A remnant of controlling force is indispensable, however, and must be provided if the system chances to be too accurately astatic, by a weak magnet fixed nearer one of the needles (underneath in Fig. 309).

Thirdly, the galvanometer is sometimes clad in a close jacket of thick soft iron which shields it from $\frac{9}{10}$ or more of the earth's field, § 511.

(3) This can be done by using a long pointer. But a long pointer is slow and heavy and cannot be borne by the suspending

fibres of silk or fused silica, which must be exceedingly thin lest their stiffness impede the feeble motions of the short magnet of a fine galvanometer, § 109. Lord Kelvin first made submarine-cable signalling practicable by using as pointer a weightless beam of light: a tiny concave mirror attached to the needle-system reflects a spot of light on to a scale about a metre away and is equivalent to a pointer 2 m. long, see § 362.

Fig. 309 is a diagram of a galvanometer embodying most of these features, the magnets are little grids built up of hair-spring; aluminium stem, magnets, mirror, and damping vane (to check swinging about) together weigh a grain or two. The coils, about the size of a penny, contain many thousands of turns of No. 42 silk-covered copper wire. The instrument can detect about a billionth of an ampère.

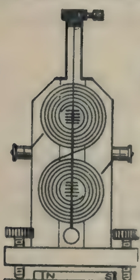


FIG. 309.

§ 612. The total **Quantity** conveyed by a current past any particular point is of course obtained by multiplying the strength of the current by the time for which it flows.

The Unit of Measurement of such Quantities of Electricity as are dealt with in Current Electricity is called the **Coulomb** and it **is the quantity conveyed by one Ampère flowing for one second.**

The ordinary laboratory way of determining the quantity of electricity that passes in a fairly steady current is therefore to take the mean of frequent readings of the ammeter and to multiply by the total time in seconds during which the current has been flowing.

For the ballistic use of the galvanometer in measuring the total quantity passing in a sudden discharge see § 600. The Quantity $=\mathrm{H}\times$ periodic time of oscillation $\times$ half first swing $\div\pi\mathrm{G}$.

§ 613: In commerce, electrical-quantity meters effect this automatically. For instance, in one pattern the ammeter pointer marks a curve of height proportional to the current on paper which is moved along steadily by clockwork so that the horizontal length of the curve is proportional to time. Then the area under the curve represents current $\times$ time, i.e. quantity.

We have seen in § 599 that the force on the coils moving in the permanent-magnet-field of an electro-motor is proportional to the current. Suppose therefore we make the motor drive a brake whose frictional resistance is always proportional to the speed.

If the machine is light and never moves very fast, so that the fractions of force involved in acceleration and deceleration are small and equal, then on the average, driving force = frictional resistance $\propto$ speed of rotation, which is therefore proportional to the current. And the total number of revolutions recorded by a revolution-counter is evidently the continued product of speed and time of turning, and therefore measures the quantity of electricity passed. The suitable brake is an electro-magnetic one; on the shaft of the little motor is a 3-in. disc of sheet copper, this passes between the poles of little steel magnets and therefore eddy-currents proportional to the speed are induced to flow in the copper disc: the electrical resistance these meet with is translated into mechanical resistance to the rotation.

Meters of this description are very largely employed in domestic supply.

It will be seen that they are, so to speak, moving-coil galvanometers in which the elastic resistance that gives way proportionally to the current, and stops, is replaced by a plastic resistance that continuously gives way.

For the electro-chemical measurement of Quantity see §§ 651, 652.

EXAMPLES.—CHAPTER LIX

2. Describe the moving-coil galvanometer and mention its advantages for workshop use over the moving-magnet form. How is it used as an ammeter for large currents? [L.]

3. Describe the construction of some form of ammeter suitable for the measurement of large currents. [L.]

4. Describe (a) the suspended-magnet galvanometer, (b) the suspended-coil galvanometer. Which galvanometer would you prefer to employ where strong electro-magnets are occasionally used, and why? [L.]

5. Describe the tangent galvanometer and explain how it may be used to determine currents in electro-magnetic units. [L.]

6. A tangent galvanometer having one turn of 31·4 cm. radius gives 45° with 10 amp. Calculate the earth's field. [L.]

7. Point out the difference between an ordinary tangent galvanometer and a galvanometer for the measurement of extremely small currents, showing how the latter is made more sensitive. [L.]

CHAPTER LX

RESISTANCE

WAYS of detecting the presence of a charge or quantity of Electricity have been described in Chapters LV, LVI, LVII. Electricity in motion can be detected by its magnetic action, Chapter LVIII.

Where such means, properly employed, detect electrical activity, there 'Electricity' is, and this statement has to serve instead of a definition of what 'Electricity' is, for that nobody knows.

Electricity travels about by processes somewhat similar to the conduction, convection, and radiation of Heat. (1) Electricity is conducted through metals (readily, as is heat) and most other conducting solids without any detectable motion of matter. (2) When it passes through conducting liquids a perceptible transference of matter goes on, different chemical substances accumulate at different places. In gases too there is detectable motion of gaseous particles. (3) And there is the electro-magnetic radiation, employed in wireless telegraphy, which can set going currents of electricity in wires suspended from distant poles. Each will be considered in turn: the present chapter deals only with conduction of the first sort.

§ 614. As stated in § 591, a complete circuit of good conducting material, preferably metal, properly insulated to prevent 'short-circuiting,' is almost indispensable in Current Electricity: when metallic connection is broken—'the circuit opened'—the current cannot pass.

Easily removable metallic connections are most commonly made by **'Binding Screws'**; a brass screw projects from the block of metal, and the wire, scraped bare of insulation and *clean*, is roughly bent into a hook half round the screw and is clamped down tightly by a milled brass nut. It is difficult to twist wire ends together tight enough to make a really reliable connection of

negligible electrical resistance, and double-ended binding screws are used for this purpose.

Of **Keys** or Switches for opening and closing circuit more quickly than by screws there are many varieties, which the reader must examine as he uses them in the laboratory.

The most reliable is the Plug Key, wherein a 6° taper round plug of brass, perhaps $\frac{1}{4}$ in. diameter, is dropped into the tapering hole which separates two brass blocks firmly fastened to an ebonite base-plate. Slight pressure and a twist to the right fits the plug in firmly, burnishing the contact surfaces and ensuring good contact: too much force soon loosens the blocks from the base-plate and spoils the key.

Not so reliably free from resistance, but very convenient, is the switch in which a brass lever moving horizontally scrapes with some pressure over flat brass studs let into the insulating base-plate. The scraping keeps the contact surfaces smooth and clean, if they get rough a little paraffin oil should be rubbed on.

Two-, three-, etc., way keys have several studs for the lever to move over. Or in plug pattern, one long brass block has a row of separate blocks beside it, from each of these it is separated by a conical hole and the plug can be dropped into any hole.

'Tapping Keys' of springy brass strip should have platinum contact points, for there is now no scraping to get rid of oxidation film, sure to form on other metals. To remove dirt a piece of paper should occasionally be drawn between the points, pressed together.

In most of the many varieties of **Reversing Keys** or **Commutators** in use in the laboratory the reader will find two movable parts, permanently attached to two binding screws of some sort, and temporarily connectible by the motion of the switch to two fixed parts. To lead a current either way into a circuit the battery is connected to the two moving parts and the circuit ends to the two fixed parts. The two-plug reversing key has four quadrantal blocks, the battery is connected to opposite quadrants and the plugs are always in opposite holes.

Mercury-cup Keys are made by boring half-inch holes into a block of wood and half filling with mercury into which the wires are led. Little arches of thick wire bridge between whichever cups one wishes to connect. The wire ends should be scraped clean and left to 'amalgamate' in mercury: there should be a rim round the block to prevent spilling, for mercury is mischievously corrosive of metal apparatus.

§ 615. Now however good the conductor may be, the passage of a considerable electric current will presently make it warm.

This is not a conversion of 'Electricity' into 'Heat,' for wherever a current-measuring instrument is put into the circuit it will show the same current. No electricity is lost.

It is a production of heat by a dissipation of electrical Energy: the electro-motive force driving the current gets lessened in the conductor: unless e.m.f. is kept up by power spent in the dynamo or chemical activity in the battery, the current stops, much as a body moving in a viscous fluid stops as soon as the driving force ceases.

Thus a sort of friction dogs the motion of electricity. In metals (solid or liquid) it is comparable to fluid friction, for the slightest electro-motive force can always cause a feeble current; in other liquids and gases it is more comparable to solid friction, for below a certain starting pressure no current will move.

In Metals indeed the most careful experiments have proved that **The current flowing in a conductor is exactly proportional to the potential difference (or electro-motive force) between the ends of the conductor, provided that the conductor is kept at a constant temperature.** This is called **Ohm's Law** of electric conduction through metals.

The better the conductor the larger the current, so that we can get rid of 'proportional to' by defining a conducting power or '**Conductance**' constant for a given conductor at a given temperature

$$\textit{Current} = \textit{conductance} \times \textit{electro-motive force},$$

though it is more usual to talk about the resisting power or '**Resistance**' offered to the passage of the current. This is evidently the reciprocal of conductance, for halving the resisting power means doubling the conducting power, etc., so that

$$\textit{Current} = \textit{electro-motive force} \div \textit{resistance}$$

$$C = \frac{E}{R} \qquad CR = E \qquad \text{or } R = \frac{E}{C}$$

§ 616. We have already defined units of current and electro-motive force, we must therefore define the **Unit of Resistance** as follows:—

When unit electro-motive force applied to the ends of a conductor causes unit current to flow through, the conductor possesses Unit Resistance.

For practical purposes, as stated in § 597, **the Volt is the Unit of Electro-motive Force** and is 100 million (10^8) times the fundamental unit there defined. [Roughly speaking, it is a little less than the potential difference between the metals of the original Volta's cell, where copper and zinc dip in salt water.]

The Ampère is the practical Unit of Current and was most unfortunately chosen as one-tenth the fundamental unit of § 597 (which is hence the decampère).

The unit of resistance is the Ohm. If 1 volt applied to the ends of a wire causes 1 ampère to flow in it, the wire has a resistance of 1 ohm.

Then $\text{Ampères} = \dfrac{\text{Volts}}{\text{Ohms}}$, $\text{Ampères} \times \text{Ohms} = \text{Volts}$, $\text{Ohms} = \dfrac{\text{Volts}}{\text{Ampères}}$

The Ohm, although a derived unit, has one decided advantage over the ampère and volt. It is the property of a portable piece of metal, and once made up there is no need to keep turning cranks and things to get it. Consequently the electrician's methods have mostly been devised to lead to measurements in terms of resistances, just as the chemist works down to his box of weights.

§ 617 : In the National Physical Laboratory stands a fine balance bearing coils of wire which hang in the magnetic field of outer coils, so that the wire would cut a very accurately calculated number of magnetic lines if it moved 1 cm. When 7 grm. has to be put in one pan to keep the balance in equilibrium **1 ampère** is flowing in all its coils.

This current passes to the coils of another machine, an elaborate pattern of Barlow's wheel, wherein a very accurately calculated number of the magnetic lines due to coils is cut by a radius of the wheel as it makes one revolution. The wheel is driven round at a steady speed and produces an e.m.f. between its axle and its rim = the accurately known number of lines cut per second. Say 100,000 lines, then the Electro-motive force = 100,000 ÷ 100,000,000 = ·001 **volt.**

The 1 ampère passes also through a certain conductor, and by trial (see Potentiometer, § 636) two points on this conductor are found between which there is exactly the same e.m.f. as the wheel produces. Then the resistance of the conductor between them = ·001 volt ÷ 1 ampère = ·001 **ohm.**

§ 618 : From this conductor other multiples can be prepared by easier processes (§ 627) than those required to realize the ampère and the volt, and for convenience the Ohm is realized as

the resistance of a certain long narrow conductor of metal. Not primarily of a solid wire—for experience has shown that even the best wire, freed as far as may be from internal strains by annealing, and protected from oxidation, alters minutely in resistance from year to year—but of a thread of liquid metal, incapable of internal strains and reproducible by anybody in pure metal :—

The resistance of a uniform thread of pure mercury at 0° C., 106·3 cm.* long and weighing 14·4521 grm. (which gives a cross-section of 1 sq. mm.) is 1 Ohm.

This is the material standard ohm agreed upon internationally. With it, or with the two machines, Secondary Standard Resistance Coils are compared at intervals. These coils are far more convenient for everyday use.

They are made of *manganin*, an alloy of 84 % copper, 12 manganese, and 4 nickel, and are constructed as in Fig. 310. A short piece of wide brass tubing is wrapped with silk, shellac varnished, and baked. A loop of double-silk-covered manganin wire, a little above the required resistance, has its ends hard-soldered to the stout copper terminal bars, and is then wound double on the cylinder, thickly varnished with shellac, and heated to 140° C. for eight hours or more by sending a considerable current through it.

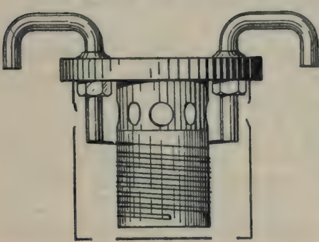


Fig. 310.

This drives off all moisture, so that there is no risk of current leaking from one turn to its neighbour, and also anneals the wire, which was 'hard' from the wire-drawing and winding. The annealing lowers the resistance 2 or 3 % and thereafter the coil remains practically unchanged for years : without it the resistance crawls down rapidly as the coil ages.

The coil is connected, through mercury cups in which its thick copper terminals rest, with an apparatus of the principle to be described in § 627, and is adjusted to equality with a primary standard by shortening one end of the wire, or by shunting part of it with a thin branch wire (§ 622).

* The original arbitrary unit was a 1 m. thread 1 sq. mm. section ; the attempt to retain something like this led to the decimation of the unit of current.

The coil, enclosed in an outer perforated case, is sunk in an oil bath at constant temperature.

The 'go and return' winding prevents the establishment of the magnetic field existing in a single-wound coil. There is no cutting of magnetic lines as the current alters, the coil is 'non-inductive,' and the current shows no tendency to run on after the circuit is broken, an action which would destroy the usefulness of the resistance for many measuring purposes.

§ 619. Ordinary resistance coils are wound in the same way on wooden bobbins two or three inches long, and are best of *manganin*, though *Eureka* alloy (copper with 40 % nickel, a white strong tough improvement on 'german silver') is good for high resistances. Low coils are of short thick wire, high of long fine wire. The resistance of both alloys is high, so that but little wire is wanted, and its change with temperature is so small as to be quite negligible in all ordinary work.

The coils are attached to the under side of an ebonite plate, and the ends of the wire are soldered to brass rods rising through the plate to brass blocks on its upper surface (Fig. 315, right). When the taper plug is inserted between the blocks the current goes through the plug, whose conductance is vastly greater than that of the coil; when the plug is taken out the current must go all through the coil: unplugging a coil brings it into use.

Boxes of coils of 1, 2, 2, 5, 10, 20, 20, 50, etc., ohms are commonly made up. In another arrangement a turning lever moves over eleven studs, between each pair of which is attached a 1-ohm coil. The current is led up to the first stud and must travel through the coils in succession till it reaches the stud on which the lever happens to rest, then it passes through the lever and goes to the first stud of a batch of ten 10-ohm coils, and so on. Either of these arrangements permits any resistance to be made up from 1 ohm to total of box: the turning-head arrangement, if the contacts are kept clean and tight, is by far the handier.

Variable resistances or Rheostats are often very useful for regulating current. A common pattern consists of bare Eureka wire, coiled tightly, with narrow air space between the turns, on a bar of slate. The current enters at one end and travels through the wire till it reaches a movable slider which carries it off along a brass rod. In the carbon rheostat the resistance of a pile of plates of carbon is reduced by squeezing them together more tightly by means of a screw.

§ 620. The resistance between the opposite faces of a 1-cm. cube of a material is called its *Specific Resistance* or **Resistivity.**

The reciprocal of Resistivity is **Conductivity,** which might also be defined, as in Heat, as the Quantity of electricity (in coulombs) that passes in 1 sec. from one face to the opposite of a 1-cm. cube when 1 volt potential difference is maintained between the faces.

Some Resistivities are :—

Material.	Resistivity.	Increase per cent per 1° C. warmer.	Conductivity.
I. *In millionths of an ohm, or microhms.*			
Silver (wire, hard)	1·65	·38	610,000
Copper („ „)	1·65	·4	„
Aluminium (commercial)	2·7	·4	370,000
Iron (mild steel)	10·5	·7	95,000
Iron (cast)	60 to 120		
Platinum (pure).......	8·2	·37	122,000
Tin (commercial)	13·3	·4	75,000
Lead („)	20·	·4	50,000
Mercury (liquid)	94·3	·72	10,600
Brass	6·3	·15	159,000
German silver	30·	·036	330,000
Eureka	45·	·000	222,000
Manganin	41·	·000	244,000
Gas-carbon	7,000	—·05	144
II. Solutions in water : *in ohms.*			
Hydrochloric acid, 20 %	·13	—1·5	7·7
Sulphuric acid, 25 % (accumulator acid)	·14	−2·1	7·2
Caustic soda	·19		5·3
Common salt 25 %	·5	−2·2	2·0
Sal ammoniac, 10 %...	·6	−1·0	1·67
Copper sulphate, 15 %.	2·4	−2·3	·42
Zinc sulphate, 25 % ...	2·1	−2·6	·48
Silver nitrate, 25 % ...	1·0	−2·1	1·0
III. Insulating materials, *in millions of millions of ohms* (*or millions of megohms*).			
Common distilled water			
Paper			
Asbestos			
Mica at 20° C.	84		14×10^{-15}
Glass at 60° C.	60		17 „
„ flint at 60° C. ...	1,000	− 5	1 „
Shellac at 30° C.	9,000		·11 „
Gutta-percha at 24° C. .	450	−20	2·2 „
India-rubber at 24° C. .	5,000		·2 „
Ebonite at 50° C.	25,000		·04 „
Sulphur			
Paraffin wax			

Notice that the increase of resistance per cent per °C. of the pure metals is very near the ·37 % expansibility of gases. Indeed the resistivity of any very pure metal is found to be proportional to the Absolute temperature, at any rate from 100° A. up to near its melting point. The temperature change for alloys is much less.

Hard graphite conducts rather better than hard carbon, a pencil streak on ground glass is sometimes used as a high resistance.

The resistivities of insulating materials in III have nothing of the definiteness of those of metals. They often diminish considerably as the voltage is increased, and very rapidly as the temperature rises.

§ 621. **Relation of Resistance to size of conductor.**

Putting 2 or 3 1-cm. cubes side by side gives 2 or 3 times the opportunity for current to flow. So that a conductor of cross-section A sq. cm. and length 1 cm. would have a conductance A times that of a single 1-cm. cube.

Its Conductance = conductivity × A
or reciprocally, ***Resistance = resistivity*** ÷ A.

Putting, however, 2, 3, etc., 1-cm. cubes 'end on,' so that the current must flow through them in succession, evidently doubles, trebles, etc., the resistance in its path, i.e. the resistance of a column of L 1-cm. cubes = L × resistivity.

Taking these two together,
For a conductor A sq. cm. cross-sectional area and L cm. long

$$\text{the } \mathbf{Resistance} = \frac{\mathbf{L}}{\mathbf{A}} \times \mathbf{resistivity}\,;$$

and inverting everything, Conductance = A/L × conductivity.

Example 1. Calculate the resistance of 1 yard (91·5 cm.) of No. 22 copper wire (diameter ·71 mm.).

$$R = \frac{91{\cdot}5}{\pi \times (\frac{1}{2} \times {\cdot}071)^2} \times {\cdot}00000165 = {\cdot}0385 \text{ ohm.}$$

Ex. 2. Calculate the resistance of an 'accumulator' in which two plates 15 cm. square are separated by ·8 cm. of sulphuric acid.

$$R = \frac{{\cdot}8}{15 \times 15} \times {\cdot}14 = {\cdot}0005 \text{ ohm.}$$

§ 622. Carrying the argument further—

The *Resistance of a number of conductors* through which the current must pass in succession or '*in series*' is the sum of their resistances

$$R = r_1 + r_2 + r_3 + \quad . \quad . \quad .$$

[NOTE.—This does not necessarily apply to classes II and III above.]

The conductance of a number of conductors by any of which current can flow from P to Q is the sum of the individual conductances, just as the traffic-carrying capacity of all the roads from one place to another is the sum of their individual carrying capacities.

$$C = c_1 + c_2 + c_3 + \quad . \quad . \quad .$$

or writing conductance as the reciprocal of resistance,

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \quad . \quad . \quad .$$

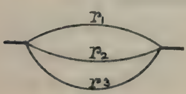


FIG. 311.

Thus the *Resistance of any number of conductors* 'in multiple arc' or '*in parallel*,' Fig. 311, has to be found by first taking the reciprocals of their individual resistances, adding these reciprocals together, and then taking the reciprocal of this.

The Currents in the various branches are proportional to their Conductances (i.e. inversely as their resistances), and are the fractions c_1/C, c_2/C, etc., of the total current flowing.

Ex. 3. Three wires of resistances 2, 4, and 6 ohms are joined in parallel and together carry 110 amp. Find their joint resistance and the current in each wire.

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} = \frac{11}{12}$$

$$\therefore R = 1\tfrac{1}{11} \text{ ohm}$$

$$\text{Current in 2 ohms} = \frac{\frac{1}{2}}{\frac{11}{12}} \times 110 \text{ amp.} = 60 \text{ amp.}$$

$$\text{,, \quad ,, \quad 4 \quad ,,} \quad = \frac{\frac{1}{4}}{\frac{11}{12}} \times 110 \text{ amp.} = 30 \quad \text{,,}$$

$$\text{,, \quad ,, \quad 6 \quad ,,} \quad = \frac{\frac{1}{6}}{\frac{11}{12}} \times 110 \text{ amp.} = 20 \quad \text{,,}$$

Ex. 4. A coil intended for a 5-ohm standard is found when tested to have a resistance 5·033 ohms ; what fine wire must be put in parallel with it (as a 'shunt') to reduce the joint resistance to 5 ohms ?

$$1/5 = 1/5{\cdot}033 + 1/x \text{ or } {\cdot}2 - {\cdot}19868 = 1/x.$$

$$\therefore x = 1/{\cdot}00132 = \underline{760 \text{ ohms.}}$$

§ 623. **Shunts.** The arrangement of conductors in parallel is often made use of to get a definite small fraction of a current, so that a galvanometer or ammeter suitable for measuring small currents may also be available for large currents. The galvanometer has its terminals connected together by a wire of resistance less than that of its own coils; most of the current arriving at A flows past the galvanometer through the shunt to B, and only a fraction traverses the coils and actuates the instrument. Thus, if the shunt has $\frac{1}{9}$th the galvanometer resistance, its conductance is 9 times that of the galvanometer, their joint conductance is 10 times, therefore $\frac{9}{10}$ of the current goes through the shunt and only $\frac{1}{10}$ through the galvanometer. If the shunt $= \frac{1}{99}$ galvanometer, only ·01 passes to galvanometer, etc.

The best type of commercial ammeter nowadays is a sensitive moving-coil galvanometer, shunted with removable wires, strips, or bars, of manganin or copper. For instance, an ammeter in use by the writer is really a very sensitive micro-ammeter (·0001 amp. moves it 100 divisions) and its moving coil has a resistance of 60 ohms. When its terminals are connected by an 8-in. piece of thick copper wire, of ·003 ohm, it requires 2 amp. to deflect it 100 divisions, and when shunted by a copper strap of ·0003 ohm it reads 20 amp. for the 100 divisions.

Ex. 5. What shunts are necessary to reduce the sensitiveness of a 500-ohm galvanometer to $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{10}$? [L]m.

The first has resistance 1/(3–1) of galvanometer, for then its conductance = 2/1 galvanometer's and it takes 2 parts of current while galvanometer takes 1.

The second has resistance 1/(5–1) of galvanometer and the third 1/(10–1). Shunts 250, 125, and 55·5 ohms.

Temporarily shunting a galvanometer with a few inches of thin wire is a precaution worth adopting when far from balance in bridge experiments, etc.

§ 624: The **'Universal Shunt.'** The foregoing necessitates a set of shunts of resistances calculated for each particular instrument. The universal shunt obviates this; it is a wire of resistance greater than the galvanometer's and is permanently connected across its terminals. The current to be measured is brought, not to the terminals as usual, but to two points on the wire. If these are close together most of the current goes through the short piece of wire between them, if slid far apart most goes through the galvanometer.

If G is the galvanometer resistance, S the whole shunt, and a

piece r is included between the two supply points, this gives two conductors in parallel of r and $G+S-r$ resistances; or conductances and therefore currents in the inverse ratio, as $G+S-r : r$. That is, the galvanometer gets $r/(G+S-r+r)=r/(G+S)$, a current proportional to the resistance between the contact points; and doubling, trebling, etc., r doubles, trebles, etc., the current through the galvanometer, whatever $G+S$ may happen to be.

METHODS OF COMPARING RESISTANCES

The comparison of resistances with one another is an important electrical operation. Some methods follow :—

§ 625. **Replacement method.** A voltaic battery and the unknown resistance are wired in series with any sort of galvanometer which will then give a conveniently large deflection. The unknown resistance is removed from the circuit, a resistance box put into its place, and resistances unplugged till exactly the same deflection is obtained.

The unknown = total of known coils unplugged.

§ 626. A circuit is made up as in Fig. 312 of a constant voltaic battery, the unknown resistance, a resistance box, and a galvanometer the *relative* values of whose scale divisions are known, (e.g. a tangent galvanometer for which current $\propto$ tangent of deflection). Let E be the electromotive force of the battery, b its resistance, w resistance of wires, g resistance of galvanometer, X the unknown. First get a reading of current C with all plugs in box, then by Ohm's law

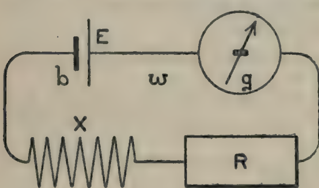


FIG. 312.

$$E=C(b+w+g+X).$$

Then unplug coils in the box to a total R till the galvanometer reading shows the current is halved.

$$\text{Then } E=\tfrac{1}{2}C(b+w+g+X+R).$$

Now b of an accumulator, w of thick wires, and g of an ammeter for currents up to 1 amp., are negligibly small, therefore nearly $X=R$. If not small, $b+w+g$ must be known beforehand.

NOTE.—A voltaic battery offers, like all conductors, a resistance to the passage of a current, even though it itself has set that

current going. This '**Internal Resistance**' of a battery is found by a slight variation of the above method. X now has to be a known resistance, then $R=X+B+w+g$ gives $B=R-X$ approx., and since in the experiment it is permissible to make $X=0$ (i.e. joining up battery and galvanometer without any other resistance), then $B=R$.

NOTE.—There is no actual need to just halve the current, it may be reduced to say p/q its value. The little extra calculation involved easily leads to

$$X=R\times p/(q-p).$$

Ex. 6. Battery of resistance 5 ohm deflects tangent galvanometer of 3 ohms to 45°. What additional resistance reduces deflection to 30°? [M.]

$$\text{Tan } 45°=1\,;\ \tan 30°=\tfrac{1}{2}\,;\ \text{i.e. halves the current.}$$
$$E=C(5+0+3+0)\,;\ E=\tfrac{1}{2}C(5+3+x).$$
$$\text{Equating these } 8C=\tfrac{1}{2}(8+x)C.\quad \therefore\ \underline{x=8 \text{ ohms.}}$$

§ 627. The **Wheatstone Bridge** is an arrangement which enables resistances to be very accurately compared. Then if one of them is known in ohms, the actual value of the other is this multiplied by their ratio.

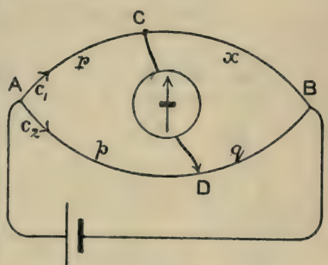


FIG. 313.

In Fig. 313 a battery circuit divides at A and rejoins at B. A is at a higher potential than B. C is at an intermediate potential, evidently there must be some point D discoverable in the other branch which is at the same intermediate potential as C. D is tried for and found when a sensitive galvanometer in the bridging wire CD shows no deviation, for if there were any potential difference between C and D it would surely drive a current through CD.

Let the resistance of $AC=r$, $CB=x$, $AD=p$, $DB=q$. Let c_1 be the current along ACB and c_2 that along ADB.

Applying Ohm's law $E=CR$ to all four sections in turn, we have, when no current flows in CD :—

Potential Difference	between		A	and	$C=c_1r$
,,	,,	,,	C	,,	$B=c_1x$
,,	,,	,,	A	,,	$D=c_2p$
,,	,,	,,	D	,,	$B=c_2q$

But difference between A and C=that between A and D, since C and D are at same potential.

$$\therefore c_1 r = c_2 p \quad \text{or} \quad p/r = c_2/c_1$$
$$\text{and } c_1 x = c_2 q \quad \text{or} \quad q/x = c_2/c_1$$
$$\therefore \frac{p}{r} = \frac{q}{x} \quad \text{or} \quad \frac{p}{q} = \frac{r}{x}$$
$$\text{hence} \quad x = qr/p$$

NOTE.—Corresponding to the easy algebraic interchange in the formula is the electrical fact that battery and galvanometer can be interchanged in the Bridge without affecting the measurements.

§ 628. In the **Metre Bridge** arrangement, Fig. 314, of the Wheatstone conductors, ADB is a straight strong wire of resistance metal stretched along a scale. The wire is quite uniform, the resistance of every cm. of it is the same, hence

Resistance p : resistance q
=length of wire AD : remaining length DB.

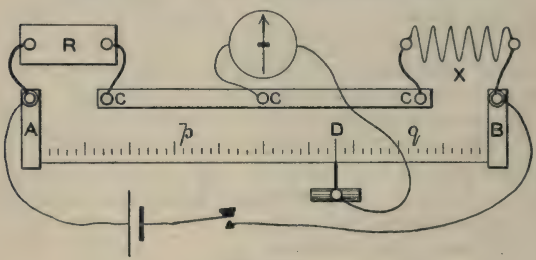


FIG. 314.

The corner points A, C, B are represented by thick straps of copper of no appreciable resistance and provided with stout binding screws. A known resistance R is connected into the gap AC by short stout clean wires, the unknown X is similarly connected into the gap CB. Then a sliding contact-maker is moved along the wire till a point is found where the delicate galvanometer in the long wire CD is not deflected at all from its rest position. Then

$$\frac{\text{resistance X}}{\text{resistance R}} = \frac{\text{length } p \text{ of stretched wire AD}}{\text{length } q \text{ of stretched wire DB}}$$

NOTE:—A very special adaptation of this bridge is the **Platinum Thermometer.** R and X are here a pair of coils, made and kept equal. Between A and end of bridge wire are the thick flexible leads to a coil of pure platinum wire, of 2·73 ohms at 0°, which is wound on a mica frame inside the end of a long porcelain tube. Between the other end of the bridge wire and D is an equal pair of leads—which is bound up with the others, is subject to all the same changes of temperature, and therefore exactly 'compensates' for them—and a box of 2·73 ohms (plus a number of tenths which come into use at higher temperatures). The bridge wire is graduated into lengths each ·005 ohm. When the platinum thermometer is warmed 1° its resistance increases 1/273=·01 ohm (§ 620), hence D must move 1 division to the left, reducing (Pt+length p) by ·005 and increasing (length q+box) by ·005, to restore balance. Thus the graduations on the wire read degrees C. When the end is reached, additional ·1-ohm coils can be unplugged in the 2·73 box and the balance point brought 100° back along the wire again.

§ 629. The accuracy of the metre bridge depends on the perfect uniformity of the stretched wire. This is difficult to maintain when the instrument has to withstand workshop and outdoor use, and the **Post Office** introduced the **Box**, Fig. 315, of resistance

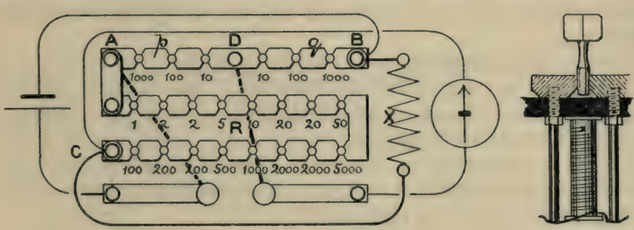


FIG. 315.

coils so arranged as to dispense with the stretched wire. In its stead is a 'bar' ADB of 6 resistance coils [A] 1000, 100, 10, [D] 10, 100, 1000, [B]. One of these either side of D being unplugged we can get ratios p/q=100/1, 10/1, equal, 1/10, 1/100. Attached at A by a link under two binding screws is a complete set of coils from 1 to 5000 ohms, enabling R to be made anything from 1 to 10,000 ohms. X to be measured is attached to the binding screws CB. The battery is connected to A and B through a tapping key

and the delicate galvanometer to C and D through another, the wires shown dotted being contained inside the box.

In use, after seeing that all the plugs fit snugly, 10 ohms is unplugged each side of D so that $p=q$. One ohm is unplugged in AC, the battery key held down, and a quick tap on galvanometer key shows a deflection towards the left, say. 1000 is unplugged in AC and a quick tap sends galvanometer to right. Making r 10 only, still to right, $r=5$ to left, 6 to left, 7 to left, 8 to right. $X : R = p : q$. $\therefore$ X between 7 and 8 ohms.

In AD unplug 100 and plug in 10, $p : q = 10 : 1$. Try $R=75$, to left, 76 to left, 77 to right. $\therefore$ X between 7·6 and 7·7.

In AD unplug 1000 and plug in the 100, make $R=760$, to left, 765 to right, 761 feeble left, 762, 763 doubtful, 764 feeble right. Therefore 7·62*5* is the nearest value.

The reader will see that the P.O. Box can not only measure single ohms by 100ths, but also resistances up to $100 \times 10{,}000 = 1$ megohm, with the same average accuracy of 1 in 500.

Turning-lever patterns of P.O. Box are made in which p and q are levers for studs of 10, 100, 1000, and R a 4-face set of 10 each, $1'_s$ $10'_s$ $100'_s$ 1000_s. See §§ 614, 619.

The binding screws are usually marked G, G for galvanometer attachment; B, B or C, Z for battery (copper, zinc); X, X or L, E for unknown (line, earth). As mentioned above, battery and galvanometer can be interchanged and sometimes this is actually desirable.

Instead of a galvanometer a telephone can be used, but then an intermittent current must be supplied. One battery wire is wound round a file along which the other is scraped: balance to faintest sound in telephone.

Liquid resistances, e.g. batteries, are measurable by using alternating current supplied from a little medical induction coil, and getting silence in a telephone.

§ 630. **Ammeter and Voltmeter.** A good practical way of measuring resistance (accurate to perhaps 1 % with good instruments) is by the use of Ammeter and Voltmeter. The Ammeter is connected in series with the resistance and the current A flowing through both is observed. The Voltmeter is connected as a shunt across the ends of the resistance and the Volts V between the ends of R observed. (A jumps up a trifle, but this is the extra current actuating the Voltmeter and must be disregarded.)

$$\text{Then R ohms} = \text{V volts} \div \text{A ampères.}$$

In this way it is easy to measure a resistance such as that of a glow lamp connected to the mains and actually working. And by using a spiral of iron wire and heating it in a flame the rise of R with temperature is strikingly shown by the fall in current.

The *very high resistances of insulators*—e.g. that between the wire core of 2 miles of cable and the water in the tank containing the cable, or between water inside and outside an inverted telegraph insulator—are measured by this same method, using a battery of several hundred cells, totalling a known 1000 volts or more, and using as ammeter a delicate reflecting galvanometer the value of whose scale readings has been previously measured.*

The resistance of an insulating sheet is measured by laying it on a pool of mercury and pouring a smaller pool on its upper surface with a rim of paraffin wax. The wires are led into the pools.

In insulation resistances it is essential to record also the applied voltage, the time it has been kept on, and the temperature. The results enable an observer of experience to judge whether the insulation will be adequate for the purpose in hand, but it is no use calculating from them by Ohm's law.

EXAMPLES.—CHAPTER LX

7. Explain how resistance of a wire depends on length and section. Compare that of 60 cm. copper wire 1 mm. diam. with 100 cm. mercury thread 1·5 mm. diam., resistivities 1 : 60. [Ab.]

8. A box contains 3 coils of 3 ohms. each. What different resistances can be obtained by coupling up any or all of these in various ways?

9. What length of manganin wire 0·253 mm. diam., resistivity 45, must be shunted across 1·03 ohm to reduce it to 1 ohm?

10. A cell of 2 volts and $\frac{1}{4}$ ohm has its terminals joined by wires of 1 and 5 ohms (a) in series, (b) in parallel. Find currents in both cases.

11. A battery of 2·1 volts in series with a tangent galvanometer and 200 ohms gives a deflection of 45°; by putting an additional 300 ohms in the circuit the deflection is reduced to 30°. Find the reduction factor of the galvanometer. [L.]

* Usually by sending current from a standard 1·02-volt cell through it and a megohm, and observing deflection.

12. How would you determine the coefficient of increase of resistance of a conductor with temperature between, say, 0° and 200° C. ? What is the change for pure metals ? Show generally the change of current from the instant of switching on (a) a carbon filament lamp, (b) a metallic filament lamp. [L.]

13. How does the electrical resistance of a metallic conductor vary with change of temperature ? Why does this render unreliable measurements of low temperatures by a platinum resistance thermometer ? [L]m.

14. State Ohm's law, and explain some method of verifying it experimentally. Deduce the theory of Wheatstone's bridge. [L.]

15. Explain Wheatstone's bridge, and show that there is no current through the galvanometer when the resistances in the arms of the bridge satisfy the condition $r_1/r_2=r_3/r$. [L]m.

16. How would you measure the resistance of an electric lamp (1) at the temperature of the room, (2) when glowing ? [L.]

Chapter LXI

ELECTRO-MOTIVE FORCE

As has been stated in Chapter LVI, Difference of Electric Potential plays the same part in Current Electricity as does difference of Temperature in the conduction of Heat, or difference of level in the flow of water. The electricity flows from the place of higher to that of lower potential; **Difference of Potential** may be regarded as the driving force, and is usually alluded to as **Electro-motive Force.**

§ 631. The unit in terms of which this is measured in Current Electricity, the Volt, has been defined in § 597 as being produced in a conductor which is cutting a hundred million unit magnetic lines per second. This definition rather suggests a hasty scramble after an elusive unit. Nevertheless, there is at least one testing outfit on the market in which this cutting of lines is made to furnish the standard electro-motive force required; it is a small magneto machine which turned by hand at a fair speed furnishes a self-regulated e.m.f. of 600 volts quite steady enough for insulation testing. But (§ 630) the testing of insulating materials admittedly calls for no very high accuracy, and a far more elaborate mechanism would be required to satisfy the demands of even ordinary electrical work for a steady pressure.

Fortunately it is found that very steady and reliable electro-motive forces arise during certain Chemical Actions, to be dealt with in Chapter LXIII, and nowadays the Volt is realized in almost as portable and handy a form as the Ohm. The potential difference between the terminals of the little ' standard cadmium cell ' to be described in § 663 is 1·0183 volt at 15° C., with a very trifling correction for change of temperature. Although the Ohm and the Ampère have been chosen by a majority at a recent congress as the International Units, owing to a lingering doubt as to the perfect purification of one of the constituent chemicals in this cell, yet the Ohm Coil and the Cadmium Cell are the workaday standards in a good many laboratories, Ohm's law,

Volts=Ampères×Ohms, affording the connecting link. The Daniell and other cells are usable as rougher standards. It makes not the least difference what size a cell happens to be, so long as the same chemical substances are present at the same concentration and temperature: the electro-motive force on 'open circuit' (i.e. when ready to send a current but not actually doing so) is the same whether the cell be made up in a thimble or in a bucket.

Methods of Comparing the Electro-motive Forces of Voltaic Cells, etc.

§ 632. Electro-motive forces are differences of potential, therefore the **Electrometers** mentioned in § 588 of Electro-statics are available for comparing them. They must, however, be of particularly sensitive construction, for the volt is only $\frac{1}{300}$ of the electro-static unit of potential.

The common gold-leaf electroscope, for instance, does not respond to the touch of the wire from a voltaic cell, but the following contrivance enables it to do so, and forms an interesting connecting link between current electricity and electro-statics. An electrophorus with a particularly flat plate stands on another flat plate of metal whose surface is well varnished with shellac; these form a condenser with a very thin dielectric and therefore of large capacity. The two metal discs are connected momentarily to the wires from the voltaic cell and receive thence charges of + and − electricity. The electrophorus is lifted, thus greatly diminishing the capacity of the condenser and raising its charge to a potential quite high enough to actuate the gold-leaf electroscope.

§ 633. Applying Ohm's law, that Electro-motive force= Current×Resistance, we can see that

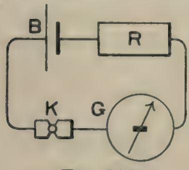


Fig. 316.

(1) To keep the current constant the resistance in a circuit must be proportional to the e.m.f. acting,

Or (2) in a circuit of constant resistance the current will be proportional to the e.m.f.

A circuit is made up as in Fig. 316 with battery (voltaic cell) B, galvanometer G, resistance box R, and a plug key K.

The circuit is made and resistance r is unplugged in R until the

galvanometer stands at some conveniently large reading g. B is removed and replaced by the other battery B′, then

(1) The resistance is altered to the r' which is found to cause precisely the same reading of the galvanometer.

$$\text{Then e.m.f. of B/e.m.f. of B}'=r/r'.$$

Or else (2) r is left unaltered and the new reading g' of the galvanometer is observed. Then if c and c' are the relative values of current represented by g and g' (e.g. for a tangent galvanometer $c/c'=\tan g/\tan g'$) we have

$$\text{e.m.f. of B/e.m.f. of B}'=c/c'.$$

The reader of § 626, in which is described an experiment only too apt to be confused with the present one, will perceive a possible defect in this method of comparing battery electro-motive forces. For the two batteries may not have the same internal resistances, and in any case if these are comparable with r and r' the ratio given above becomes only a rough approximation.

e.g. r for an 'accumulator' was 10 ohms, r' for a Leclanché was 5 ohms, hence e.m.f.'s apparently as 2 : 1. But with a more sensitive galvanometer r was 1000 ohms and r' 700 ohms, e.m.f.'s as 2 : 1·4. The discrepancy was due to the Leclanché having an internal resistance of 2 ohms while that of the accumulator was insignificant, this made the actual ratio of resistances in circuit in the first case 10 : 7, while in the second case 1000 : 702 does not differ appreciably from the accepted 1000 : 700.

§ 634. A way of avoiding this difficulty is to keep both batteries in circuit always. At first they are connected properly in series (i.e. circuit to zinc, carbon to zinc of second, second carbon to circuit).

$$\text{Then } E+E'=cr.$$

Subsequently one cell is disconnected, turned round, and put back, so that they are now joined in opposition (carbon to carbon, zincs to circuit),

$$\text{then } E-E'=cr' \qquad \text{or else}=c'r.$$

Adding these two equations, and then subtracting them, gives

$$\frac{E}{E'}=\frac{r+r'}{r-r'} \qquad \text{or else}=\frac{c+c'}{c-c'},$$

This '*Sum and Difference Method*' seems to find favour with examiners, but it is open to the very grave objection that few voltaic cells submit to having a current driven through them backwards without increasing in e.m.f. to oppose it.

§ 635. Undoubtedly the best plan is to swamp variations in battery resistance by using a galvanometer with which is incorporated a high constant resistance. This combination forms a **Voltmeter**, it is method (2) above in portable form.

A rather elaborate voltmeter in use by the writer consists of a micro-ammeter (as in § 606) of 60 ohms shunted by 1·2 ohms ; in series with this combination is a manganin coil of 999 ohms enclosed in the same box. Thus 1 volt applied to the instrument's terminals drives ·001 amp. through the resistance, $\frac{49}{50}$ of this passes through the shunt and $\frac{1}{50}$, i.e. ·000020 amp., through the actual galvanometer.

The graduations 20, 40, 60, 80, 100 microampères have therefore been relabelled 1, 2, 3, 4, 5 Volts, and the instrument reads pretty exactly the electro-motive force of any ordinary voltaic cell to which its terminals may be wired. And it is immediately made available for voltages above 5 by putting in series with it an appropriate number of thousands of ohms ; e.g. for maximum range 20 volts put 3000 ohms in series, 20 volts sends the same current through the total 4000 ohms as 5 through the 1000, and the graduations are now to be read 4, 12, 16, 20 volts.

Thus a Voltmeter is a high-resistance galvanometer with a scale graduated to read volts pressure between the terminals instead of the magnitude in ampères of the current passing through, just as a spring balance employed for weighing parcels might be graduated to read cost of postage instead of weight in pounds avoirdupois.

NOTE.—The place of an Ammeter is *in* the main circuit ; a Voltmeter is placed as a shunt across the two points of the main circuit between which the e.m.f. is required.

§ 636. The best of voltmeters has its limitations, however. The exceptionally high resistance voltmeter just described would read only ·6 volt if connected to a standard 1·018 volt cadmium cell. For the cell has an internal resistance of about 600 ohms and demands a far more perfect method of comparing electromotive forces than those described above. Such a method is afforded by the **Potentiometer.**

The Potentiometer in its simplest form, Fig. 317, consists of a long thin wire of resistance metal, stretched beside a scale of equal parts. An accumulator, which is a particularly constant sort of voltaic cell, maintains a steady current through the wire. A point on the wire near the end connected to the + terminal

of the accumulator is at a potential nearly 2 volts higher than a point near the other end. If the wire is perfectly uniform the fall of potential takes place perfectly uniformly along it; the potential

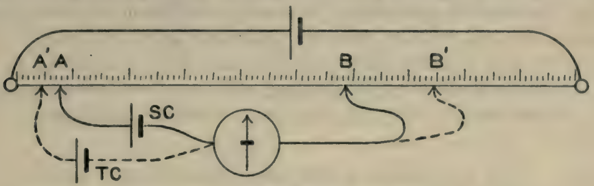


FIG. 317.

difference between two points 10 cm. apart is the same wherever the pair is located on the wire, the potential difference between any pair of points 20 cm. apart is twice as much, and so on.

Just in the same way, if water were flowing along a straight uniform channel the water-level would fall uniformly with distance, the difference of level between two points 3 miles apart being 3 times that between any two points 1 mile apart, and so on.

At a point A near the upper (+) end of the long wire a sliding contact-maker makes connection with a wire from the + end of a standard cell SC. The − end of the cell is connected through a sensitive galvanometer to a second sliding contact-maker lower down the long wire, and by trial there is found for this a position B such that when the contacts at A and B are made there is no deflection of the galvanometer. Then the difference of potential between A and B is equal to the electro-motive force of the standard cell.

For, returning to the water analogy, it is almost as if the standard cell were a weir, in a back-water represented by the wires joining it to A and B. *No current flows in the back-water*, it is at one stagnant level above the weir and at a lower stagnant level below the weir. Evidently these levels are those of A and B on the stream, or else water would flow in or out of the back-water there. Hence the sudden difference of level at the weir is the difference of level between A and B.

Now substituting (as suggested by the dotted lines) for the standard cell any one TC of the cells to be tested, a length A′B′ is found on the potentiometer wire such that again the galvanometer remains undeflected when the contacts are made there.

$$\text{Then } \frac{\text{e.m.f. of test cell}}{\text{e.m.f. of standard cell}} = \frac{\text{length } A'B'}{\text{length } AB}$$

and any number of cells being tested in turn, their e.m.f.'s are proportional to the distances required on the wire to balance them.

Since no current flows in cell, connecting wires, or galvanometer when the desired position of balance is attained, therefore their resistances are wholly without influence on the result. The standard cell of 600 ohms can be accurately compared with a dry cell of half an ohm, the leading wires may be miles in length, the galvanometer may be the first sensitive instrument that comes to hand.

§ 637 : For compactness the long potentiometer wire may be lapped to and fro on a board, but that is a poor arrangement. A better plan is that adopted in the Crompton and other potentiometers, now in widespread use in commercial work ; nine tenths of the wire are wound into little resistance coils, only the remaining tenth is stretched along the scale. Contact A is made on one of the studs separating the coils (see § 619), B is on the wire ; there is no disadvantage in this, it is like using a foot rule with only the last inch subdivided.

In these instruments the main current is adjusted by an external variable resistance until a reading 1018 balances the standard cell, they then read straightaway in millivolts (·001 volts) without any rule of three.

In the laboratory two resistance boxes in series sometimes constitute the potentiometer 'wire.' Their total resistance is kept always 10,000 ohms, the driving accumulator is kept on the extreme ends, the branch circuit containing cell, galvanometer, and tapping key is connected to the terminals of the first box only, resistance is unplugged in one box and an equal amount plugged in the other till balance is obtained; then e.m.f.'s of successive test cells are directly as balance values of resistance in first box.

§ 638: **Further uses of the Potentiometer.** The e.m.f. to be measured need not necessarily be due to a voltaic cell, it may be that between the ends of a conductor through which a current is flowing, and this enormously increases the usefulness of the potentiometer.

For suppose we want to measure an E.M.F. greater than the 2 volts or so which the driving accumulator maintains between the ends of the wire, say the pressure somewhere in a nominal 120-volt lighting system. Connect across the mains a 10,000-ohm resistance, select two points on this 100 ohms apart, the fall

of potential between them is only 1 % of the whole drop, lead wires from these points to the potentiometer and measure their 1·2 volts in terms of the standard cell.

Or if a large Current is to be measured, it is sent through a standard low resistance, say 700 amp. through a broad plate of manganin of ·001 ohm resistance. The fall of potential between the ends of this will be ·001 of that over 1 ohm, i.e. by Ohm's law $\cdot 001 \times 700 = \cdot 7$ volt; wires are brought from the ends of this low resistance up to the potentiometer, and the standard cell supplants the ammeter in the measurement of current.

The Potentiometer finds further employment in the accurate comparison of Resistances, and competes successfully with the Wheatstone bridge. A current is sent through the two resistances in series, and wires from the ends of first one, then the other, are brought to the potentiometer; the ratio of the readings obtained is that of the potential drops, and that is the ratio of the two resistances. The resistances may be very much smaller than can be dealt with by the Wheatstone bridge.

EXAMPLES.—CHAPTER LXI

1. How would you show deflection of the compass needle by the current from a Wimshurst and diverge the leaves of an electroscope by a voltaic cell? [L]m.

2. Two cells are joined in series and give ·044 amp. through a resistance. One being now reversed they give ·013 amp. If one has e.m.f. 1·08 volt calculate that of the greater. [L]m.

3. What are the essential features of an ammeter and of a voltmeter? How is each connected to the circuit to be tested? [L.]

4. Describe some form of potentiometer, and show how it may be arranged to compare a small electro-motive force such as that of a thermo-couple with the electro-motive force of a standard cell. [L.]

5. Explain how you would calibrate a low-range voltmeter by means of a potentiometer. [L.]

6. How would you use the potentiometer to measure a large current or a high voltage? [L.]

Chapter LXII

ENERGY AND THERMAL EFFECTS OF CURRENT

The great utility of Electricity lies in the power it gives us of doing work at a distance. That is, an electric current carries Energy.

§ 639. It was explained in § 572 that to raise a Quantity of Electricity through a Difference of Potential involved the doing of an amount of work equal to the product of charge and potential difference. In that paragraph the unit of electrical quantity was the electro-static unit defined in § 569, and the unit of potential was such that their product was one erg of energy. Now, in Current Electricity, although very different-sized units are employed, the fundamental relation, *Quantity of electricity $\times$ potential difference, i.e. quantity $\times$ electro-motive force = Energy,* of course still holds. The primary unit of quantity is that carried by the decampère in one second, and to raise this through the small unit potential difference also defined in § 597 demands the expenditure of one erg of energy.

[For if a wire carries 1 decamp. across a field of such strength that if the wire moved 1 cm. sideways it would cut 1 unit magnetic line, a force of 1 dyne is acting upon it and therefore 1 dyne $\times$ 1 cm. $=1$ erg of work would be done in the motion. Conversely, if a wire moved 1 cm. and cut 1 line in 1 sec., unit e.m.f. would be induced in it, and if it required 1 dyne $\times$ 1 cm. of work to cause the motion, it is a logical necessity that 1 decamp. has been made to flow in the wire for the 1 sec. by this unit electro-motive force.]

The practical unit of quantity, the coulomb, carried by one ampère flowing for one second, is one-tenth, and the volt is one hundred million times the corresponding primary electro-magnetic unit; their product is therefore ten million ergs, the **Joule** of § 23.

To drive one Coulomb of electricity against a potential difference of one Volt requires one Joule of work to be done.

Conversely, *when an electro-motive force amounting to one Volt between point P and point Q in a circuit has driven one Coulomb of electricity from P to Q one Joule of work has been done.*

Whether any of this electrical work has been converted into useful mechanical work or whether it has all been dissipated in heat depends on the nature of the circuit between P and Q. An electro-motor in PQ could give us most of this as mechanical energy, a mere resistance wire converts it at once into heat with perhaps a little light.

§ 640. To measure the expenditure of electrical energy we adopt the arrangement of Fig. 318. The Ammeter A measures the current through PQ, the watch T measures the duration of the current in seconds, AT is the number of coulombs. The Voltmeter V applied from time to time as a shunt over the points P Q measures the electro-motive force or potential difference between them, in Volts ; VAT Joules f work have been expended in PQ, whether it be motor, resistance wire,* lamp, electrolytic cell, or whatnot.

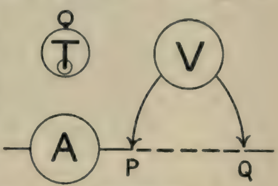


FIG. 318.

Of course there is in practice a wide choice of quantity- and pressure-measuring instruments.

In experiments of high accuracy most observers nowadays would probably employ a 'silver voltameter,' § 652, for coulombs, and a potentiometer and standard cadmium cell for volts.

Or, taking the commonest instance of all, the quantity of electricity entering your house is measured on the spot by a meter such as was described in § 613, the pressure is measured at the Electric Supply Station by a voltmeter, a little loss of pressure in the mains is allowed for, and you are charged for the (coulombs × volts) you 'consume.'

§ 641. Your quarterly account, however, contains no mention of either of these things, but is reckoned on the number of 'Board of Trade Units' (B.T. Units, or simply, Units) at a few pence each. When you inquire of the engineer what these may be you hear that they are 'Kilowatt-hours.'

* Recollect that 4·2 joules of work are equivalent to 1 calorie of heat.

The **Watt** is the unit of Power adopted in electrical measurements, i.e. the unit rate of doing work. A power of one Watt does one Joule of work per second.

$$\textit{Watts} \times \textit{seconds} = \textit{Joules}.$$

One horse-power = 746 watts; one kilowatt = 1000 watts = about $1\frac{1}{3}$ h.p.

That is, the B.T.Unit is the amount of work done by a $1\frac{1}{3}$-h.p. engine working for an hour, and it = 1000 × joule/second × 3600 seconds = 3,600,000 joules.

$$\text{Also} \quad \textit{Watts} = \textit{Volts} \times \textit{Ampères}.$$

Multiplying together the readings of voltmeter and ammeter in Fig. 318 gives the power in watts in the circuit at that moment.*

Instruments called Wattmeters are sometimes employed, they effect this multiplication automatically. Their construction is that of the iron-less alternating-current instruments of § 607; the main current flows in the fixed coil and produces a field proportional to itself; the shunt current, which is proportional to the voltage, traverses the moving coil, the force on which is therefore proportional to ampères × volts, i.e. to watts.

Thus the Power is the product of Volts and Ampères, a large current at a low voltage carries no more power than a quite small current at a high voltage; 1 amp. from a 2-volt accumulator runs a bicycle lamp, $\frac{1}{4}$ amp. on a 250-volt supply runs a 50-c.p. lamp; 373 amp. at 2 volts would be necessary to drive the 1-h.p. motor that $1\frac{1}{4}$ amp. drives at 600 volts, $\frac{1}{16}$ amp. at 12,000 volts or $\frac{1}{80}$ amp. at the quite practicable 60,000 volts.

§ 642. **Heat produced in a resistance.** In the particular case of PQ being simply a resistance of some sort obeying Ohm's law we can find another expression for the energy expended in it, now in the form of Heat (incl. light).

$$\text{For} \quad \text{volts} = \text{ampères} \times \text{ohms}$$

$$\therefore \textbf{Energy } VAT = ARAT = A^2RT \text{ joules}$$

which expresses **Joule's Law** that *The dissipation of energy as heat in a resistance is proportional to the resistance, the time, and the square of the current.*

$$\text{Watts} = (\text{ampères})^2 \times \text{ohms}$$

$$\text{Joules} = (\text{ampères})^2 \times \text{ohms} \times \text{seconds}$$

and since 4·2 joules = 1 calorie, § 187,

$$\textbf{Heat } \textit{produced, in calories} = (\textit{ampères})^2 \times \textit{ohms} \times \textit{seconds} \div 4{\cdot}2.$$

* And dividing volts by ampères gives the Resistance of PQ, § 630.

To test this law experimentally an open coil of eureka wire (the resistance of which does not appreciably change with temperature) is wound, and its resistance measured. It is immersed in paraffin oil in a calorimeter, the total water-equivalent of which is known, and the current from a strong battery of several cells is sent through for a definite time. The current is regulated by adjustable resistance and measured by an ammeter.

§ 643. Heating can be localized in a circuit by introducing short pieces of high resistance, and this local heating is made the greatest possible use of in everyday applications of electricity.

'Hot-wire' Ammeters and Voltmeters are actuated by the expansion of a fine wire which is heated by the current to be measured, either continuous or alternating. The wire sags, a thread attached to its middle and wound round the axle of the pointer is pulled back by a spring, and the pointer moves over the scale.

Resistances of coiled iron wire or ribbon, used for regulating considerable currents (e.g. for starting motors) have to be well ventilated. On the other hand, electric car- and room-heaters, ovens, flat-irons, warming-pans, kettles, saucepans, etc. etc., are designed to make the best use of the heat generated in wires, strips, or films of resistance metal usually embedded in insulating enamel and forming part of their walls.

The surgeon's electric cautery and the tobacconist's cigar-lighter alike consist of a short piece of thin platinum wire heated to white-heat by the current from a few cells.

The electric incandescent lamp was at first also a fine platinum wire, enclosed in a vacuous glass bulb. The platinum wire was soon superseded by a carbonized vegetable fibre which withstood a higher temperature and being of higher resistance enabled more economical voltages to be used (see below). Nowadays the carbon filament is obtained by carbonizing a squirted thread of chemically prepared cellulose and then precipitating a glossy coating of graphite on it by heating it to redness in a hydrocarbon vapour. The vacuum in the bulb is made a very good one. Carbon filaments are now largely superseded by exceedingly fine threads of the metals tantalum or tungsten; the latter especially endures a much higher temperature than can the carbon thread, and is therefore a three times more efficient light giver (§ 501).

Incandescent lamps as bought are marked to take (A) ampères and give (C) candle-power at voltage (V) of the mains. This means that striking a balance between cost of power and cost of renewals the best economy will be obtained by using that particular voltage.

The power consumption, in ' Watts per candle ' VA/C, is too great at lower voltages, for the lamp may then carry quite half the current and be only dull red hot; it diminishes rapidly as the voltage is increased, but the life of the lamp shortens faster. The candle-power of a carbon lamp is proportional to nearly the sixth power of the voltage, but the metal filament, whose resistance rises with temperature instead of decreasing, is less affected; a tungsten lamp will stand double voltage for a minute or two with apparent impunity while the carbon lamp has become the blackened remnant of a glorious firework.

Lamps are always arranged in parallel between the mains and the highest candle-power lamps have the lowest resistances; for V being constant, A is inversely as R, halving R doubles A, the heat production A^2R is doubled. Or simply, A is proportional to the conductance, and so therefore is VA the power consumed.

The high temperature necessary for efficient luminosity (§ 500) and consequently the intense radiation (§ 496) means that much energy must be dissipated per square millimetre of radiating surface. The same resistance is offered by a wire ·06 mm. diam. and 4 m. long and by a wire ·03 mm. and 1 m. long, but the former has 8 times the radiating surface and will hardly glow with a current that raises the latter to the brilliance of the tungsten lamp. This is a 230-volt 32-c.p. lamp; it is the finest filament the lamp-makers had succeeded in producing in 1911. Suppose they attempt a 16 c.p.-lamp at the same voltage: it is no use shortening the filament, for that reduces its resistance, A^2R increases and so do temperature and candle-power: it is no use lengthening the filament, for that diminishes A^2R and temperature and is ruinous to efficiency. That is why the more resistant carbon still holds its own in high-voltage lamps of small candle-power.

The Electric Furnace is a trough packed at start with a poorly conducting mixture of coke, ore, etc. Several hundred volts is applied between carbon blocks at the ends, current starts, warms the mass and increases its conductance; VA rapidly increases, and partly by pure resistance, partly by arc formation, the whole contents are presently boiling somewhere between 2000° and 3500° C. Such furnaces are coming into commercial use in steel-

casting in this country, and even in iron-smelting in Norway; at Niagara coke and sand yield the intensely hard carborundum, from which the highest temperatures distil everything to leave pure soft graphite.

One is sure to hear it asked, "Why cannot an electric current be regulated like water or steam by partly turning off a tap?" The sharp constriction in a half-closed tap causes violent eddies in the fluid, and it is the dissipation of energy in these that absorbs driving pressure and slows the stream. No such eddies are produced in the electric stream, and the whole resistance of a short sharp constriction, say the points of contact in a nearly opened switch, is but small. Further, a stream of fluid of great heat-carrying capacity conveys away the frictional heat generated in the tap, but the electric stream cannot, and the constricted part runs the risk of getting melted. **Fuses** are short bits of thin wire introduced into a circuit, being short they normally absorb no energy worth mentioning, but when from a 'short-circuit' or an 'overload' a current too great for safety begins to flow, the fuse wire melts and stops it.

§ 644: It is in consequence of Joule's law that the electric supply companies adopt such dangerously high voltages: they are forced to by economic considerations of the loss of energy in the mains. As an instance, suppose it is desired to drive a 150-h.p. motor half a mile away. At the old-fashioned supply pressure of 110 volts the motor requires 1000 amp., for 150 h.p. = about 110 kilowatts. A copper conductor 1 sq. in. cross-section is usually allowed for 1000 amp., go and return mains would therefore contain 63,360 cu. in. of copper, weighing 20,000 lb. and costing about £1000 sterling. Their resistance would be ·04 ohm, and to drive 1000 amp. through this absorbs $1000 \times \cdot 04 = 40$ volts, by Ohm's law. That is, 150 volts pressure must be maintained by the generator to keep 110 at the motor; the power represented by 1000 amp. driven by the difference, 40 volts, (about 50 h.p.) being absolutely wasted in merely warming up the mains.

But at 5500 volts the required 110 kilowatts would be carried by 20 amp. ($5500 \times 20 = 110 \times 1000$). A copper wire only $\frac{1}{50}$ sq. in. section need be allowed for this; weighing only 400 lb. and costing £20 for copper and probably £100 or £200 for insulation. Its resistance would be 50 times as much as before, i.e. 2 ohms; to drive 20 amp. through this takes $2 \times 20 = 40$ volts. This is as great a fall of pressure on the way as before, but now

it is only an insignificant addition to 5500 volts—less than 1 %—only $40 \times 20 = 800$ watts = about 1 h.p. is now wasted in the mains. Thus the economy of transmission improves about proportionally to the voltage. Long-distance transmissions are working at 120,000 volts, but now the engineer is confronted with the new and serious difficulty that at these pressures electricity begins to leak into the air around his overhead line.

§ 645. The heating effect above described is the same whatever the direction of the current, it is the result of the frictional drag that we call electrical resistance. In fact the resistance in ohms might be defined as the dissipation of energy in joules per second when 1 amp. is passing through the conductor.

There are other effects, to which the title **Thermo-Electric** is usually reserved, which are reversible, depending on the direction of the current, and are not at all of the nature of a frictional drag, for they may produce active electro-motive force.

Peltier discovered in 1834 that when a current was sent through the junction of two different metals, there was a local heating quite distinct from the Joule heating, for it did not depend on the cross-section of the conductor, but only on the current, to which it was directly proportional (not to C^2). And further, when the current was reversed, there was a local cooling in place of the heating.

In the first case the current evidently has to leave a proportion of its energy at the place where it enters the second metal, in the second case heat-energy is returned to the current at the place where it returns to the first metal. It is as if there were a very small difference of potential between the metals: the current has to climb against this and lose electric energy in the one case, in the other it is assisted by it.

It is therefore to be expected that if we kept on supplying heat from without to the junction from which the current abstracts heat, and if we arranged to take away the heat it evolves at the other junction, we might keep a current going by this means alone. And this tallies with Seebeck's experimental discovery in 1821 that if in a circuit composed of two different metals one of the junctions is heated and the other kept cold an electric current is caused to flow round the circuit.

This **thermo-electric current** is usually very small, but by reducing the resistance of the circuit it may become pretty large. An old demonstration apparatus consisted of a flat horizontal bar of bismuth a few inches long, with a fairly large compass

needle pivoted on its middle. A stout copper strap arched over the needle and had its ends soldered to those of the bismuth bar.

When one soldering was heated by a match the compass needle deflected showing the circulation of a current which went from bismuth to copper at the 'hot junction.' Or a piece of ice on one end produced a current from copper to bismuth at the 'cold junction.'

In an ordinary circuit thermo-junctions can easily be made by putting in a length of iron wire with its ends twisted or soldered to the copper wire, one can be kept in boiling water, and then inserting different resistances the feeble current will be found inversely proportional to them, i.e. it is a constant Electro-motive Force that is being produced in the circuit.

This thermo-electro-motive force depends on the metals in contact. Of common metals bismuth and type-metal (lead containing antimony) are most effective, if the junctions are at 0° and 100° C. they produce an e.m.f. of about ·01 volt. Copper and iron give about $\frac{1}{8}$ as much, copper and eureka ·0037 volt. Between copper and lead, solder, mercury, and platinum the thermo-electro-motive force is very small; between copper, phosphor-bronze, and manganin it is practically nil. At the hot junction the current flows from bismuth or eureka to copper or lead.

The e.m.f. is not simply proportional to the difference of temperature of the junctions, the connection between them is parabolic, the e.m.f. may be likened to the height above the ground of a thrown stone and the difference of temperature to the horizontal distance it has travelled. Immediately after leaving the hand the stone's path is nearly straight, its rise is proportional to its distance. Many pairs of metals never get beyond this condition, their thermo-electro-motive force is practically proportional to the difference in temperature. Such are bismuth and lead until they melt, copper and eureka to 800° C.

Later the stone ceases to rise; copper-irons at 0° and 137° attain a maximum e.m.f.

Ultimately the stone falls to the starting level and quickly passes below it, if possible—heating the copper-iron junction above 275° reverses the current and sends it from iron to copper. This is easily shown by attaching iron and copper wires to an ordinary sensitive galvanometer, twisting their ends together and heating them with a match; the needle swings gently to the left, returns to zero, and swings wide to the right.

Thermo-electric junctions are now extensively used as ther-

mometers and pyrometers; copper and eureka up to 800°, platinum and platinum-rhodium alloy up to 1600° C. All that is necessary is a couple of thin wires welded together at their points and enclosed in a little tube sheath, insulated flexible leads, and a sensitive voltmeter graduated by trial and calculation to read temperatures direct.

The thermo-electric contrivances employed in studying radiation have been described in §495.

The experiment Fig. 319 illustrates both the Peltier and Seebeck effects.—

To both ends of two steel knitting pins are soldered copper wires, the pins are laid side by side separated by thin silk and wrapped up together. The wires from one pin go to a reflecting galvanometer, those from the other to a battery which sends a few ampères through. The 'Joule effect' warms the whole pin uniformly, the 'Peltier effect' warms one end and tends to cool the other, heat passes through the silk at different rates at the two ends, the junctions on the first pin are warmed to different temperatures and a feeble current soon begins to show in the galvanometer.

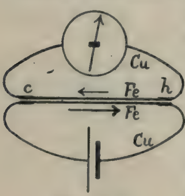


FIG. 319.

A third small thermo-electric action is the 'Thomson effect.' Electricity and heat both flowing in a metal bar interfere with each other to a slight extent. In copper and most metals electricity flowing from a hot part to a cold assists the flow of heat, but in iron it retards the flow and exaggerates differences of temperature.

EXAMPLES.—CHAPTER LXII

1. Distinguish clearly between quantity of electricity and quantity of electrical energy. What current transmits 1000 horse-power at 500 volts, and what fall of volts will occur in the ·01 ohm mains? [L.]

2. A metal cylinder is kept spinning with uniform angular velocity ω' between the poles of an electro-magnet, the axis of rotation being perpendicular to the magnetic force (H). Show that the work per second required to maintain the rotation is proportional to $H^2\omega'^2$. [L.]

3. How would you find the watts per candle required by an incandescent electric lamp? [L.]

4. A carbon-filament glow lamp carries ·3 amp. at 200 volts and gives 16 candle-power, a tantalum lamp carries ·3 amp. at 100 volts and gives 21 candle-power, a Nernst lamp carries ·25 amp. at 240 volts and gives 32 candle-power. Compare the working efficiencies of these three lamps. [L]m.

5. Prove theoretically that amount of heat produced by a current is proportional to (1) square of the current, (2) resistance through which the current passes, (3) time. [L.]

6. Describe a frictional and an electrical method for measuring Joule's Equivalent, explaining how to calculate out. [M.]

7. How can you concentrate the heat production into a small portion of the circuit ? [L]m.

8. State laws of production of heat in electrical circuit. How prove them ? Compare calories produced per second in A of 20 ohms with 30 volts over ends, and in B of 30 ohms carrying 15 amp. [M.]

9. State Joule's law of heating by an electric current, and deduce it from the principles of energy. An incandescent lamp receives $\frac{1}{2}$ amp. from a dynamo. If the voltage between the dynamo terminals is 111 and between the lamp terminals 110, find rate of consumption of energy (1) in lamp, (2) in leads. [L.]

10. If two wires are in parallel prove that more heat is developed in the thicker wire. Consider the filaments of lamps of different c.p. [L]m.

11. Two wires of the same material, but of different lengths and diameters, are joined in parallel and connected to a battery so that they are heated to a high temperature. What must be the relation between the lengths and diameters in order that the two wires may have the same temperature ? [L.]

12. State Joule's law for heat produced in a circuit and explain its connection with Ohm's law. Three equal conductors are put in a circuit carrying a fixed current (a) in series, (b) in parallel, compare heat produced. [Ab.]

13. How would you prove experimentally that the heat produced in a wire is proportional to the square of the current. Explain how to determine the resistance of a wire in electro-magnetic units with the aid of a standard ammeter. [L.]

14. A battery supplies 250 incandescent lamps in parallel, resistance of each 300 ohms. If voltage between the lamp terminals is 120, but rises to 122 when 100 lamps are switched off, calculate internal resistance of battery plus leads. Also find watts absorbed by each lamp in the first case. [L.]

15. If a 14-c.p. lamp uses 4·2 watts per candle, in how long will it heat a litre of water 10° C. ? [L]m.

16. A copper wire ·02 cm. diam. carrying 1 amp. reaches a steady temperature 100° C. If its resistivity is $2{\cdot}1 \times 10^{-6}$ and $J = 4{\cdot}2 \times 10^{7}$, calculate the Emissivity of a copper surface at 100°.

17. On passing 1 amp. through a platinum wire, its temperature rises 10° C. above surrounding objects, which are at 0° C. Assuming rate of loss of heat proportional to difference of temperature, calculate

temperature of wire when 2 amp. is passed. Temperature coefficient of resistance of wire 0·004. [L.]

18. Lamps aggregating 1 ohm resistance are supplied through leads of 0·02 ohm from a source at 51 volts. The voltage is subsequently raised to 250 and the lamps replaced by high-voltage lamps consuming the same total energy. Calculate the saving per thousand hours at fourpence per kilowatt hour. [L.]

19. Why are high potential currents usually employed in transmitting electrical energy to a distance? Explain the use of transformers in such cases, and point out the sources of waste of energy. [L.]

20. Explain a thermopile and how to use it to find the quantities of heat emitted by two hot bodies. [L.]

CHAPTER LXIII

THE PASSAGE OF ELECTRICITY THROUGH LIQUIDS

§ 646. When an electric current passes through a liquid (not being a molten metal), chemical changes are observable at the places where it enters and leaves the liquid. These changes began to be studied just over a century ago. In 1800 Carlisle and Nicholson decomposed water into oxygen and hydrogen, in 1801 Wollaston deposited copper and silver on baser metals and laid the foundation of the art of electro-plating, in 1807 Davy decomposed moist caustic soda and potash and discovered the metals sodium and potassium. Subsequently the subject was taken up by Faraday and he introduced various terms now employed in it. The chemical decomposition he called **Electrolysis** (λυσω, λυαω, unloose), the conducting substance the **electrolyte**, the plates leading the current in and out are the **electrodes** (ὁδος, a threshold), the entering plate the **anode** (ἀνα, up), the leaving plate the **cathode** (κατα, down from).

§ 647. The commonly accepted **Ionic Theory** affords the easiest explanation of most of the observed facts of Electrolysis and may as well be introduced at once.—

In a solution that can conduct electricity some of the molecules of dissolved substance are already split into constituent 'radicles,' e.g. HCl into H and Cl, $CuSO_4$ into Cu and SO_4, Na_2SO_4 into Na, Na, SO_4, etc. Support is lent to this view by the nearly doubled or trebled osmotic pressure, boiling-point rise, or freezing-point depression observed with conducting solutions (cf. Chapter XXIX). That these half-molecules do not immediately act on the solvent water as would be expected (e.g. Na to form NaHO and hydrogen) is explained by their carrying charges of electricity which in some way prevent chemical action—in all probability chemical action is an electric process. Characteristically hydrogen and the metals carry + charges, and -Cl, $-SO_4$, $-Fe(CN)_6$, etc., — charges.

When charged electrodes are dipped into the liquid the oppositely charged radicles—**Ions** (wanderers, 'ηιον, going)—very slowly travel to them and give up their charges; the negatively charged **anion** going up to the higher potential anode, the positively charged **cation** going down stream to the cathode. Thus the electric current passes through the liquid by a sort of convection, but the liquid itself does not move, only the free ions in it.

Having given up their charges to anode and cathode the ions are now in the usual condition of free chemical substances, or rather in the 'nascent' state, and produce a variety of effects according as they are chemically incapable or capable of attacking the materials in the liquid or the electrodes.—

§ 648. **Examples of Electrolysis with no secondary actions.**

For the experiments which follow the wires from a battery of two or more strong voltaic cells should be twisted round the ends of two pieces of arc-lamp carbon, the lower ends of which will serve as unalterable electrodes.

Dipping the electrodes into a beaker of strong hydrochloric acid, streams of fine gas-bubbles arise from each. Collecting the gases in inverted tubes in the usual way, the gas from the cathode is light and inflammable hydrogen, the colour and odour of the anode gas are those of chlorine; HCl has split into ions $\overset{+}{H}$ and $\overset{-}{Cl}$ which have given up their charges to the $-$ and $+$ electrodes respectively and become free gases.

Dipping the electrodes into a crucible of fused lithium chloride the choking smell of chlorine arises and the cathode when withdrawn shows little shining globules of metallic lithium;

$$LiCl \rightarrow \overset{+}{Li} + \overset{-}{Cl}.$$

Aluminium is commercially produced by similarly electrolysing aluminium hydroxide dissolved in melted aluminium fluoride, aluminium appears at the cathode, steam and oxygen rise from the anode. $2Al(HO)_3 = 2Al + 3H_2O + 3O.$

§ 649. **Examples of electrolysis accompanied by secondary actions on the liquid.**

With less concentrated hydrochloric acid the anode gas is less in quantity and less coloured, and after standing some time over water leaves an insoluble residue answering to the usual test for oxygen; the 'nascent' chlorine has attacked the solvent water to re-form acid and drive out oxygen.

With dilute acid of any sort, say sulphuric, two volumes of hydrogen are given off from the cathode and about one volume of oxygen from the anode.

$$\begin{array}{r|l} HH & SO_4 \\ H_2 \text{ off at cathode} & +H_2O=H_2SO_4+O \text{ at anode.} \end{array}$$

The oxygen when freed from its ionic charge is ' nascent,' it has not yet collected into its customary molecules O_2 of two atoms each, and a small part of it usually gathers into triatomic molecules O_3 of Ozone, recognizable by its odour. Up to 20 % ozone has been obtained by using a very small anode. In electrolysing cold potassium acid sulphate part of the anode oxygen goes to precipitate persulphate

$$2KHSO_4+O=H_2O+2KSO_4.$$

In the blue solution of copper sulphate the cathode is quickly covered with a red coat of copper, while oxygen bubbles off from the anode.

$$\begin{array}{r|l} Cu & SO_4 \\ \text{deposited on cathode} & +H_2O=H_2SO_4+O \text{ off at anode} \end{array}$$

so that unless stirred up the cathode liquid becomes paler by loss of copper and the anode liquid becomes strongly acid.

The electrolysis of strong brine has become an important commercial process. The sodium attacks the water round the cathode to form caustic-soda solution, chlorine accumulates in solution around the anode.

$$\begin{array}{r|l} Na & Cl \\ H \text{ off}+NaHO \leftarrow H_2O+ & (\text{and partially } 2Cl+H_2O=2HCl+O \\ [\text{cathode alkali}] \quad [\text{neutral}] & \qquad\qquad\qquad [\text{anode acid}] \end{array}$$

For the manufacture of caustic soda the cathode is a pool of mercury in which much of the sodium is temporarily retained as an amalgam, the metal is circulated into an adjoining tank of pure water and the amalgam slowly decomposes there to produce pure caustic solution.

If the whole liquid is gently stirred and kept cool, chlorine and caustic interact to produce sodium hypochlorite, bleaching and disinfectant ; if warm the interaction produces sodium chlorate, from which the potassium chlorate used in explosives can be obtained.

' *Pole-finding paper* ' is impregnated with sodium sulphate and phenolphthalein ; when moistened and laid across the ends of a broken circuit it turns crimson on the negative wire, owing to the alkali set free at the cathode.

§ 650. **Examples of electrolysis accompanied by secondary action on the electrodes.**

In electrolysing hydrochloric acid, using pieces of platinum foil as electrodes, the chlorine at the anode not only attacks water to set free oxygen, but also dissolves the anode to form the orange solution of $PtCl_4.2HCl$.

In electrolysing copper sulphate between copper plates, pure copper is deposited as before on the cathode, but now the 'sulphions' at the anode almost exclusively attack copper instead of water, the anode loses weight as fast as the cathode gains; or rather faster, because the impurities in its metal also fall away.

This electro-deposited layer of copper is used in almost all good Electro-Plating as a foundation for subsequent layers of nickel, silver, or gold. In Electro-Typing the medal, engraved wood-block, etc., is first brushed over with a thin conducting coat of blacklead and is made the cathode, deposition takes place on this and is continued to a considerable thickness. Model and mould are separated, and if much printing is to be done from the copper block a thin hard film of iron is now deposited on its face from an iron-sulphate bath. Copper for electrical purposes is usually refined electrolytically, as the impurities of the anode ingot, whether they fall away or dissolve in the liquid, do not get deposited on the cathode. The latter is sometimes a large revolving cylinder, and the deposit on it is kept uniform and free from liquid inclusions by the action of mechanical burnishers; when thick enough it is marked by a cutting wheel and torn off as a rough square wire requiring but little subsequent wire-drawing.

Silver plating is done in a bath of a so-called double cyanide of potassium and silver, more accurately potassium argenti-cyanide $KAgCy_2$. This splits into cation K and anion $AgCy_2$. At the cathode the potassium atom attacks the solution thus,

$$K+KAgCy_2=2KCy+Ag$$

and by this action the silver is deposited in a smooth layer (whereas directly deposited from silver-nitrate solution it is in separate granular crystals). At the anode the $AgCy_2$ attacks the ever-present excess of potassium cyanide and the silver anode plate itself, and re-forms the argenti-cyanide :—

$$AgCy_2+2KCy+Ag=2KAgCy_2.$$

Gilding is similarly done from a gold-cyanide bath.

§ 651. **Faraday** enunciated the quantitative **Laws of Electrolysis.** Having first satisfied himself that the current was the same all round the circuit, he discovered and stated—

Law I. *The amount of chemical action taking place in one and the same electrolyte, as measured by the mass of some particular constituent set free, is proportional to the quantity of electricity passed through, measured electro-magnetically* [*as in* § 612].

This is to be proved by comparing the weights of copper, for instance, deposited on the cathode when the ammeter reads 1 amp. for 60 min. or 3 amp. for 20 min., they should be the same; or by comparing the volumes of hydrogen or oxygen given off from dilute acid by different currents. On this law is based the use of the Voltameters described below for measuring the total quantity of electricity carried through by any current, however variable.

The mass of a substance set free by one coulomb of electricity is called the **Electro-Chemical Equivalent** *of the substance.*

That of hydrogen is ·00001035 grm., of silver ·001118 grm., of copper from blue copper sulphate ·000328 grm.

Law II. *The mass of an element set free by the passage of a given quantity of electricity is proportional to its chemical Combining Weight in the compound being electrolysed.*

To investigate this, voltameters (see below) containing solutions of dilute acid or alkali, of copper sulphate, ferrous sulphate, ferric chloride, silver nitrate, gold cyanide, platinum chloride, etc., are connected in series and a current passed, necessarily conveying the same quantity of electricity through each. Calculating the weights of the volumes of hydrogen and oxygen collected, and weighing the various cathode deposits, they will be found in the ratio, hydrogen 1, oxygen 8, copper 31·5, iron from green sulphate 28, iron from yellow solution 18·3, silver 108, gold 66, platinum 49, etc. These are the same proportional weights that ordinary chemical analysis shows capable of combining with or of replacing unit weight of hydrogen.

Notice at once the completely different meanings of *Electro-chemical equivalent* and *Chemical combining weight* (sometimes called *chemical equivalent weight*). But notice also their proportionality. Notice further, their possible variation, in the case of some substances, between different classes of compounds; iron would be deposited with greater economy of current from green salts than from yellow: the chemist knows how to explain

this on the Atomic Theory, but be careful not to confuse the Combining Weight disclosed by analysis with the Atomic Weight derived by the subsequent application of theory (though probably you will nowadays actually recollect the latter and get the former by dividing by the valency).

§ 652. The utilization of Faraday's first law of electrolysis for the measurement of quantities of electricity proves extremely convenient in practice. The electrolytic cells used for the purpose are called **Voltameters,** or sometimes Coulometers, and many varieties of them have been devised.

The oldest pattern of Gas Voltameter resembles Fig. 320; there are two electrodes of platinum foil immersed in weak sulphuric acid, the leading wires are covered with waterproof insula-

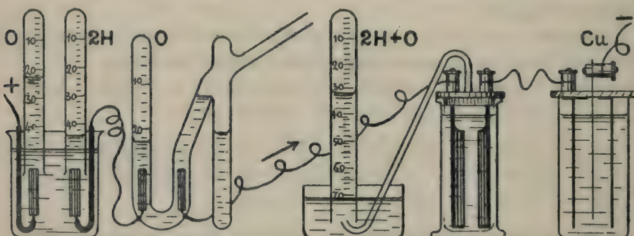


FIG. 320. FIG. 321. FIG. 322. FIG. 323.

tion. Rising above each is a glass tube graduated in cubic centimetres; the hydrogen rising into the cathode tube and the oxygen into the anode tube displace the liquid with which both are filled at first. Another pattern, easy to refill, is shown in Fig. 321, in it either hydrogen or oxygen can be collected according to direction of current, hydrogen preferably because the volume of oxygen is apt to be unduly diminished by its greater solubility in water and by its partial ozonization.

The Mixed-gas Voltameter, Fig. 322, is a small jar provided with an air-tight bung and leading tube and containing caustic-soda solution in which dip two large electrodes of sheet nickel. The action can be represented—

$$\begin{array}{r} 2H_2O+ \\ H_2\text{ off}+2NaHO \leftarrow \end{array}\left\{\begin{array}{c|c} Na & HO \\ Na & HO \end{array}\right\}\rightarrow H_2O+O\text{ off}$$

The nickel plates are not attacked, and the oxygen is free from

ozone. The mixed gases are collected in a graduated tube over water; they are of course explosive.

An instance of the Calculation necessary with gas voltameters has been given in § 205. It remains only to point out that since 1 coulomb liberates the electro-chemical equivalent ·00001035 grm. of hydrogen, the weight ·00541 grm. there calculated indicates the passage of ·00541/·00001035 = 522·5 coulombs. (If this were collected in 240 sec. during which the current was kept steady by galvanometer, that would indicate 522·5/240 = 2·18 amp., etc.)

[NOTE.—Roughly 1 coulomb produces $\frac{1}{8}$ c.c. of hydrogen.]

The mixed-gas voltameter is occasionally used as a house meter for electric supply; instead of catching the gases the fall of level in a graduated tube as the water is electrolysed away is used as the measure of quantity.

The Copper Voltameter, Fig. 323, consists of a small tank of fairly strong blue solution of copper sulphate slightly acidified with sulphuric acid; in it dip a couple of anode plates of sheet copper and between them a thin removable cathode plate. The latter is scoured, rinsed, dried, and weighed at the start, and rinsed, dried, and weighed at the finish, the grammes gain in weight divided by ·000328 = coulombs passed. The copper voltameter is very largely used for commercial testing purposes, it has the objection that the acid liquid attacks the newly deposited copper to a minute extent depending on temperature, concentration, time and area of contact, but tabulated correction for this is easily made and the results can be relied upon to $\frac{1}{4}$ %.

In a 'prepayment meter' the insertion of a coin enabled a definite length of thin copper sheet to be unrolled so as to dip into the liquid where as anode it was gradually eaten away.

The Wright house meter electrolyses mercurous-nitrate solution between a pool of mercury and a metal thimble, the mercury globules drop from the thimble into a graduated tube whence they periodically siphon over into a second wider graduated tube. The whole is hermetically sealed and is reset for use by inverting it for a moment, when the mercury runs back into the pool.

The Silver Voltameter is the most accurate that we possess, being reliable under prescribed conditions to one part in 10,000. A platinum bowl cleaned with nitric acid, dried and weighed, holds about an ounce of a 10 to 20 % neutral solution of silver nitrate, in which is suspended horizontally an anode of pure silver

plate the size of a half-crown. The anode is wrapped in filter-paper to prevent any specks of a black powder which usually appears on it from falling into the bowl. The silver is deposited in streaks of little granules on the platinum and is well rinsed with warm distilled water before drying and weighing the bowl. ·001118 grm. of silver is taken in international practice as the deposit for 1 coulomb.

A way, that will appeal to the chemist, of measuring small quantities, is to electrolyse potassium iodide between platinum electrodes and titrate the iodine with thiosulphate.

Figs. 322, 323 represent the arrangement of an experiment on Law II, for comparing the electro-chemical equivalents of hydrogen and copper, and so determining the latter's combining weight.

The reader will easily see that $1/\cdot00001035 = 96{,}600$ *coulombs deposit the combining weight in grammes of any substance.*

§ 653. **Electrolytic Polarization.** In the gas voltameter we start with a couple of plates of platinum immersed in weak acid. As soon as electrolysis begins one of these plates gets covered with oxygen and the other with hydrogen, and we have now virtually a plate of oxygen and a plate of hydrogen dipping in the liquid, both of them in a very active chemical condition, e.g. the oxygen is probably endeavouring to oxidize H_2O into H_2O_2. Now if chemical action is electrical in character it is surely probable that very different electro-motive forces may arise locally at these two plates and either assist or impede the passage of current through the cell (together they will impede it or we might get a perpetual motion). And this is actually found to be the case, a 1·1-volt Daniell cell is quite unable to drive a current through a water voltameter ; no gas *appears* on the electrodes, but we know the power platinum possesses of occluding gases, and the merest traces of them suffice to **polarize** the electrolytic cell. Even a 2-volt accumulator can scarcely produce a visible bubble, whereas with two accumulators in series the decomposition goes on merrily.

This back electro-motive force of **Electrolytic Polarization** is easily observed directly ; by a two-way switch the battery is thrown out of the voltameter circuit and a galvanometer put in, a strong deflection lasting several seconds is obtained while the polarizing gases pass back into the solution as charged ions again ; with large plates there is no difficulty in ringing an electric bell.

§ 654. **The Capillary Electrometer** affords an instance of polarization put to practical use. In a simple pattern, Fig. 324, a slightly sloping capillary tube joins two little reservoirs. That at the lower end is partly filled with mercury which also rises up the capillary, but not to the full level, for it is held down as in § 252 by the surface tension in the meniscus separating it from the weak sulphuric acid in the rest of the tube and second reservoir. At the bottom of this latter is a broad pool of mercury; wires are connected to both lots of mercury.

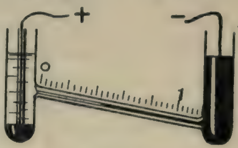


FIG. 324.

When a fraction of a volt is applied between these wires, so as to tend to drive a current in the capillary from acid to mercury, the meniscus surface of course polarizes and stops it. But *the polarization increases the surface tension* and drives the mercury farther down the capillary tube, past a scale which can be graduated either by preliminary trials with known fractions of a volt or by calculation from the known parabolic relation between potential difference and tension at a mercury-acid surface.

Thus the instrument is a sensitive electrometer or voltmeter for anything below ·9 volt, unlike a voltmeter it takes no current, unlike a quadrant electrometer it is easy to make and manage, and it finds much favour in physiological work. At higher voltages hydrogen bubbles form.

§ 655. A curiously exaggerated sort of polarization occurs with aluminium electrodes and is utilized for 'rectifying' alternating current, i.e. stopping out the back flow and transmitting only the direct rushes of current, so that accumulators may be charged, etc. The **Rectifier** is simply a lead plate and an aluminium rod in a jar of 1% sodium phosphate solution; when the aluminium is cathode the 1 volt back e.m.f. of hydrogen upon it is easily overcome by the 20 to 100 volts in the mains, but when reversed current makes it anode it is instantly overspread by a non-conducting oxide film, of exceeding thinness, but quite capable of preventing current being driven back, even by 100 volts.

§ 656. Suppose that we could accumulate much larger quantities of oxygen and hydrogen on the plates of our electrolytic cell, the polarization e.m.f. would drive a current for us for quite a useful length of time. Something towards this may be done by coating the electrodes with platinum black, which has a great

power of occluding gases, but nothing of practical value. To store adequate quantities we shall have to get them into some easily decomposed chemical combinations, liquid or solid. Now the monoxide of lead is a substance very readily reducible to metallic lead by the action of hydrogen, and quite easily oxidizable to higher oxides (red lead and brown peroxide). And lead is only too ready to re-form its monoxide, while the peroxide is a very active oxidizing agent used in safety matches: here then is the very material for our purpose.

The **Accumulator** or **Storage Battery** has two (or two sets of) plates which, whatever substances the maker chose to employ in their manufacture, are after the preliminary electrolysis of sulphuric acid (sp. gr. 1·2) between them, composed the one of porous lead and the other of porous lead peroxide. When these are joined by a wire circuit, current flows from the brown peroxide 'positive' plate through the wire to the grey lead 'negative' plate, for in the liquid the following process is taking place—

$$\begin{array}{r|l} \text{grey } Pb \leftarrow \overset{--}{SO_4} & \overset{++}{H_2} \rightarrow \overbrace{O \;\vdots\; PbO} \text{ brown} \\ =PbSO_4 & \qquad\quad\;\; \vdots\; +H_2SO_4 = PbSO_4 + H_2O \end{array}$$

with the ultimate result that both the plates have all their active surfaces converted into sulphate of lead while the acid becomes diluted to sp. gr. 1·18. Chemical action and current now cease of course; the accumulator has been discharged. It is recharged by again electrolysing the acid by a current driven through in the opposite direction, so as to carry hydrogen ions to the grey plate and reduce it to lead and oxidizing ions to the other plate to reconvert it into peroxide thus—

$$\begin{array}{r|l} Pb \;\vdots\; SO_4 \leftarrow \overset{++}{H_2} & \overset{--}{SO_4} \rightarrow PbSO_4 \\ & \qquad +2H_2O = PbO_2 + 2H_2SO_4 \end{array}$$

while the acid regains its 'full charge' gravity 1·2.

§ 657. The earliest accumulators were laboriously 'formed' up from two sheets of lead by charging, discharging, and recharging oppositely, over and over again, so as to corrode the surfaces deeply and get a fair amount of porous active substance. Some positive plates of lead, cast with multitudes of narrow and deep grooves to increase the surface, are still 'formed' in this way, but most positive plates are constructed by packing a paste made from red lead and nitric acid into the meshes of a leaden grid, while negative plates are packed with lead powder; the

whole idea being to get a large active surface with the least possible bulk and weight.

There may be several + plates connected by a leaden cross-bar in each cell, with intermediate (and 1 more) —plates similarly bridged together. The plates are kept apart by insulating distance pieces of glass, ebonite, or celluloid.

To charge the accumulator, *current is sent into its positive terminal* (red, marked +, connected to brown plate). Any number can be connected in series, i.e. — to + throughout, so long as the supply voltage provides at least 2·5 volts per cell. *A suitable adjustable resistance must be put in circuit* to keep the charging current down to what the makers prescribe, for too great a current, either on charge or discharge, disintegrates and buckles the plates; and then falling fragments lodge between them, and short-circuit and ruin the cell. When full the cell 'gasses' freely, for the lead and peroxide plates are not further alterable and the hydrogen and oxygen must come off; there is no fear of overcharging. The voltage is now 2·3.

With an alternating supply a Rectifier also must be put in circuit, with its aluminium rod connected to the + terminal of accumulator.

During **discharge** the current must be kept within the prescribed value (usually one-sixth the capacity in ampère-hours). The voltage quickly drops to 2 and there remains very constant till the battery is nearly exhausted, when it quickly falls. Below 1·8 volts the action becomes feeble and there is a tendency for the electrolytic variety to change into the common white insoluble variety of lead sulphate, which permanently chokes the cell.

Accumulators should be charged until they gas freely, every month, whether they have been used or not. Neglected discharged plates are sure to sulphate and spoil, especially when dry; if removal of liquid is necessary the cell must be fully charged, then the acid emptied out and the plates rinsed with soft water.

Only pure acid and soft water must be used; accumulators containing traces of metallic impurity, as from corroded terminals or interfering iron tools, are leaky and unreliable.

It is often recommended to test an accumulator's working condition by voltmeter only. In my experience this is too much like gauging a man's kicking power by feeling his pulse. A boxed-up accumulator in ordinary use is liable to so many ailments. Attach a short wire to one terminal and strike the

loose end quickly across the other terminal—a snappy sputtering spark means all is well, reject a weak sparker. Strike quick, or you will burn your fingers and may damage the battery, for an accumulator's internal resistance is very small and hence on short-circuit it sends a current so large as to speedily ruin the plates. I write this with some feeling, and left-handed; lately an accumulator's failure broke my arm; on subsequent test it satisfied the voltmeter, but its spark was feeble.

The Edison accumulator has a negative plate packed with spongy iron and a positive of electrolytic peroxide of nickel, the electrolyte is caustic-soda solution. Its e.m.f. is only 1·5 volt per watt-hour, it is scarcely lighter than a lead battery, but is said to withstand rougher treatment.

§ 658. **Primary Batteries** produce currents as soon as they are put together, without any previous 'charging with electricity.'

The earliest was the invention of Volta (whence primary batteries are often called Voltaic Cells) about 1796; it consisted of a plate of zinc and a plate of copper dipping in salt water or weak acid. The zinc dissolves, tiny bubbles of hydrogen overspread the copper, and a charm compass shows the passage of a current from copper to zinc along a wire joining them outside the liquid.

The action may be described in this way:—Both the metals tend to send out positively charged ions of themselves—to evaporate so to speak into the liquid—up to a definite 'solution pressure,' like a saturated vapour pressure; this tendency is by far the greater with the zinc. The numerous zinc ions meet and combine with the negative 'sulphions' or 'chlor-ions' of the acid employed; the now superabundant positive H ions give the liquid a positive charge which drives back all copper ions and then many give up their + charges to the copper electrode and form gas on its surface.

But this accumulation of hydrogen **polarizes** the electrode, the hydrogen soon competing as keenly for the oxidizing negative ions as does the zinc, the current stops, and the simple voltaic cell has no practical value.

The hydrogen must be got rid of somehow; merely scrubbing the copper with a wire brush has some effect, better was Smee's device of covering its surface with the absorbent platinum black; but voltaic cells were not really a success until chemical means were employed to remove the hydrogen.

§ 659. **Daniell** surrounded the copper with a blue solution of its own sulphate, kept from mixing with the weak sulphuric acid round the zinc by means of a 'porous pot.' The hydrogen ions diffusing through the pot on their way to the copper cathode attack the sulphate and displace its copper ions, and *these* give up their charges and are deposited as metal on the cathode. Here is a zinc-copper cell that never gets choked with hydrogen

$$\text{Zinc, Zn} \rightarrow SO_4 \;\Big|\; H_2 \rightarrow SO_4 \;\Big|\; \text{Cu} \rightarrow \text{Copper}, +$$

The Daniell Cell may have a variety of forms, the porous pot may be a round or flat vessel of unglazed earthenware, or a canvas bag, the zinc may be inside (Fig. 325, left) or out, according to choice. In 'gravity' patterns (Fig. 325, right) the pot is dispensed with, the copper plate lies at the bottom of a deep dish under a layer of blue copper sulphate crystals, three or four inches depth of weak sulphuric acid is poured on and the zinc plate is supported horizontally near the surface; the great slowness of liquid diffusion (§ 259) prevents the copper solution from reaching and seriously contaminating the zinc for weeks; if the cell may have to be moved about it can be half filled with sand.

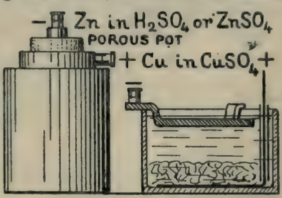


FIG. 325.

Whenever zinc is to be used in acid it must first be 'amalgamated' by rubbing it over with mercury under weak sulphuric acid. For common zinc rapidly dissolves to waste, but zinc amalgam, like pure zinc, does not dissolve in weak acid until an electric current is permitted to pass from it to the acid by closing the circuit of the voltaic cell.

The Daniell cell may, however, be set up with unamalgamated zinc in zinc-sulphate solution and works perfectly well.

$$\text{Zinc, Zn} \rightarrow SO_4 \;\Big|\; \text{Zn} \rightarrow SO_4 \;\Big|\; \text{Cu} \rightarrow \text{copper}, +$$

an action that stultifies the popular assertion that 'the zinc naturally dissolves in acid and so drives the cell,' and necessitates a fuller explanation on the lines given above. Evidently this cell is reversible, a current forced in at the copper would gradually

remove the deposited copper and redeposit the dissolved zinc, but the zinc comes down in incoherent crystals and makes the cell impracticable as an accumulator.*

The Daniell produces a steady electro-motive force of 1·07 to 1·10 volt, according to concentrations of solutions, its internal resistance even with a very porous pot is seldom less than an ohm, consequently it gives steady currents of no great power. It was formerly largely employed in telegraphic work, but requires too frequent attention.

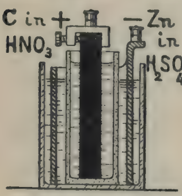


FIG. 326.

§ 660. In the **Bunsen** battery, Fig. 326, the hydrogen ions from amalgamated zinc in weak sulphuric acid are destroyed by strong nitric acid inside a porous pot containing the carbon cathode or 'positive plate' from which the current passes out into circuit. Lower oxides of nitrogen result, turning the nitric acid bluish green and presently coming off in brown fumes destructive alike of metal and mucous membrane.

The Bunsen cell's electro-motive force nearly reaches 2 volts, nitric acid is a ten times better conductor than copper-sulphate solution, consequently its internal resistance is low; Bunsen batteries can therefore send large currents and work very hard, but they must be kept outdoors, and be taken to pieces as soon as finished with, or the nitric acid will diffuse through and destroy the zinc.

FIG. 327.

§ 661. Chromic acid does not have such an instantaneously destructive effect on amalgamated zinc as does nitric, it is therefore permissible to dispense with the porous partitions, in the Chromic-acid or **Bichromate Battery,** Fig. 327, in which a plate of amalgamated zinc and twin plates of carbon dip in a solution of sulphuric acid 15 % and chromic acid, or more usually bichromate of potash, about 10 %. Here the hydrogen ions reduce the bichromate to a salt of chromium

$$6H+K_2Cr_2O_7+5H_2SO_4=2KHSO_4+Cr_2(SO_4)_3+7H_2O$$

* With potassium cyanide in place of copper sulphate the cell naturally works backwards with e.m.f. about ·5 volt, the copper dissolving to colourless cupricyanide in which it is anionic.

blackening the wine-red liquid and ultimately turning it green, when it must be renewed.

The bichromate cell produces at best nearly 2 volts, its internal resistance is low and it can work very hard, but the bichromate in the narrow spaces between the plates is soon used up and the cell polarizes unless shaken up. Though in this respect inferior to the Bunsen, it is frequently preferred for its freedom from fumes and ease of management; all that is necessary in laying the cell by is to withdraw the zinc from the solution, either by a sliding rod or as in some portable hand-lamps by turning the sealed cell upside down.

In the recent Benko pattern fresh liquid constantly oozes through the carbons under slight pressure; a steady 50 amp. is obtainable.

The Bleek-Love battery uses a zinc cylinder in caustic soda in a porous pot, surrounding which is a moulded carbon cylinder in a strong mixture of sulphuric acid, sodium bichromate, and common salt, smelling objectionably of chlorine. The whole volt e.m.f. between the soda (HO ions) and acid (H ions) gives it the high total of 2·7 volts.

§ 662. The **Leclanché Battery** stands always ready to yield moderate currents of short duration, and requires attention only once a year or so to replace water lost by evaporation and to put in a pinch of fresh sal-ammoniac; hence it excels for electric bells, telegraphs, and telephones. The containing jar is usually square, to stand close, and of glass so that the liquid level can be seen; the zinc is a plain rod, and the solution is a saturated one of ammonium chloride (sal-ammoniac) which has no action whatever on zinc until the circuit is closed. Then it attacks the zinc to form zinc chloride (which crystallizes as the double chloride of zinc and ammonium), sets free ammonia (which remains in solution and can be smelt if the liquid is warmed), and produces the hydrogen ions. These are oxidized by the solid depolarizer, black oxide of manganese, which, in granules mixed with carbon, is either packed round the carbon positive plate in a porous pot (Fig. 328, left), or is strapped on to it in baked blocks, or is incorporated in the hollow positive cylinder of baked carbon itself.

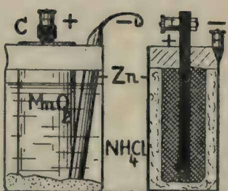


Fig. 328.

$$Zn + 4NH_4Cl = ZnCl_2.2NH_4Cl + 2NH_3 + 2H$$

The black MnO_2 gets partially reduced to a mixture of lower oxides. It is a rather slow oxidizing agent and this unfits the Leclanché for sending strong currents.

The electro-motive force of the Leclanché is 1·45 volts. The internal resistance of the pint size with porous pot is about an ohm, and this size will very steadily maintain the ·2 amp. for a bell, but fails when larger currents are demanded.

So-called Dry Cells are Leclanchés in disguise. Their outer case of zinc (Fig. 328, right) contains a thick cream of sal-ammoniac solution and paper-pulp or plaster, surrounding the carbon plate baked in a lump of carbon and MnO_2; the cell is sealed off with melted brimstone and pitch. Large dry cells of good make will maintain 3 or 4 amp., but cheap dry cells are not particularly trustworthy.

§ 663. **The Standard Cell.** The Daniell cell was formerly employed as a Standard of Electro-motive Force, with saturated sulphate solutions at 64° F. and pure metals its e.m.f. is 1·094 volt. But it must always be fresh set up, lest the copper sulphate diffuse and contaminate the zinc, and its e.m.f. depends to some extent on temperature (though less than most cells') because the sulphates are more soluble in warmer water.

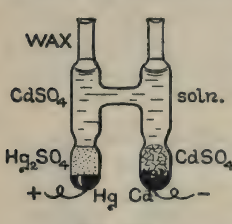


FIG. 329.

The modern **Standard Cadmium Cell** uses cadmium in place of zinc and mercury instead of copper, pairs of closely related metals but possessing useful dissimilarities. The mercurous sulphate forms a plaster-like plug above the mercury in Fig. 329, it is nearly insoluble, but if any does diffuse over and reach the cadmium it can do no harm, for the cadmium is already mixed with mercury into an amalgam. Cadmium sulphate is no more soluble hot than cold, the solution filling the cell remains of invariable strength, and the e.m.f. is very nearly **1·0183 volt** at any temperature.

The cell made up and sealed in an H tube can be sent by post without derangement. Its internal resistance is about 650 ohms, consequently it is useless except with the potentiometer. If permitted to send more than ·0001 amp. it polarizes, but recovers in a few minutes.

§ 664. We have two remaining questions to consider: what is **the source of the electro-motive force** and the energy of a voltaic cell, and how can we obtain the best output of energy from a battery of cells?

The old view, that the e.m.f. arose at the actual contact of the two metals (say, where the copper wire coming from a copper plate was fastened to the zinc), was apparently supported by the experimental use of a copper electrophorus and a zinc plate in the experiment of § 632; when these were separated after contact the electroscope opened, without any extraneous charging whatever, and this happened even in vacuo. But we saw in § 645 that a contact e.m.f. between metals exists indeed but is extremely small, the merest fraction of that of a voltaic cell. And we know nowadays that mere pumping does not remove the film of moisture and air condensed on surfaces. The experiment has been tried of putting iron and platinum plates in a vessel of hard glass, exhausting the air, and while red-hot washing out over and over again with hydrogen (to which both metals are quite permeable when hot); and this treatment effectually destroyed the ·37 volt contact-potential-difference usually existing between iron and platinum. It is now accepted that the e.m.f. of the cell arises at the metal-liquid contacts and partly at the liquid-liquid contact in a two-fluid cell.

The energy comes mainly from the heat of solution of the zinc in the acid, and the Electro-motive Force of a cell can sometimes be calculated thus:—

1 chemical combining weight in grammes ($32\frac{1}{2}$) of zinc dissolves to form zinc-sulphate solution with the evolution of 54,230 calories.

The removal of 1 combining weight in grammes of oxygen from nitric acid to form nitrous acid is found to require 9150 calories.

In voltaic cells the energy appears as electrical energy instead of heat, 96,600 coulombs cause the deposition or solution of 1 combining weight in grammes, § 651. Hence the output of this quantity of electricity from a Bunsen cell is accompanied by $(54{,}230 - 9150) \times 4{\cdot}2 = 45{,}080 \times 4{\cdot}2 = 189{,}000$ joules of energy, or 1·96 joules per coulomb. And since joules$=$coulombs$\times$volts (§ 639), therefore 1·96 is the voltage of the Bunsen cell.

Those varieties of cell whose e.m.f.'s are much affected by temperature grow hotter or colder during action, and require

a difficult thermo-dynamic correction to the foregoing simple calculation.

§ 665. The problem of **how best to arrange a number of voltaic cells** so as to drive the greatest current through a given circuit—cautery, coil, bell, telegraph, etc.—involves some knowledge of the relative values of the resistance of the external circuit and the internal resistance of the cells—that resistance which the current meets with as it crosses the layer of liquid (and perhaps the porous partition) between the plates.

The larger the plates, provided their distance apart is not increased, the less this internal resistance, hence large cells are capable of giving a greater current on short circuit than small ones, and of course of maintaining it, because there is a larger supply of active materials in the big cell. But for a high resistance, e.g. a telegraph circuit, small cells do just as well as large, for their e.m.f. is just as great and their few ohms extra internal resistance is insignificant compared with the total.

A battery of cells *all of the same sort* connected **'in parallel,'** i.e. with all their carbons wired together and to one wire of the circuit and all their zincs wired together and to the other end of the circuit (Fig. 330, P), is simply an imitation of one big cell. Its e.m.f. is that of a single cell, its internal resistance is that of all the individuals in parallel, calculated precisely as in § 622, e.g. if the N cells are all the same size it is 1/N that of a single one.

This is evidently the arrangement to adopt when the external resistance is smaller than that of a cell, for otherwise the greater part of the e.m.f. would be exhausted in getting the current through the battery itself.

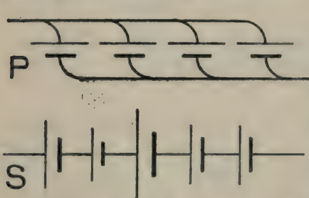


Fig. 330.

When the external circuit's resistance exceeds a cell's, the e.m.f. of a single cell may be inadequate to drive the requisite current. Then cells must be connected **'in series,'** i.e. circuit to zinc of first, carbon of first to zinc of second, carbons to zincs all through, last carbon to circuit (Fig. 330, S). The joint e.m.f. is the sum of all the cells, the joint resistance is the sum of the individual resistances. This is the only way in which cells of odd sorts can safely be connected up.

When 'in series' the whole current passes through each cell,

and the total consumption of zinc, etc., is much greater than when 'in parallel,' with each cell contributing only a fraction of the current.

When the joint resistance approaches the external it may be time to stop building in single series and to arrange in 'series parallel,' i.e. to couple pairs of cells in parallel and then to arrange these pairs in series, or even to parallel in threes and then arrange these groups in series. But when one comes to make arithmetical calculation it is difficult to find any justification for the trouble involved in this, unless one is dealing with an academic host of cells too small for their work.

EXAMPLES.—CHAPTER LXIII

1. A battery is joined to an electrolytic cell which contains (a) copper sulphate solution, (b) sugar solution. Describe what happens in each case. [L]m.

2. What is the connection between the volumes of the gases evolved by electrolysis of dilute sulphuric acid and the current passing through the electrolyte? Explain the action of the acid which enables the current to pass through the solution. [L]m.

3. What is the electro-chemical equivalent of an element? Describe how that of oxygen or hydrogen may be determined. [L.]

4. State the laws of electrolysis. If unit current decomposes $\frac{1}{3}$ grm. water per hour, how long would 10 units take to deposit 1 kg. silver? [M.]

5. Describe how to compare the electro-chemical equivalents of copper and hydrogen.

6. State Faraday's laws of electrolysis. A current which deposits ·85 grm. Cu in 25 min. [e.ch. equiv. ·000328] passes through 2 ohms, and a voltmeter attached to this reads 3·5 volts. Find its error.

7. Explain polarization in an electrolytic cell. [L]m.

8. Describe the electrodes you would use to pass electric currents through organic tissues, and give reasons for their use. [D]m.

9. Describe the effect of connecting the poles of a Daniell cell to (a) two platinum wires dipping into dilute sulphuric acid, (b) two copper wires dipping into copper sulphate solution. [L.]

10. Two lead plates immersed in dilute sulphuric acid are connected to the poles of three Daniell cells in series, and after some time are disconnected from these and joined together by a wire. Describe and explain the various processes taking place. [L.]

11. Describe a battery entirely suitable for operating a cautery. [L]m.

12. Describe the construction of some form of cell suitable for a standard of electro-motive force, and explain the chemical reactions when a current is passing. [L.]

13. Explain the construction and mode of action of any one form of secondary battery. What are the advantages of secondary batteries as compared with primary batteries ? [L.]

14. Describe the parts of a storage cell or accumulator, and state the changes that occur in them during the process of charging and discharging. Why is it important that the voltage of the cell should not be allowed to fall below 1·9 ? [L]m.

15. Three Daniell cells in series are used to charge an accumulator. Find the weight of copper deposited while 1 grm. of PbO is converted into PbO_2. Equivalent weights : copper, 31·8 ; lead, 103·5 ; oxygen, 8. [L.]

16. What is meant by the internal resistance of a voltaic cell ? How would you measure it, and what difficulties are met with ? [L.]

17. When two batteries, A and B, are joined in turn to a galvanometer, A gives the greater current ; but when another galvanometer is employed B gives the greater current. Explain. [L.]

18. What determines the maximum current obtainable from a cell ? Find out all you can about a cell which sends $\frac{1}{3}$ amp. through a wire of 1·8 ohms, and $\frac{1}{4}$ amp. through a wire of 2·9 ohms, connected directly to the poles of the cell. [L.]

19. Four cells, each of 2 volts and 0·5 ohm, are connected in two groups of two cells each, in 'series-parallel.' Calculate the current the battery will maintain through a coil of wire with a resistance of 0·7 ohm. [L.]

20. A Daniell cell has e.m.f. 1·07 volts, resistance 2 ohms. Its ends are connected by 3 and 4 ohms in parallel. If electro-chemical equivalent of copper is 0·000328, find weight deposited per hour. Also find heat developed per hour in cell and each wire. [L]m.

21. A battery of 12 cells each of internal resistance 2 ohms is to give maximum current through a cautery of 4 ohms. What arrangements of cells will give maximum current, and which consumes less zinc ? [L]m.

22. Compare the quantities of heat developed in a Grove cell for each gramme of zinc consumed when the poles of the cell are (1) connected by 50 ohms, (2) short circuited, resistance of cell being 1·5 ohms. [L.]

23. Five cells each of 1·8 volts and ·2 ohm are in series. Through what external resistance will they send 1 amp. and what is then the p.d. at the battery terminals ? [M.]

24. Two wires, one of 5 ohms and one of 15, are joined in parallel to the terminals of a cell of electro-motive force 2·1 volts and internal resistance ·45 ohm. Calculate total current given out by cell and potential difference between its terminals. [L]m.

25. Two batteries of 10 and 8 volts, and internal resistance $3\frac{1}{2}$ and 2 ohms respectively, are in parallel, and their poles are joined by 14 ohms resistance. Find current through each.

26. + poles of 2 cells are joined by thick wire A and the negative poles B and C by a 5-ohm wire. Find p.d.'s between A and B, B and C, C and A, if cells are 1·5 volts, ·75 ohm, and 2 volts, ·5 ohm. [L]m.

CHAPTER LXIV

THE PASSAGE OF ELECTRICITY THROUGH GASES

THOSE making frictional electrical experiments learn that air is a most reliable non-conductor until they put too great an electrical stress upon it—as by holding a charged knob at high potential within too short a distance of the knuckle at zero potential—and that then it suddenly 'breaks down' and conducts away the charge through an **electric spark.** This disruptive discharge, though so easily obtainable, is really very complex, and we must first study a much quieter transmission of electricity through air or other gases.

§ 666 : Suppose that we contrive to put a great electrical stress on the air in such a way as to give no opportunity for sparks. Look at the pointed end of the oval conductor in Fig. 286 ; the lines are crowded together, showing an intense electric field. Roughly, one can think of the end as the sphere of § 574 ; the potential this produced was inversely as the radius from its centre. Let a sharp needle-point with a hemispherical end perhaps ·001 cm. radius be attached to a conductor at, say, 50 e.s. units of potential. At radius ·01 cm. around the point the potential is roughly one-tenth of this, a drop of 45 units in ·009 cm., at the average rate of 5000 e.s. units (1,500,000 volts) per cm., a field of electro-static strength 5000, and this is more than the air can sustain. Fix a needle on the prime conductor of an electrical machine and turn the handle, electricity makes a quiet or slightly hissing escape ; there is no spark, but in the dark a tiny bluish glow is seen at the point.

Now (1) any insulated conductor, e.g. an electroscope cap, held near the point, gets a charge [remember the use of sharp-pointed combs to collect charge from the plates in electrical machines, § 568]. Evidently the air is conveying electricity.

(2) A candle flame held near is blown aside by a wind from

the discharging point. And the reaction between point and wind is sometimes illustrated by a little 'Barker's mill' of four wires radiating from a cap mounted on a pivot on the conductor; their sharp ends are bent tangentially forward. The mill is driven round backward by the repulsion between the wires and the electrified air driven away from their points.

From these it would appear that the transmission of electricity through a gas is an actual convection of charged particles, analogous to the motion of the ions in electrolysis, but much faster in the more mobile medium.

§ 667: In an electrolyte, however, there is evidence that the production of charged ions is a spontaneous process (as is their recombination into neutral molecules). In a gas the ions are not spontaneously produced, they have to be manufactured somehow, some drastic treatment of the gas is necessary to enable it to conduct (and then it soon loses its conductivity as the ions spontaneously recombine). Again in gases, unlike electrolytes, + and — ions are formed independently of chemical nature.

Some treatments that can ionize a gas and so give it conducting power are:—

(1) The presence of a *very intense electric field*, already referred to.

(2) *Chemical action, especially combustion.* Freshly manufactured hydrogen is transiently conductive. A flame is a good conductor, swept over an electrified plate it removes the charge forthwith: a little leyden jar held in the hot fumes just above the flame is soon discharged. Meteorologists use a long wire with a bit of spirit-soaked tow burning on the top and an electroscope at the lower end; electric charge present in the air passes in through the flame, and the electroscope leaves open out to show the potential of the atmosphere around the flame.

(3) *Solids heated to incandescence* ionize the air near them, and will lose sometimes a +, sometimes a — charge, depending on the nature of the solid and its temperature.

(4) *Ultra-violet light shining on some oxidizable metals* enables them to lose a negative charge. A clean zinc plate is laid on a charged electroscope cap; close above the plate is an earthed wire gauze. An arc lamp shines on the plate through glass without effect: removing the glass which stopped the ultra-violet the leaves collapse if they were — charged, not if +.

The alkali metals are sensitive lower in the spectrum, even to red light.

(5) *Röntgen rays* passing through a gas ionize it. Their presence puts a stop to all frictional electrical experiments.

(6) *Radium and other radio-active materials* ionize gases. An old uranium salt powdered and spread on an electroscope cap enables the charge to leak away to an earthed plate held just above: a mere trace of modern more active preparations collapses the leaves quickly.

C. V. Boys showed that the leakage from a charged body supported on a short rod of fused silica was little more than from one on a long fine fibre of it; that therefore the support was not always to blame for leakage, but that air itself was not a perfect insulator. It is now known that it is the traces of radio-active material in earth, bricks, etc., that account for this extremely feeble continuous ionization.

§ 668: **The current through an ionized gas.** When Röntgen rays, for instance, are passing athwart the air space between two small metal plates which are connected to a charged condenser and attached electrometer, a discharging current immediately flows across. As the voltage between the plates is raised (by having used more and more cells to charge the condenser) the current increases, but *not* proportionally (as Ohm's law would require). In fact, after a certain limiting potential difference has been reached (never more than 1000 volts per cm. of air gap) the current does not increase at all, and is called a **saturation current,** Fig. 331.

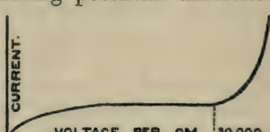


Fig. 331.

And now if the air gap is lengthened and the voltage per centimetre length kept the same, the saturation current *increases* almost proportionally to the quantity of air between the plates and exposed to the ionizing influence.

A satisfactory explanation is that the + and − ions as soon as formed begin to move towards the − and + plates with speeds about proportional to the forces acting on them, i.e. to the field, the 'volts per centimetre.' In weak fields the motion is slow and the majority get time to spontaneously recombine into neutral molecules, hence only a few give up their charges to the plates and the current is small. Strong fields drag the ions out so fast that few get the chance of recombining; since the total

production of ions depends on the external ionizing influence it is pretty obvious that a still stronger field will gather no more ions ; the current is 'saturated.'

The larger the space the more ions, hence the greater the maximum current obtainable.

In another experiment a very strong field is put on at a definite interval after the ionizing rays have been cut off, and the total discharge obtained is measured on an electrometer. Its diminution with increase of interval enables the rate at which the ions have been naturally recombining into neutral molecules to be calculated.

The sudden increase of current shown near 30,000 volts per cm. is explained by (1) above ; fields of this strength actually *produce* abundant ions and sparking soon ensues.

§ 669. To return to the electric spark. Between the mile-long lightning flash, the sparks that Franklin drew from the lower end of the wet string that tethered his kite in the rain cloud, and the sparks of electrical machines, induction coils, etc., there is no distinction other than that of size. Their 'crackly' sinuous shape is familiar to all. Short sparks, however, are straighter, and when one examines a small spark under the microscope one finds it by no means a uniform streak of light.

It happens that a much more convenient way of magnifying the structure of the small spark is to reduce the pressure of the gas.

The apparatus may be a long wide tube, Fig. 332, with plates and wires of aluminium inside, brought out by platinum wires through the glass to connections with a Wimshurst machine or an induction coil. The tube is sealed on to a mercury pump.

Going to the logical extreme and using the best vacuum attainable (far from perfect), discharge will not pass at all.

§ 670. Admitting a very minute trace of air, there appears a fluorescent green patch on the glass directly opposite the negative terminal or *cathode*. If obstacles are enclosed in the tube their shadows are thrown in the patch of light, and make it plain that rays of some sort are streaming out from the cathode in straight lines. These are the **Cathode Rays.**

The stream pays no attention to the anode, which may be anywhere in the tube.

If the cathode is towards one's left and the N. pole of a magnet is pushed up to the front of the tube the light patch on the right moves downwards ; the straight stream of cathode rays, crossing

the magnetic lines at right angles, has curved, and in the opposite way to that in which a wire carrying a current would have moved, i.e. it behaves as a flexible conductor carrying negative electricity.

That the rays do carry negative electricity has been proved by screening them down to a narrow stream through perforated plugs (as in Fig. 333) and then deflecting the bright fluorescent spot into a metal cup inside the tube connected to an electroscope outside. The electroscope moves when, and only when, the spot touches the cup.

That they carry a great deal of energy is shown in the very high vacuum tubes, Fig. 335, used for producing **Röntgen rays.** In them the concave cathode focusses its discharge on to an 'anticathode' plate of platinum or tantalum, and will make it red-hot and even melt a hole if the plate is too thin. The Röntgen rays start from this focussed spot on the anticathode.

The cathode rays do not escape from the tube.



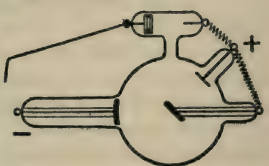



FIG. 332. FIG. 334. FIG. 335.
FIG. 333.

§ 671 : **Electrons.**

The mechanical force exerted per centimetre length on a current-carrying conductor = magnetic field × current (§ 595). Assuming the cathode rays to be composed of flying particles—**electrons**—carrying a negative charge, the 'current' that each one represents = its charge $e \times$ its speed v [think of the 'current' of water that a bucket passed from hand to hand towards a fire is equivalent to].

Hence the force deflecting each electron as it crosses a magnetic field = Hev, this makes it take a curved path of radius r and is equal and opposite to the centrifugal force mv^2/r (§ 35).

$\mathrm{H}ev = mv^2/r$, therefore ratio of charge to momentum of electron, $e/mv = 1/\mathrm{H}r$, and r is easily computed from the observed displacement of the green spot excited by the rays where they strike a fluorescent screen at the end of the tube Fig. 333, after passing across the magnetic field of the coil hh.

Side plates ff can be put into the tube, and a strong electric field F maintained between them, tending to drag the electrons towards the + plate with a force Fe and so deflect the stream upwards. The magnetic field H is adjusted till its down-bending action just compensates this, then $Fe = Hev$ or

$$v = F/H$$

This speed is found to be between 10,000 and 30,000 km. per sec.

And from the two equations, $e/mv = 1/Hr$ and $v = F/H$,

$$\frac{e}{m} = \frac{F}{H^2 r}$$

and this ratio of charge to mass of the electron is found to be 180,000,000, in coulombs/grammes.

The charge e on an electron in the cathode rays has not been measured, but the evidence goes to show that the mode of production of ions in a gas by the passage of Röntgen rays, etc., is that a single electron, or sometimes two, is torn from one atom and enters another, leaving the former as a + ion and making the latter a — ion.

If a moist dust-free gas is ionized and suddenly cooled by expansion, so as to become about 4 times supersaturated, the water condenses on the negative ions present and they fall as a miniature shower of rain. The total weight of the shower is calculable from the volume (perhaps 1 litre), etc., of the expansion chamber; the size of individual drops is calculated from their rate of fall through the gas, hence the number of drops is known. The total charge carried down is found by receiving the shower on an insulated plate connected to an electrometer; hence the charge per drop is known, and in all probability this is the electronic charge e,

$$e = 1{\cdot}6 \times 10^{-19} \text{ coulombs}$$

and since $e/m = 1{\cdot}8 \times 10^8$, $\therefore$ m the electronic mass $= 9 \times 10^{-28}$ grm., which is about $\frac{1}{1500}$ of the estimated mass of the hydrogen atom.

The electron is at present the indivisible unit charge of electricity.

§ 672. At these low pressures there is no luminosity in the gas, only a thin glow enveloping the cathode, and the distant fluorescence on the glass, green or blue according to the latter's chemical composition.

Note.—The cathode can be forced to emit also atomic, or larger, fragments of itself. Glass, wax, lace, etc., can be coated

with a film of gold or other metal by placing them in front of a cathode of the metal and driving a heavy current through.

As further traces of gas are admitted there appears a glow in the anodal part of the tube, lavender-hued in air, pink in hydrogen. This luminosity probably indicates places where electrons are leaving or entering atoms, i.e. places of ionization or recombination. In the denser gas now present the cathode electrons cannot escape ' collision ' with atoms ; they do not reach the glass now, and its fluorescence fades.

With rising pressure the glow moves towards the cathode, becoming condensed and brighter. The **' Crookes dark space '** occupied by the cathode stream still separates it from the cathode and the **' Faraday dark space '** appears on the anode side of it.

Still increasing the pressure, towards 1 mm. of mercury, the Crookes dark space hardly visibly separates the glow from the cathode, the Faraday dark space remains prominent, and there advances from the anode a long **positive column** of luminosity, often beautifully marked with transverse striations, steady or flickering. In a long tube this will occupy practically the whole length. All these are shown in Fig. 332. The discharge has ceased to be a stiff straight radiation and is a flexible stream between cathode and anode, only locally distorted by a magnet.

In common ' vacuum tubes ' (Plücker or Geissler), Fig. 334, at 1 cm. pressure the positive column squeezed in the capillary portion is continuous and brilliant, and is used to give the spectrum of the gas in the tube (H, CO_2, N, A, He, etc.).

At 2 or 3 cm. pressure the discharge shrinks to a fuzzy line of light, and nearing atmospheric pressure becomes the disruptive noisy spark.

§ 673. The Spark.

Small sparks under the microscope show the cathode glow, the Faraday space nearly dark, and the long positive column. Lengthening the spark simply lengthens the positive column, of which therefore long sparks almost entirely consist. It shows no striation, but is a compressed continuous line, and in air is complicated by the combustion of the nitrogen in the oxygen, which causes an enveloping line of flame. This flame is easily seen by gently blowing on a stream of sparks, and it produces choking fumes of nitrogen peroxide only too perceptible near sparking apparatus.

If the discharge is strong and rapidly periodic the air may

become so hot that this auto-combustion is continuous, and flame being a good conductor the noisy rattle of bright sparks dies down into a quieter wriggling line.

The potential difference required to produce sparks in air at atmospheric pressure is about 100 e.s. units=30,000 volts per cm. length of spark between knobs 2 cm. diameter, and it is roughly proportional to the air pressure.

§ 674: The Arc.

In § 667 (3) it was stated that incandescent solids ionized the gas near them. For instance, a hot lime cathode will discharge electrons at a tenth or less of the minimum voltage (about 450) necessary to work a vacuum tube with a cold cathode.

Now when a circuit carrying a few ampères is broken, the small points last in contact are sure to be heated strongly, for they represent a high resistance interposed in the path of a current which self-induction keeps flowing on for the moment. Both points will therefore probably ionize the air between them, and the current will pass in a glowing stream across the gap. If current $\times$ voltage, i.e. the power in circuit, is small, the heated points will be minute, and conduction back into the masses of metal will soon cool them; and if in addition the air gap widens quickly, the 'spark at break' is soon quenched. But if there is adequate power, with more than a certain minimum voltage (5 to 50, according to material), and if the gap is kept small, the discharge continues as the **electric arc.**

The typical carbon arc is formed between two rods of gas carbon in a circuit of about 60 volts. The points are originally in contact but are pulled $\frac{1}{8}$ in. or more apart directly after the current is turned on.*

The heated negative carbon (cathode) is regarded as emitting a blast of electrons which strike the positive carbon and keep it brilliantly incandescent, and it in turn sends forth +charged particles, *atoms*, of the gas close to it. [That the hot cathode is responsible for the initiation of the arc is illustrated by the re-kindling of an arc in a 500-volt circuit, after a second's extinction, without the need of touching the hot carbons together again.]

* This is done automatically. The current passing round a small solenoid draws in an iron core linked to a clutch on the carbon rod. As the carbons burn away and the arc lengthens the voltage between the carbons increases, and increases the small current through another solenoid, of fine wire, placed as a shunt across the arc; this presently releases the clutch and lets the carbons drop closer.

The + carbon wears away rather rapidly, presenting a concave crater to the pointed — carbon. The crater is filled with the vapour of boiling carbon, for the temperature of the arc is the boiling point of the carbon 4000° A.: greater current makes a larger but not hotter crater. It is the high temperature of this boiling point that makes the carbon arc so much more luminous than arcs between volatile metal poles.

The carbon vapour gradually drifts out of the arc and burns, so that the carbons waste away (the + twice as fast as the —, hence it is made thicker). Excess of current causes hissing, the flame creeping out round the sides of the carbons and burning them away badly. Enclosed in a small oval nearly air-tight globe of translucent porcelain the carbons last as much as 100 hours, quite ten times as long as usual.

A current of 1 or 2 amp. maintains a diminutive arc between carbons of slate-pencil size, 10 to 15 amp. runs a street lamp of 1000 c.p., 100 amp. and more is used in search-lights and lighthouses. A wire resistance coil is always put in series with the arc, or else the current is too unstable. Thousands of ampères are used in electric furnaces.

High voltages maintain longer arcs.

Of late the arc itself has been made long and very luminous (instead of short, bluish, and scarcely visible) by impregnating the carbons with salts of sodium, calcium, etc., which distil out and provide abundance of very easily ionized vapours (see §§ 415, 502).

The mercury arc takes place in a tube, in mercury vapour only, between two small pools of mercury. The cathodal pool has to be heated locally to start the discharge; this is effected by running a stream of mercury across and breaking it. The blue-green arc of intense luminosity (§§ 415, 502) flames through the tube, which usually keeps below 300° C.

§ 675: **Globe lightning**. There are many records of the descent, during thunderstorms, of balls of fire from a few inches to a couple of feet in diameter, and of the destruction they have wrought to things they happened to touch in their wanderings, as they drifted about in the air, gradually dwindling.

The silent discharge from pin-points, sharp edges, etc., ozonizes the air strongly, and is in commercial use for that purpose: it has recently been suggested that the fiery ball is a volume of ozone which has been formed in the thunder cloud and, being half

as dense again as air, has sunk to the ground. The unstable ozone O_3 is steadily returning into oxygen, and shining with the ionic glow. Coming into contact with any easily oxidizable material the decomposition would be accelerated, and it is calculated that a ball a foot in diameter could work at the rate of many hundred horse-power for a second, i.e. would explode with some power.

§ 676. **Radio-activity.** This most recent and remarkable development of our subject may be said to have begun with the discovery by Becquerel that a piece of pitch-blende, the mineral from which the salts of the metal uranium are prepared, could strongly affect a photographic plate when laid upon it in the dark. Unlike fluorescent substances the pitch-blende required no previous treatment whatever, and further, the action took place even through black paper or thin aluminium. The whole of this power concentrated in the uranium oxide when this was extracted from the mineral; any uranium salts prepared a few years ago show it readily.

It was soon found also that the air close by became ionized and conducting, just as if traversed by Röntgen rays. If a little powdered uranium nitrate is spread on the plate of a charged electroscope and a metal plate connected to earth is held half an inch above it, the leaves steadily collapse.

Further chemical treatment proved that the power was really the property of an unknown chemical element present in small traces. This element M. and Mme. Curie, by a long and painstaking process of concentration, succeeded in separating, as a bromide, and named it Radium.

How this substance, not unlike barium bromide in appearance and general chemical character, is self-luminous in the dark, can excite more brilliant luminosity in powdered zinc-blende, affects photographic plates even through thick metal screens, discharges electroscopes instantly, has a speedy destructive action on the skin and tissues, keeps itself warm by the spontaneous production of heat at the rate of 100 calories per gramme per hour—all these things are well known to everybody, though perhaps it is the high price, necessitated in part by the long labour of concentrating a substance present in such minute traces, that has most impressed the public.

§ 677. It is now believed that in radium and a number of other radio-active elements—polonium, actinium, thorium,* etc.—we

* A thoria gas mantle separated from a plate by thin paper or aluminium, and all clamped together, will photograph itself in a month.

are witnessing stages in the transmutation—the Evolution—of the Chemical Elements, an evolution for the most part proceeding with unthinkable slowness through countless æons, but here accelerated into centuries, or even seconds. For from the solid radium bromide there can be pumped a heavy gas—'radium emanation'—at a rate which shows that half the metal radium would have become converted into this new chemical substance in about 1400 years. And the bright luminosity of this gaseous emanation dies away to half in $3\frac{1}{2}$ days, as the gas forms a new substance, a solid deposit on the walls of the tube; and this solid deposit has been traced through half a dozen more transformations.

It is suggested that the heavy chemical atom is a complex system of electrons in orbital motion, that in course of time, by the loss of energy in some unknown way, this motion is slowed down to the point of instability, that thereupon an atomic convulsion ensues and results sometimes in the expulsion of electrons either singly or in small stable atomic groups, that the residue forms a new grouping usually capable of persisting a long time, but that in radium and its successors we have lit upon a series of groupings so unstable that they persist only a few centuries, or days, or seconds.

§ 678. The atomic rearrangement causes the emission of what are called alpha, beta, and gamma rays. The distinctive characters of these are best illustrated by an experiment on the lines of Fig. 336. On a narrow strip of photographic plate stands a diminutive lead cannon loaded with a speck of radium bromide; 2 or 3 cm. above is a second photographic plate. Thick lead absorbs almost all the rays striking it, consequently it is only those ejected in the direction of the muzzle that come under observation. The little contrivance is placed between the pole-pieces of an electro-magnet so that it stands in a magnetic field the lines of which go down perpendicularly to the plane of the paper.

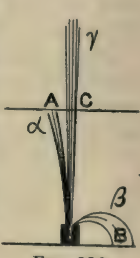


FIG. 336.

Without the magnetic field, development of the plates after a short exposure shows a black spot in the direct line of fire at C.

With a strong field there appear in addition blackenings at A and B. If a piece of thin mica or paper were laid on the muzzle, or if the upper plate were more than 5·5 cm. away in air, spot A

is missing. If a thicker plate lay over the muzzle, spot B is weakened.

A is therefore caused by a stream which is deflected like a current as it crosses the magnetic field. It is a stream of positively charged α particles, its very slight deviation shows that the electro-magnetic force makes but little difference to their momentum, they are heavy particles travelling fast. That they are separate particles is proved by the distinct flash of light as each strikes a zinc-blende screen under the microscope, and by the scattered black dots visible under high magnification on a photographic plate which has been exposed to them. Subsequent research has shown that they are atoms of helium (At. Wt. 4), their speed is about 20,000 km. per sec. They are stopped entirely by any solid* except the thinnest films of glass or mica, they can traverse only 5·5 cm. of air, but they make that air highly conductive, each producing 110,000 ions in its short flight.

The sharp curling over the opposite way to produce spot B is evidently that of a stream of negatively charged particles possessing comparatively little momentum. These β particles must be smaller than the α particles, for with far less momentum they penetrate some thickness of solid. They must be travelling faster than cathode electrons, for the latter cannot penetrate the glass of their tube. They are electrons travelling at 100,000 km. per sec., several times faster than the average cathode particle.

The γ radiation causing spot C cannot be deviated and is capable of penetrating several inches of metal. Since it is the impact of cathode electrons on a solid obstacle that originates the undeviable and penetrative Röntgen rays, it is believed to be the impact of the swifter β particles on the bromide crystals lying in their track that gives rise to the γ radiation, so like Röntgen rays of exaggerated 'hardness.'

* Probably it is their stoppage by the radio-active material itself that causes its evolution of heat.

Chapter LXV

ELECTRIC SIGNALLING TO A DISTANCE

The chief use, besides those hitherto mentioned, to which electricity is put, is the old-established one of signalling to a distance.

§ 679: The first successful **Electric Telegraph** was a form of ' needle ' instrument. An installation of these nowadays consists of (a) a battery of Leclanché or Daniell cells supplying about 1 volt per mile of line; (b) a sending key which is some pattern of quickly worked reversing key; (c) the familiar insulated line of galvanized iron or copper wire; (d) the receiver, a rough sort of galvanometer, with a vertical magnet closely surrounded by a pair of coils. The magnet is weighted to keep it vertical when at rest, and its horizontal axle projects through the dial of the instrument and bears the ' needle.' Two little cylinders of thin steel, like penholder ends, are usually fixed on the dial; struck by the needle in its movements to right and left they tinkle in different keys, and save watching the needle; (e) the earth return, the current being turned adrift from a buried metal plate to find its way back to the sender or to mingle with, and be neutralized by, other stray earth currents.

The Morse apparatus has a simple key which the sender depresses sharply or more steadily, and sends currents to line for about $\frac{1}{8}$ sec. or $\frac{1}{4}$ sec. The receiver is a spring bar holding an armature just over an electro-magnet. The bar moves noisily between the tips of two screws so that a couple of taps signalize the start and stop of current. In a recording instrument the bar has a long tail bearing an ink-wheel which presses against a running tape of paper and converts the short and long currents into ' dots and dashes.'

The Wheatstone recorder has lighter moving parts and ironless coils and is worked very rapidly from an automatic transmitter through which a previously punched tape is running at 400 words per minute.

On long lines the joint effects of leakage and capacity enfeeble the currents so much that they can work only miniature instruments, called relays. The relay then makes and breaks the current from a local battery to actuate the sounder, etc., or on very long lines to pass on to the next relay 200 miles or so ahead.

With long submarine cables still more sensitive instruments must be employed. The superintendent of the first Atlantic cable, impressed by its length, used induction coils and transformed up his sending pressure above 10,000 volts. That the cable spoke for six weeks was high testimony to the excellence of its gutta-percha covering. The next cable was worked by William Thomson, Lord Kelvin, with a Daniell cell and the mirror 'speaking' galvanometer he invented for the purpose. His siphon recorder, now in use on all cables, is a large moving-coil galvanometer with a capillary pointer through which ink oozes on to a running tape: friction between pen and paper is avoided by keeping the tape trembling, so that the record is really a string of fine dots, waving $\frac{1}{8}$ in. right or left according to the current received.

§ 680: **Duplexing** a line makes it possible to send messages both ways at once and doubles its earning capacity. One way of effecting it is to wind each receiving instrument with two equal coils. The sender's current divides and passes opposite ways through his own instrument, half then passes out to line and round a coil of B's instrument and the other half passes through a rheostat and back to battery: the sender keeps the rheostat adjusted so that the halves are equal, and then his own instrument does not respond, while B's does. But if B, simultaneously pressing his key, connects his battery to line, he chokes the line and both 'line' coils against A's current, and both instruments now respond to the currents through the 'rheostat' coils, just as if current had come by line to both.

§ 681. In the 'microphone transmitter' of the **Telephone** the sound waves set into vibration a thin plate, apparently made of hard compressed carbon and graphite; 5 mm. behind this is a fixed plate, and between them is a heap of grains of very hard carbon. Encircling this black glittering sand is a wall of soft felt, and dispersed among it are some studs of felt which hinder it from 'packing,' see Fig. 337 (front view on left). The slightest compression of this granular mass by the vibrating diaphragm reduces its resistance and so alters the current, from a couple of

Leclanchés, flowing through it between the front and back plates. The current goes to a miniature transformer, § 603, from which a higher pressure secondary current, faithfully copying all its variations, goes out to line and to the distant receiver.

The line is double, for an earth return causes disturbing noises, and if they travel alongside telegraph wires the go and return wires twist round each other once in every four post-lengths, which prevents their inductively picking up the telegraph signals.

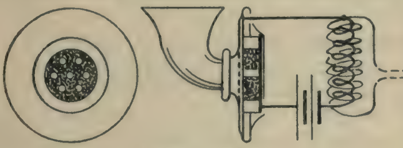


FIG. 337.

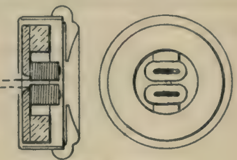


FIG. 338.

In the Receiver the current traverses coils of fine wire wound round the small pole-pieces of a ring-shaped steel magnet (Fig. 338, section and front view), modifies the latter's pull on a plate of thin iron about $\frac{1}{4}$ mm. in front of the poles, and so sets the plate into vibration. [This pull is proportional to the square of the strength of the magnet, m^2 becomes $(m+dm)^2=m^2+2mdm$, the difference in pull $\therefore \propto m$, hence the necessity for a strong 'permanent' magnet.] Thus the fluctuations of current set up by the motion of the transmitter diaphragm are retranslated into sound waves.

Bell's original telephone had no battery or carbon microphone, the transmitter and receiver were alike. When the thin iron plate is driven towards the magnet by the voice, some of the magnetic lines running in air across between the poles will move out, to run across in iron instead, cutting through the little coils as they move and inducing currents in them. Thus the forced vibration of the iron plate gives rise to alternating currents able to work a receiver over a short line and reproduce perhaps a ten-thousandth of the original sound, but too feeble to cope with the losses and disturbances of a long line.

§ 682 : **Electrical oscillations.**

Let a charged leyden jar be discharged through a circuit possessing self-inductance (§ 604), a thick copper wire, say, twisted into two or three large rings. The suddenly rising current has to establish its encircling system of magnetic lines, and these cut through the wire rings as they move out, and cause a back e.m.f.

which the current has to expend energy in overcoming. This energy is restored to the short rush of current as it dies away, for the returning lines cut back through the circuit and now produce a direct e.m.f. which forces the current to continue until it has charged the jar oppositely. The jar will discharge back, and so on, and a swinging to and fro of electricity will go on, just as water will oscillate in a bath or a U-tube into which more has been poured at one end. The resistance of the circuit ultimately subdues the oscillation, just as fluid friction brings the water to rest.

The periodic time of oscillation can be shown to be

$$2\pi \times \sqrt{(\text{capacity of jar} \times \text{inductance of circuit})},$$

both measured in electro-magnetic units.

The spark of a jar to a short discharging wire is a single flash, but when the discharging circuit is a bulky coil of stout thickly insulated wire a photograph of the spark taken with a rapidly moving lens shows it to consist of a score or more flashes, an oscillatory discharge with a period of perhaps ·00005 sec. Large jars and coils will even produce a spark oscillating slowly enough to be heard as a short dull musical note. If an inductance coil and large condenser in series are connected as a shunt across an arc lamp the arc likewise oscillates and ' sings ' drearily.

§ 683 : **Electro-magnetic Waves.**

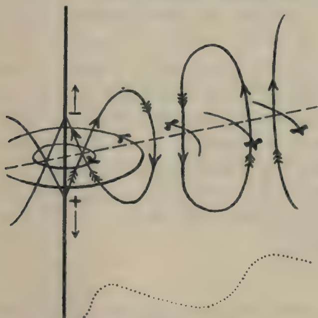


FIG. 339.

Suppose electric charges are oscillating in the conductor, Fig. 339. As two opposite charges separate and move off to charge the ends oppositely, lines of electric force spread out between them. And the movement of + electricity downwards and — upwards is of course equivalent to a double current flowing down, and sends forth circular lines of magnetic force. Thus at any external point there will be an electric force in the plane of the wire (the feathered arrows) and a magnetic force at right angles to it (the broad arrows).

During the return swing this electro-magnetic system is gradually withdrawn and replaced by a reversed system. But if the oscillations become very rapid there comes about a remarkable change.

Suppose a piston is being worked up and down in an open cylinder. The air near by moves to and fro, its motion is not perceptible ten feet away. But let the piston move a few hundred short strokes a second, and strong sound waves are 'radiated out' and can affect the ear or other detectors at long distances.

Similarly, when the electric oscillations become very rapid the electro-magnetic lines no longer quietly return to the wire to be replaced by a reversed system, but are driven out and away at great speed as successive waves, each wave bearing in its front an electric force parallel to the conductor and a magnetic force at right angles to it, and in its back equal reversed forces. Each pair of oscillating charges originates, per wave, a pair of closed loops of electric force, formed as in Fig. 339 by the crossing of the line of force joining them when the charges rush past each other, and the subsequent breaking apart at the knot.

§ 684. The apparatus of Fig. 340 is employed to produce electro-magnetic oscillations of high frequency. Current is sent from a battery B through a key K and a vibrating break of some

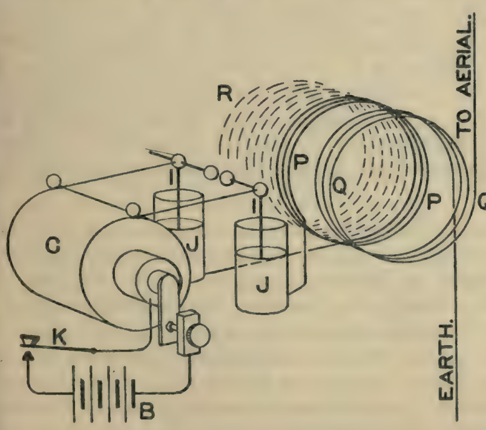


FIG. 340.

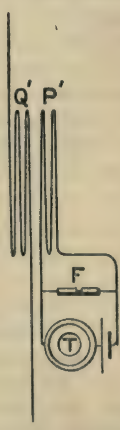


FIG. 341.

sort into an induction coil C which at every ' break ' overcharges the inner coatings I I of two leyden jars. They discharge through the short spark gap shown. This circuit II has very little self-inductance and the spark is a single short discharge: various means, such as burnishing the knobs, and enclosing them in an atmosphere of coal-gas, are employed to increase its abruptness.

The charging of I I with + and − involves the charging of the outer coatings J J with − and + charges, which travel round through P, a coil of two or three wide turns of stout copper wire.

After the spark the charges on J J are left in the circuit JPJ, which possesses capacity in J J and small self-inductance in coil P; consequently they oscillate to and fro in an alternating current of high frequency through P.

§ 685. The Lodge ignition for gas engines is a little modification of the above; it has a sparking plug connected as a shunt across the greater part of coil P. The comparatively slow charging of J J brings current round through P, but the sudden discharge jumps the plug in a blazing oscillatory spark, it being easier for it to do this than to force its way all round P against the back e.m.f. which is always called into existence by a sudden attempt to establish the large magnetic-line system of an inductive circuit.

The ' high-frequency ' apparatus of the medical electrician is the above with the addition of a prolongation R of the coil P, usually a winding of a few dozen turns of bare wire on a varnished wooden post perhaps 20 cm. diam. and 50 cm. high. This long coil ' resounds ' electrically to the oscillation in P, just like a resonance pipe to a whistle, and high-frequency discharges of varying intensity can be drawn from different parts of it, through heavily insulated leads, to suitable electrodes. In some patterns R is separate from P and then functions as does Q, below.

§ 686: For Wireless-Telegraph purposes another coil Q of a few open turns is placed coaxial with P, so that P and Q form primary and secondary coils of an ironless transformer, § 603. One end of Q is connected to an earth plate, the other runs up to the ' aerial ' of several highly insulated wires stretched from mast to mast.

This extensive system—Aerial, coil Q, earth—possesses about the same capacity and inductance as circuit JPJ. These circuits are therefore ' in tune ' and the oscillations in P readily induce similar oscillations in the wider-spread system, and it gives off long electro-magnetic waves into space, each containing pairs of

half-loops of electric force, their cut ends travelling on the surface of earth and sea (suggested by the broken line, Fig. 339).

When these come across a similar 'aerial' system an electric line finds itself meeting an insulated wire in two places, and forthwith contracts and disappears, drawing together + and − charges at its ends, i.e. causing a small current to flow in the receiver's coil Q′, shown edge-on in Fig. 341.

This induces a similar small current in P′, which the receiver has switched on to a circuit containing a **coherer,** F. This instrument consists of some sort of 'loose contact,' so bad that the ½ volt derived from a local battery sends no current through it. There are many forms, that shown consists of a heap of 2 or 3 cubic millimetres of fine silver and nickel filings lying between metal plugs in a sealed tube.

Immediately the feeble but high-pressure oscillation-current traverses the bad contacts of the filing heap, etc., this begins to conduct quite easily, and the battery sends a sudden current large enough to make an audible signal in a telephone T. 'Decoherence' after the waves have passed is effected by a quick tap from an automatic hammer on the filing tube. Thus the receiver's telephone murmurs as long as the sender keeps his coil in action by holding down his Morse key, and the message is spelled out in dots and dashes.

§ 687: The same principle is being worked out in connection with the more rapid succession of signals which go to build up vocal sounds, but there is also another quite distinct variety of wireless telephony. A thin smear of the element selenium, spread on a surface so as to join the edges of metal electrodes, possesses a fairly high resistance which is greatly diminished during its exposure to light. In a photophonic transmitter a strong beam of light is reflected from a thin silvered diaphragm to the distant receiver, where a lens concentrates it on the selenium 'cell' which is in circuit with a battery and telephone receiver. The vibrations of the transmitter diaphragm when spoken to, scatter the beam and so alter the brightness of illumination of the selenium, this varies the current and the telephone reproduces speech. There are obvious limitations, and no great distance has been reached.

The selenium cell is in use in the transmitter and receiver of a successful telectrograph, for sending pictures over a telegraph wire, and both principles have been employed in a 'wireless telectrograph.'

§ 688 : **Relation between electro-static and electro-magnetic units.**

The reader has found two completely different sets of definitions of electrical units, the Electro-static units of Chapter LVI which start with the unit quantity repelling unit quantity 1 cm. distant with a force of 1 dyne, and the other the Electro-magnetic units of Chapters LVIII, LIX, where the unit quantity is that carried past in 1 sec. by a current whose conductor experiences a force of 1 dyne per cm. length as it crosses unit magnetic field. The latter quantity is enormously greater than the former. Both systems originate quite naturally, and it is worth while inquiring how many electro-static unit charges go to make up the electro-magnetic unit (the deca-coulomb).

The measurement is made experimentally by the use of a parallel-plate air condenser, § 580, of calculable electro-static capacity $S/4\pi t$. This is charged from a battery n (say 50) times a second and as frequently discharged through a differential galvanometer, in which this outflow is balanced against the current sent from the same battery through a known high resistance R. Then

$$\begin{gathered}[\text{Electro-static}]\ (S/4\pi t)\times n\times \text{voltage of battery}\\ =\text{voltage}/R\ [\text{Electro-magnetic}]\end{gathered}$$

or $\dfrac{nRS}{4\pi t}$ electro-static units of quantity $=1$ electro-magnetic unit.

The number proves to be 3×10^{10}, thirty thousand million, and hence the Coulomb is three thousand million electro-static units of charge.

Now the question can be put in another way, and the answer reveals that this value is no mere accident of numbers, but has a physical meaning of the greatest interest. Instead of asking how vast a horde of electro-static units must be driven past a given point in a second, give a single unit a centimetre length of circuit all to itself—say a ring 1 cm. circumference—and give it the task of imitating 1-cm. length of unit (electro-magnetic) current—of producing the same magnetic effect as 1 decamp. flowing round the ring. How fast must it move ?

Evidently it must pass a given point 3×10^{10} times per sec., consequently its speed must be 3×10^{10} cm. per sec.

The perfectly natural way in which this number has arisen suggests that this actually is the speed of free movement of an

electric charge in a conductor. Now each charge has its lines of electric force, and the attached ends of these must keep pace with it ; any standing out perpendicularly to the conductor must therefore move perpendicularly to themselves at this same speed.

The lines of Fig. 339 as they spread are moving at right angles to themselves at this speed : *it is the speed of travel of an Electro-magnetic Wave.* Turn to § 481, *it is the speed of Light.*

§ 689 : The length in centimetres of an electro-magnetic wave can be shown to be $2\pi\sqrt{LC}$ where L is the self-inductance of the circuit radiating it, defined as in § 604 (the number of unit magnetic lines linked with the circuit when 1 decamp. flows in it) and C is the capacity of the circuit, measured as in § 578.

The length of the waves employed in wireless telegraphy is $\frac{1}{4}$ to 1 km. For experiments in rooms shorter waves are produced by the much simpler oscillating circuit with which Hertz first realized the theoretical predictions of Maxwell. It consists of a couple of zinc plates in the same vertical plane with a horizontal brass rod joining them, except for a small spark gap in its middle. The plates are charged by induction coil, to the ends of which they are directly wired, and they discharge across the gap in an oscillatory spark. A pair of plates 1 ft. diam. would have a capacity somewhere about 5, and $1\frac{1}{2}$ ft. of $\frac{1}{4}$-in. straight rod a self-inductance about 600, hence an oscillator of these dimensions radiates waves about $2\pi\sqrt{600\times 5}$ $=350$ cm. long. As detector (resonator) one would use a yard of wire, held parallel to the oscillator, and interrupted in the middle by a very minute spark gap or by a coherer. Shorter waves are obtained from an oscillator which is merely a brass knob, the opposite sides of which receive + and − charges through sparks from smaller knobs, and on which these charges then oscillate [cf. the oscillating drops of § 253].

These waves have been diffracted, reflected, focussed, refracted in prisms of brimstone or pitch, shown to be polarized, etc. ; they thus possess the general characters of light waves of exaggerated size.

Waves only ·3 cm. long have been obtained from oscillating circuits which were short lines of silver on glass.

' Heat ' waves ·03 cm. long have lately been detected (see § 503) by the energy they bring to the bolometer : no one doubts that these waves of ' dark heat ' travel at the same speed as light.

Light is nowadays regarded as the electro-magnetic radiation

from charges (electrons) oscillating in an atom of the luminous substance: the small dimensions have shortened the waves to ·0008–·0004 cm.

Röntgen radiation has been experimentally found to travel at the same speed as light. It starts from the anticathode of dense metal in which it is believed that the electrons flying from the cathode (§ 670) are very abruptly stopped. This is a much more violent change of velocity than occurs among the vibrating electrons of the atom, and the radiation consists of electromagnetic pulses much shorter than light waves. These sudden pulses can therefore sweep over atoms and be gone without setting the contained electrons into sympathetic vibration; the latter therefore cannot affect their speed: Röntgen rays suffer no refraction.

§ 690. **Electricity in Medicine.**

With Gilbert, the physician of Colchester, the science of Frictional Electricity began: to Galvani, the anatomist of Bologna, must be credited the first discoveries in the quieter realm of Current Electricity. In 1791 he published the results of his studies in a *Commentarius de viribus Electricitatis in Motu Musculari*, and it is only appropriate that a branch of electrotherapeutics should still be distinguished as Galvanism, while the work of Volta of Pavia, developing along physical lines—showing e.g. that salt water was as effective as living tissue—is commemorated in the title of Voltaic Electricity applied to the whole science of currents produced by chemical means.

The armament of the **Galvanist** consists of a battery of several small cells—dry cells nowadays—of which any number can be brought into use by a 'collector' switch so as to yield steady electro-motive forces from 10 to 70 volts. There is a variable resistance or 'rheostat' in circuit and the leads terminate in electrodes—pads and plates of metal, moist leather, sponge, etc.—which are applied to the patient's skin. A steady current of about 1 milliampère (·001 amp.) per sq. cm. soon produces local tingling and burning sensations, with a reddening, due to increased vascularity, of the skin. With a small electrode and heavy current (15 to 250 milliampères), destruction of the tissues ensues. If in **Electrolysis** a negative needle is plunged in (as for the removal of hair) while the positive electrode is a large surface plate, the tissue round the needle breaks down into a frothy alkaline liquid; if it is at the positive electrode that the

current density is great, acid is produced, the tissue contracts, bleeding is stopped, etc.

Three or four large dry cells will run the 4 or 6 volt electric lamps so useful to the surgeon for the exploration of dark cavities. Or they maintain the glowing heat of the Cautery, a little loop of resistance wire beaten into the semblance of a knife, which sears a bloodless and aseptic way through the tissues.

Faradism is a name applied to treatment with the jerkily discontinuous, small, high-pressure currents from little induction coils or from rotary medical magneto machines. The currents are weakened by withdrawing the iron core of the coil, or by ' shunting ' part of the lines from the magnet of the machine through a soft-iron armature bar. An induction coil too feeble to produce visible sparks is quite capable of causing most painful tetanic contractions, a fact which your friends have doubtless taken care that you should learn beyond fear of forgetting.

A gently alternating current like that from A.C. supply mains is occasionally applied, and is described as **sinusoidal.**

Some practitioners employ the tingling sparks from Wimshurst machines (§ 568) under the title of **Static treatment** or **Franklinization.**

Not very different from this in the sensation it produces is the modern fashionable **High-Frequency treatment**, § 685.

Fresh varieties of these keep making their appearance of course. It is an open question how much of their curative value some of them owe to the popular mystery that clings round all matters electrical, mystery still able to procure a sale for those ancient quackeries the electropathic belt and the anti-rheumatic ring.

The production of the **Röntgen rays** (from a tube like Fig. 335 attached to the secondary of a large induction coil) and their use, whether by photographic record or by visual observation with the fluorescent screen (a card blackened on the object and tube side, and coated with barium platino-cyanide or calcium tungstate on the observer's side) has been invaluable in surgery. The rays are also largely employed therapeutically in regulated ' doses ' for the cure of ringworm and other superficial infections. They are apt to ' burn ' the skin, but the intractable dermatitis that afflicted operators before lead-glass shields (opaque to the rays) came into use is a secondary effect due to the invasion of repeatedly burnt areas by streptococci.

The **ultra-violet radiation** present in sunshine accounts for the greater part of its germicidal power. Fortunately a more

dependable source is found in the arc, or in strong coil sparks between iron points, and the radiation from these (filtered if necessary from its heat by passing through water) when concentrated by quartz lenses and compressors is an established remedy for some cutaneous diseases. The silica-glass mercury vapour lamp gives out a flood of ultra-violet and is coming into use as a convenient and powerful sterilizer for water : ten seconds' stirring within ten centimetres of the lamp is ample.

The β and γ radiations from radio-active materials coagulate and kill morbid tissue through which they pass, and as they are easy of application and highly penetrative much is hoped for from their use.

ANSWERS

Chapter II, pp. 12–14

2. m.p.h. $\times$ 44·704.
3. $37 \times 1{\cdot}152 = 42{\cdot}7$, i.e. knots to m.p.h. add about 1 in 7.
4. Knots $\times$ 51·43.
5. 20·4 sec. and 30·6 sec. The easiest way of finding your speed in the train.
6. 32·2.
9. 9 sec.
10. 110 ft.
12. 10 cm./sec.^{2}. 15, 25, 35, 45 cm./sec.
13. 105 ft./sec.^{2}
17. 1 ft./sec.^{2} 6·2 tons wt. $= 6{\cdot}1 \times 10^9$ dynes.
18. $2{\cdot}24 \times 10^8$ dynes $= {\cdot}228$ ton wt. 33 ft.
20. 10^{11} dynes.
22. (i) ${\cdot}7 \times 10^9$ dynes.
23. 83·3 cm./sec.
24. 3480 cm./sec.
27. $3{\cdot}75 \times 10^9$ dynes $=$ about $3\frac{3}{4}$ tons.

Chapter III, p. 21

2. From food-reserves of body, to be spent in acquiring gravitational potential energy, and in friction. Gets hot for physiological reasons.
5. 18 : 1, ·4. Absurdly strong man, nearly $\frac{2}{3}$ h.p.
6. 2·1 h.p.
7. 10·5 m.p.h.
10. 1·12, ·4.
11. 80 lb. wt., 1·625 ft./sec.

Chapter IV, p. 26

1. 985·5 cm./sec.^{2}
2. 30 ft./sec.^{2}.
5. 70 cm./sec.
6. $32\frac{1}{4}$.
7. 32.
8. 61 and 62 grm.

Chapter V, p. 32

1. Super-elevation of outer rail interferes to some extent.
3. 99·4 cm. $=$ 39·14 in., seconds pendulum.
4. 1·1075 sec.

5. 1,973,000 dynes, about 4 times the weight.
6. 313 cm./sec.
7. 3·385 dynes.
8. ·588 dyne.
9. About 100 m. per sec.
10. 23,660.
11. $17\frac{1}{2}$ tons per sq. cm., practically breaking strain of hardest steel.

Chapter VI, p. 46

16. $\frac{1}{2}$ W.
17. 8 lb. in string, 5 lb. on wall.

Chapter VII, p. 54

1. $1{\cdot}804 \times 10^{10}$ grm. cm.^{2} units.
2. $\cdot 243 \times 10^{10}$.
3. $\frac{1}{2} \times (2{\cdot}047 \times 10^{10}) \times (2\pi \times 240 \div 60)^2 = 6{\cdot}44 \times 10^{12}$ ergs $= 86\frac{1}{2}$ h.p. for 10 sec.
5. $2000 \times 60 \times 981$ ergs $= \frac{1}{2}I(28{\cdot}8 \div 3)^2$.
6. They are equal; rolling hoop has twice the energy of one slid along ground.
7. Speeds equal; § 54.
8. 1·57 sec.; 1·64 sec.
9. 337, 334·2 [neglecting breadths 333·3], 4·2.
10. As $\sqrt{(337+554)} : \sqrt{337}$, 16·25 sec.
12. Machine going North rotor points W., falling to E. gives a rotor pointing N. ∴ resultant rotor points W. by N., i.e. wheel slews round towards East.

Chapter VIII, p. 65

1. 101 lb. per sq. in. = total force ÷ area of valve.
2. 155 lb.
4. 1615 ft.
5. 1033; 1,016,000.
6. 113·3 m.
9. 179 grm./cm.^{2}
10. 40 oz.
12. 312 lb.
13. 10 m.
14. 432,000 ft. lb.

Chapter X, pp. 78, 79

1. 0·74.
3. 8·02.
5. $B \times 5{\cdot}5 = b \times 13{\cdot}6 + (B - b) \times 1{\cdot}46$; $b/B = 1/3$.
6. 80 grm. transferred.
7. 1·2 tons.
8. As 8 : 11, 64 and 88 c.c.
9. 1·08 cm., 20 cm.
11. Sp. gr. liq. $(x-z)/(x-y)$, sol. $x/(x-y)$. Density w times as much.
12. Sol. 2·40, liq. ·81.
13. $12/15 = \cdot 80$.
14. $\frac{5}{6}$ oz.
15. 30 cu. in.
16. $B \times 1{\cdot}4 = (B + 5 \text{ in.}) \times 1{\cdot}2$; 30 in.
17. 1·54.
18. 2·1.

Chapter XI, p. 84

1. 28 : 1.
2. 3·7 : 1.
3. P must be in dynes, 1414 cm./sec.
4. 1085 cm./sec., 195·3 litres.
5. About 1,200,000 dynes or 1·2 kg.
6. $(1\cdot57 \times 3000)$ grm. $\times 3000$ dynes $= 14$ kg.

Chapter XII, p. 94

3. 200,000 and 1,200,000 kg./cm.^{2}.
4. 50 grm., $=49,000$.
5. 0·17 in. below top.
6. 40 ft. 5 in.
7. 1 in.
8. 28·15 in.
10. 1·20 ft.
11. 7 ft., 10,500 lb., 367 cu. ft.
12. 500 m. per sec.

Chapter XIV, p. 112

6. 801·085.
7. Quartz loses in air ·0450, platinum ·0056 grm. $\therefore$ put 39·5 mg. with quartz when testing.

Chapter XV, pp. 129, 130

4. 43 tons.
14. ·00001.
15. $6\frac{1}{2}$ in.
16. Increases ·0045 c.c.
17. 0·918 c.c.
20. 26 in.
21. As in Ex. 8 calculate increased area; then mercury must increase its own total height *twice* $4/3 \times 42 \times \cdot000012$ in. How much will do this? 8·4 in.
22. ·0165 in.
23. 4·4 tons.
24. ·000825.
27. ·0001815.
30. ·00366.
31. 15·5°.
33. $d_\circ/(1-at)-d_\circ/(1-at')$. Viscous forces.

Chapter XVI, p. 145

9. ·0358 cm.
10. 258·6° C.
12. 122° F., $-40°$ F.; $13\frac{1}{3}°$ C., $-17\cdot78°$ C.
13. See especially under Radiation.
15. 8·13 atmos.
16. 31,200.
17. 14·22 litres.
18. R$=25$, Ab Zero$=-270°$ C.; 1330° C
19. 775 c.c.
20. 21.55.

Chapter XVII, p. 151

6. 45·2°.
7. 5·5 grm. of water.
8. 13·6.
10. 31·7°.
11. ·203.
12. ·034.
13. 1 hour.
14. 1·2.
15. 1·85 tons.
16. 900°
17. 0·3.

Chapter XVIII, pp. 157, 158

9. 80 cals. per grm.
10. 79·53.
11. 625 grm.
12. 10°.
13. 10°.
14. 146,800 cals. per hour.
15. 940,000 tons.
16. $-1·45°$.
17. 9·15 cm.
18. 529 cals. per grm.
19. 5 lb.
20. 2420 and 5345 tons.
21. 87 kg.
22. 1·63 grm.
23. 19 grm.
24. 11·1 grm.
25. 727,000 cals.
26. 91·33, sp. ht. not required.
27. 57·5 kg.
28. 116·7 kg.
29. 344 kg.

Chapter XX, pp. 170, 171

6. V : S = 34 : 53.
7. ·0002.
8. 825 cals.
9. 134°.
10. $7\frac{1}{2}$ million cals.
11. ·0003.
12. 8 grm.
13. ·000217.

Chapter XXI, pp. 174, 175

2. 4·23 joules per calorie.
3. 39·3°.
6. 40·0°, 40·1°.
7. 91,600 cm./sec.
8. 28,400 cm./sec.
9. 87° F.
10. $3\frac{3}{4}$ grains.
11. $4·18 \times 10^7$ ergs.
12. One-twentieth degree C.
13. $7\frac{1}{2}$ km. per sec. (most are 3 to 6 times as fast).
14. 15 ft.-tons.
15. 1 oz.
16. ·243.
17. 14 lb.
18. ·092.
19. ·00147, contractor quite content, ·0000275.

Chapter XXIII, p. 206

2. Consult the tables.
4. Double the partial pressure of the air. 65·5 cm.
5. Figs. 83, 102.
6. 2·37 atmos.
7. ·987 lit.
14. 0·2875 grm.
15. 0·248 grm.

Chapter XXV, p. 216

3. 73·73 cm.
4. Use Fig. 83. Doubled.

5. 7/760 of 9/14·4 of mass of 1 cu. m. air at 20°. 7 grm.
7. Improbable, because so much moisture forming mist between 42° and 32° and checking radiation. If dew-point below 32° frost probable.
8. ·20, arid.

CHAPTER XXVIII, p. 240

2. 1·03 cm.
3. $\frac{1}{200000}$ atmo.
6. 3 cm.; $2\frac{1}{3}$ m.
7. ·12 cm.

CHAPTER XXIX, p. 255

5. 94·5 cm. mercury.

CHAPTER XXXIV, p. 295

3. 1100 ft./sec.
4. 324 m./sec.

CHAPTER XXXV, p. 304

2. 109.
3. $\cdot 01n = 1(n+10/3)$, 333 min./sec.
10. 270 puffs per sec., see Chapter xxxvi.

CHAPTER XXXVI, pp. 317, 318

1. Time reduced.
3. A.
5. 666 or 1332, e'' or e'''.
6. Nil (unless fork sounds trace of octave) : — : nil : resounds : nil.
9. 192.
10. 3·5.
11. 4·1 : 1.
12. 1·13 : 1.
13. 258.
15. 10^{12} dynes/cm.^{2}.

CHAPTER XXXVII, p. 330

6. S/10, S/8.

CHAPTER XXXVIII, p. 338

8. 10 in. from smaller between the lamps; and 58 in. beyond smaller.
9. 60° from perpendicular.

CHAPTER XXXIX, pp. 351, 352

18. 1·734; $35\frac{1}{4}$°.
19. 46°.

CHAPTER XL, p. 370

2. $7\frac{1}{2}$ in.
3. $f=20/34$ ft.; $b=20/33$ ft.
4. 20/21 ft. = 11·43 in.
9. Identical with curve for distances from focus in mirrors, and obtainable similarly.
10. 12 in. concave.
11. Concave 17·4 in. or convex 13·2 in. focus.

42

12. 12·7 in. from object.
13. ·2 or ·8 m.
14. 2·41 in.
16. 1·33.
18. 1·1 in.
19. 6·6-in. radius. 1½ in. or 3 in.

Chapter XLI, p. 377

5. One upright virtual apparently ¼ diam. below surface, other inverted real actually ¼ diam. in front of back surface.
6. Both become larger and come forward. No.
7. 6 ft. No real image.
8. 4·3 ft.
10. 25 cm.
12. 30 cm.
13. Compare Fig. 178 viii, 1·538.

Chapter XLIII, p. 385

1. 43 cm.
3. A distance beyond the second lens (a) 3 in., (b) 2·73 in., (c) zero, (d) infinity.

Chapter XLV, p. 407

1. The farther edge.

Chapter XLVI, pp. 415, 416

5. 6 in. concave.
6. 25 cm. concave or −4 Diopters.
7. 9·6 in. concave ; 18 in. convex.
8. −5D ; 20 cm. and infinity.
11. −200 cm., 8·5 cm. ; ·123.
12. If very near surface, cf. § 368.

Chapter XLVII, p. 438

1. 1·6 in. from lens.
2. 64·3 in. square. 450 : 1.
5. Drawn out.
9. 4½ and 1½ in.
12. A convex lens at whose principal focal distance is a concave mirror the centre of curvature of which is at the optical centre of the lens. See Fig. 229. In use as a reflex rear-light on bicycles.

Chapter XLIX, p. 451

1. Does not.
4. 300,000 km./sec.
5. 3×10^{10} cm./sec.
6. 315,000,000 m./sec.

Chapter LIII, p. 497

7. $I=10$, $k=20$.
19. 0·98.
20. ·0075 dyne.
21. $2{\cdot}78-1{\cdot}02+{\cdot}207=1{\cdot}967$ dyne.
22. 62·6 units.
23. 0·1437 dyne.

Chapter LIV, p. 509

4. As $35^2 : 40^2 = 1 : 1{\cdot}31$; $1 : 1{\cdot}86$.
10. $67\frac{1}{2}°$.

Chapter LVII, pp. 542, 543

6. 6 (cm.).
8. 890,00 up to 106,500 ergs ; again of 17,500.
11. (1) P.D. constant, charge and energy gained ; (2) charge constant, P.D. and energy lost.
13. 2690 (cm.).
14. Calculate capacity of each, then charge $= 10 \times c_1$; calculate $\frac{1}{2}Q^2/c_1$ and $\frac{1}{2}Q^2/(c_1+c_2)$. 40,000 ergs.
15. 157 ergs.
17. $2\pi e^2 A/k = 2\pi Q^2/kA = \frac{1}{2}Q^2/Ct$.
18. 5·6.
19. Field 0·1 ; force ·0004 dyne/cm.^{2} ; field same, mechanical force increased K times.

Chapter LVIII, pp. 564, 566

5. See 6.
9. Field due to C decampères in long straight wire at distance r from it is $2C/r$.
10. Field due to one current $= 2 \times {\cdot}1/2 = {\cdot}1$ line per sq. cm. The other carries ·1 decampère through 100 cm. of this; force $= {\cdot}1 \times 100 \times {\cdot}1 = 1$ dyne.
11. Couple $2 \times 750 \div \sqrt{2} \times 4$ dynes cm.
13. Between poles, at right angles to direction of motion, then curling round outside to return to starting point. These 'Eddy Currents' cause a heavy drag on the motion of the plate, and heat it.
16. ·00009 volt.
17. Vertical component lines cut per sec. $= 1200$ cm. $\times 80{,}000/108 \times {\cdot}2 \times 2 = {\cdot}00355$ volt. None. (a) Not at all. (b) Diminishes to zero on magnetic equator, there changes sign and increases.
18. Faces of coil are N. and S. poles.
21. Like a miniature earth inductor.
27. (b) No effect.
30. The inductance flash ; the drawing-in of iron, etc.
33. See 13.

Chapter LIX, p. 573

6. $H = 0{\cdot}20$.

Chapter LX, pp. 589, 590

7. 1 : 44·4.
8. 1, $1\frac{1}{2}$, 2, 3, $4\frac{1}{2}$, 6, 9.
9. 3·81 m. of wire 9 ohms per metre.
10. ·32 and 1·84 amp.
11. Factor $= {\cdot}007$; galv. resist. $= 100$ ohms.
12. Resistance of carbon filament falls to half when hot, that of metal filament increases about 6 times.

Chapter LXI, p. 597

1. Use sensitive galvanometer ; and § 632.
2. 1·98 volts.
5. Work voltmeter by a battery and adjustable resistance and bring a pair of wires from its terminals up to the potentiometer.

Chapter LXII, pp. 606–608

1. 1492 amps., 14·92 volts.
3. By voltmeter, ammeter, and photometer.
4. 1 : 2·6 : 2.
8. 1 : 150.
9. 55 watts, ·5 watt.
11. Diameter : square of length = constant.
12. 9 : 1.
14. $A_1 = 120 \div 300/250$, $A_2 = 122 \div 300/150$; $A_1(B + 300/250) = A_2(B + 300/150)$. $\therefore$ B = ·0515 ohm. 48 watts.
15. 12 min.
16. A^2R/J per cm. $= 1 \times \cdot021 \div (\pi \times 4\cdot2)$, hence per sq. cm. ·025 cal. per sec.
17. As $1\cdot04R : 10° :: 4(1 + \cdot004t)R : t°$, 45·5°.
18. Losses in mains 50 watts and 2 watts (Note—wires unchanged) ; 16 shillings.

Chapter LXIII, pp. 627, 628

1. (b) Non-conductor.
4. 50 hours.
6. ·04 volt high.
15. $\cdot285 \times 3 = \cdot856$ grm.
17. B and second galvanometer very different in resistance from A and first.
18. $E/(R + 1\cdot8) = 1/3$, $E/(R + 2\cdot9) = 1/4$. $\therefore$ E = 1·1 volt, R internal = 1·5 ohm.
19. $E = 2 \times 2$, $R = \frac{1}{2}(2 \times \cdot5)$, 3·3 amp.
20. Total $R = 2 + 1/(\frac{1}{3} + \frac{1}{4})$, E = 1·07. $\therefore$ ·284 amp. which deposits ·335 grm. $\cdot284^2 \times 2 \times 3600 \div 4\cdot2 = 138$ calories in cell. 50·5 in 4 ohms, 68 in 3 ohms.
21. In two series of six each,
    voltage = 6V, resistance in cct. $= (2 \times 6 \div 2) + 4$.
    In three series of four each,
    voltage = 4V, resistance in cct. $= (2 \times 4 \div 3) + 4$.
    Latter consumes only $\frac{2}{3}$ as much zinc.
22. As voltages dissipated in cell, viz. $1\cdot5/51\cdot5 : 1 = \cdot029 : 1$.
23. 8 ohms, 8 volts.
24. 0·5 amp., $2\cdot1 - \cdot5 \times \cdot45 = 1\cdot875$ volts.
25. No current through 8-volt battery, $\frac{4}{7}$ amp. elsewhere.
26. AB 1·56, BC 0·40, CA −1·96 volts.

INDEX

(TO PAGES)

Aberrations of mirrors and lenses, 401
Acceleration, 6
Accumulators, 618
Activity and reactivity, 10
Air pumps, 62
Altitude and temperature, 218
Amagat, 92
Ammeters, 567
Ampère, the, 577
Ampère's rule, 546
Amplitude, 257
Aneroid, 69
Angle, measurement of, 105
Animal calorimetry, 156, 163
Anode and cathode, 609
Anomalous dispersion, 394
Anticyclones, 224
Aperture in optical instruments, 439
Apparatus of precision, 95
Approximations useful in calculation, 112
Arc, electric, 636
Archimedes, principle of, 72
Area, measurement of, 104
Astigmatic beams, 403
— eye, 413
Astronomical masses, 25
Atmolysis, 243
Atomic heat, 151
Atwood's machine, 22

Balance, 108
— ratio of arms, 109
— rider, 109
— sensitiveness, 109
— wheel, compensated, 119
Balancing columns of fluid, 75
Barker's mill, 83
Barlow's wheel, etc., 547
Barometer, 67
— heights by, 71
— temperature correction, 126
Batteries, electric or voltaic, 620
Beats, 296, 299
Black body, 465
Blue sky, 398
Boiling, 193
Bolometer, 464
Bourdon gauge, 70
Boyle's law, 90
Brownian motion, 246
B.T.Unit, 599
Bulk modulus, 86

Calorescence, 395
Calorie, 146
Calorimeter, 148
— ice, and steam, 155
Camera lenses, 420
— lucida, 431
Capillarity, 231
Catadioptric lantern, 418
Cathetometer, 104
Cathode rays, 632
Caustics, 401
Cautery, 601, 651
Centimetre-gramme-second system, 2
Centre of gravity, or mass, 36
Centrifugal force, 28
Charles's law, 127
Chladni's figures, 320
Chromatic aberration, 404
Chronograph, 107, 298
Clocks, 106
Coherer, 647
Colloids and crystalloids, 245
Colour, 386, 392
Comparator, 117
Compound bar, 119

Compound harmonic motion, 258
Compressibility, 86
Conduction, 162, 165
Conductivities, thermal, 170
— electrical, 580
Conservation of energy, 18
Constraints, 95
Convection, 162, 164
Cooling, Newton's law, 159
Coronæ, 397
Coulomb, the, 572
Couples, 47
Critical state, 201
Cryophorus, 199
Curvature, 104
Curve, movement in, 27
Cyclones, 222

Damping, 264
Decampère, 553
Density, 73, 110
— of gases, 81, 111
— of vapour, 188
Depressions, 222
Dew, 218
Diagonal scale, 101
Dialysis, 245
Diamagnetism, 479
Diathermancy, 461
Diffraction, 275
— colours, 397
— grating, 277, 388, 446
Diffusion, 241
Diffusivity, 242
Dilatometers, 121
Diopters, 359
Dispersion, optical, 404
Dissipation of energy, 18
Doppler's principle, 285, 328, 391
Double refraction, 453
Dust, atmospheric, 217, 398
— tube, 303
Dynamo, 558
Dynamometers, 20
Dyne, 8

Ear, 323
Earth inductor, 556
Ebullition, 186
Echo, 288, 290
Effect of pressure on melting, 179, 181
Effusion, 81
Ejectors, 83
Elasticity, 85
— of gases, 90
Electric arc, 636
— field, 527
— furnace, 602
— incandescent lamps, 601
— lines of force, 515
— shielding, 516
Electrical capacity, 534
— condensers, 534
— — energy of, 539
— machines, 523
— oscillations, 643
— potential, 529
— supply, high pressure, 603
— surface density, 532
— units, practical, 577
Electricity, frictional, 510
— meters, 572, 615
Electro-chemical equivalents, 613
Electrolysis, 609
— medical, 650
Electro-magnet, 555
Electrometers, 512, 540, 592
Electrometer, capillary, 617
Electro-motive force, 591
— — standard cell, 624
Electro-motor, 555
Electrons, 633
Electrophorus, 521
Electro-plating, etc., 612
Electro-statics, 510
Emissivity, 161
Emulsions, 246
Energy, conservation of, 18
— dissipation of, 18
— kinetic, 15
— potential, 17
— of charged condenser, 539
— of electric current, 598
— of liquid surface, 235
— of vibration, 261
— of waves, 271
Equilibrium, 32
— stable, etc., 45
Equivalent lens, 382
Erg, 16
Evaporation, 162, 186
— and boiling, 193
— cooling effect of, 198
Expansibilities, linear, 116
Expansibility, absolute, 124

Expansibility, apparent, 121
— of liquids, 120
Expansion, thermal, 113
— area and volume, 120
— of solids, 114
— of gases, 127
— compensating contrivances, 119
Extensibility, 86
Eye, 408, 443
— astigmatic, 413
Eye-pieces, 425

Falling-plate experiment, 22
Faradism, 651
Fibres, 237
Filter-pump, 83
Flames, sensitive and manometric, 301
Fluids, laws of, 56
— in motion, 78
Fluorescence, 395
Focal lines, 403
Foot-pound, 16
Force, 8
— muscular, 12
Franklinization, 651
Freedom, degrees of, 95
Freezing mixtures, 183
Frequency, 257
Friction, solid, 9
— — coefficient of, 10
— fluid, 228
Frictional electricity, 510
Fuses, 603
Fusion, 177

Galvanism, medical, 650
Galvanometers, 567
Gas equation, 144
— thermometer, constant pressure, 129
— — — volume, 141
Gases, expansion of, 127
— liquefaction of, 207
Geometrical optics, 358
Glaciers, 182
Globe lightning, 637
Gravitation, 22
— law of universal, 24
— Newtonian constant of, 24
Gravity, 22
Greenhouse, 471
Gyroscope, 53

Halo, 400
Hardness, 90
Hare's apparatus, 75
Harmonic motion, simple, 257
— — compound, 258
Harmonics, 311
Heart, work done by, 64
Heat of combustion, etc., 156
— latent, 152
— specific, 147
— unit of quantity, 146
Heating, electrical, 598
High-frequency apparatus, 646, 651
Hoar-frost, 218
Hooke's law, 85
Human body, heat loss of, 163
Humidity, 212
Hydraulic press, 57
— ram, 61
Hydrometers, 77
Hydrostatic balance, 76
Hygrometry, 212

Ice, 180
Images, brightness of, 439
Impact and impulse, 11, 16
Inclined plane, 43
— — and roller, 23
Induction coil, 562
— electric, 513
— electro-magnetic, 544
— magnetic, 477
— self and mutual, 561
Inertia, 7
— moments of, 47
Injectors, 83
Interference, 273
— colours, 396
— tube, 300
Intrinsic brilliance, 469
Ionic theory, 609
Ionization of gases, 630
Iron, 183
Isothermals, 202
Isotonic solutions, 247

Jet pump, 83
— reaction, 83
Joule, 16, 598
Joule's equivalent, 172
— law, 600

Kater's pendulum, 24, 50
Kilowatt, 19
Kinetic energy, 15
— theory, 12, 92
— — liquid-vapour, 204
— — gas viscosity, 230

Lamps, efficiencies of, 469
Lantern, optical, 421
Latent heat, 152
Laws of motion, Newtonian, 7, 8, 10
Lens combinations, 381
— focal power, 359
— in water, 382
— magnifying power of, 423
— optical centre, 356
— problems, 366
— telephoto, 436
Lenses, 353
— achromatic, 405
— echelon, 417
— pocket, 424
— principal planes, 383
— thick, 383
— thin, 354
— and mirrors, practical methods, 378
Lenz's law, 552
Levers, 37
Leyden jars, 536
Light, speed of travel of, 448
Lighthouses, 418
Lissajous' figures, 261, 297
Lubrication, 100, 228

Machines, 42
Magic lantern, 421
Magnetic adhesion, 489
— compass, 508
— declination, 499, 503
— dip, 500, 502
— fields, 486
— — comparison of, 490
— induction, 477
— lines of force, 484
— moment, 492
— permeability, 479, 489
— pole strengths, 486
— shielding, 476
— susceptibility, 489
Magnetism, 474
— of Earth, 499
Magnetism of iron ships, 506
Magnetization, intensity of, 489
Magnetizing methods, 478
Magnets, molecular, 483
Magnification, 365
Magnifying power of lens, 423
— — of telescope, 427
— — of microscope, 431
— — normal, 440
Manometers, 66
Manometric flame, 302
Mass, measurement of, 108
Mechanical equivalent of heat, 172
Medical electricity, 650
Melde's experiment, 313
Melting point, 176
— — and pressure, 179, 181
Membranes, semi-permeable, 248
— vibrating, 322
Meteorology, 217
Metre bridge, 586
Metric and English measures, 3
Micrometer screw, 102
Microphone, 642
Microscope, compound, 430
— fine adjustment, 98
— micrometer, 433
— numerical aperture, 432, 444
— simple, 422
— substage illuminators, 419
— ultra-, 398
— ultra-violet, 445
Mirage, 347
Mirror problems, 375
Mirrors, inclined, 342
— parallel, 342
— plane, 341
— spherical, 371
Moduli of elasticity, 86, 93
Moment of a magnet, 492
Moments, principle of, 35
— of inertia, 47
Momentum, 6
Monochord, 315
Motion, circular, 27
— linear of particle, 4
— three kinds, 4

Nicol prism, 453
Night glasses, 440
Nodes and antinodes, 284

Ohm, 577

Ohm's law, 576
Opera glasses, 435
Ophthalmoscope, 436
Optical instruments, 417
— — aperture in, 439
— lever, 340
Optometer, 410
Oscillations, 257
— electric, 643
— forced, 262
Osmosis, 245
Osmotic pressure, 246
Overtones, 311

Parabolic mirrors, 402, 417, 419
Peltier and Seebeck effects, 604
Pendulum, 23
— conical, 29
— simple, 30
— compound, 49
— Borda's, 50
— Kater's reversible, 24, 50
— compensated, 119
Period, 257
Periodic motion, 256
Personal equation, 107
Phase, 257
Phosphorescence, 395
Photometers, 337
Pipes, 308
Pitch, 228, 289, 296, 300
Planetary temperatures, 472
Polarimeters, 457
Polarization of light, 452
— electrolytic, 616
Pole-finding paper, 611
Polygon of vectors, 4
Porous-plug experiment, 207
Post-office box, 587
Potential energy, 17
— electric, 529
Potentiometer, 594
Power, 18
Pressure of fluid, 56
— of vapour, 188
Prism, deviation in thin, 281
— direct-vision, 406
— minimum deviation, 348
— spectroscopes, 387
— — resolving power of, 445
Pulleys, 43
Pumps, 61
— jet, 83
Pumps, Sprengel vacuum, 82
— work done by, 63
Pyrometers, 119, 467, 468

Quantity of electricity, 572
— of heat, 147

Radiation, 162, 458
— pressure, 472
— selective, 469
— Stefan's law, 465
— theory of exchanges, 459
— Wien's law, 468
Radiometer, 464
Radio-activity, 638, 652
Rainbows, 398
Recoil, 16
Rectifier, electrolytic, 617
Reeds, 309
Reflection, laws of, 280, 339
— at spherical surfaces, 371
— total, 281, 349
Refraction, atmospheric, 347
— at spherical surfaces, 409
— laws of, 281, 344
Refractive indices, 345
Refrigerating machinery, 210
Regelation, 181
Resistance boxes, 579
— coils, 578
— comparisons, 584
— insulation, 589
— internal of battery, 585
Resistivities, table of, 580
Resolving power, 441
Resonance, 263, 305
— in a medium, 282
— tube, 291, 307
Rigidity, 87
Roller on incline, 23
Röntgen rays, 633, 651
Rotation, 47
Rotors, 51

Saturated vapour pressure, 191
Saturation current, 631
Screw gauge, 103
Searchlights, 417
Sensitive flame, 301
Sextant, 340
Shunts, 583
Simple harmonic motion, 30, 256
Sinusoidal current, 651

Siphons, 59
Solutions, boiling points of, 252
— freezing points of, 253
— vapour pressures of, 251
Sonometer, 315
Sound, speed of, 289
Spark, electric, 635
Specific gravity, 73
— heat, 147
— inductive capacity, 537
Spectacles, 411
Spectra, 386
Spectroscope, 387, 445
Spectrum, extent of, 391, 467
— top, 415
Spherical aberration, 402
— mirrors, 371
Spheroidal state, 186
Spherometer, 103
Spinning tops, 52
Sprengel pump, 82
Stars, 441
Statics, 32
Steam engine, etc., 199
Stress-strain diagram, 88
Strings, 313
Stroboscope, 299
Sublimation, 187
Sunlight power, 471
Supersaturated solutions, 178
— vapour, 197
Surface tension, 231
Syren, 299

Table of approximations in calculation, 112
— — conductivities for heat, 170
— — — — electricity, 580
— — diffusivities, 242
— — elastic constants, 93
— — electro-chemical equivalents, 613
— — expansibilities by heat, 115
— — isotonic or isosmotic solutions, 247
— — magnetic permeabilities, 479
— — — — feeble, 480
— — metric and English equivalents, 3
— — moments of inertia, 48
— — musical ratios, 326
— — pendulum corrections, 30
— — radiation (Fig. 242), 468
Table of optical refractive indices, 345
— — resistivities, 580
— — saturated vapour pressure (Fig. 83), 191
— — specific inductive capacities, 537
— — spectra (Fig. 185), 389
— — thermal data for solids and liquids, 185
— — — — — gases, 211
— — viscosities (Fig. 98), 226
— — wet and dry bulb hygrometer (Fig. 94), 215
Telegraphs, 641
Telephone, 642
Telescopes, 426
— terrestrial, 433
Temperature, 131
— absolute, 143
— scales, 133, 139
Tension in revolving rim, 31
Thermo-electricity, 604
Thermopile, 463
Thermometers, 133
— differential, 463
— gas, 129, 141
— platinum, 587
Thomson effect, 606
Tide machine, 260
Time, measurement of, 105
Tonometer, 300
Torque, 47
Tourmaline, 453
Transformer, 560
Turbines, 83
Twisting, 100

Ultra-microscope, 398
Ultra-violet microscope, 445
— radiation, 391, 467, 651
Undercooling, 177
Unilens, 436
Units, electro-static and electromagnetic, 648

Van der Waals, 93
Van't Hoff's law, 249
Vaporization, 186
Vapour density, 188
— pressure, 188
— saturated, 190
— supersaturated, 197

Vapour, unsaturated, 197
Vectors, 4, 6
Velocity, 1, 5
Vena contracta, 82
Vernier, 101
Vibration, 257
— frequency, 296
— microscope, 297
Vibrations of bars, plates, etc., 319
— longitudinal of rods, 316
— torsional, 317
Virtual work, 39
Viscosity, 225
Visibility and invisibility, 350
Vision, 410
— colour, 414
Volt, 577
Voltaic cells, 620
Voltameters, 614
Voltmeters, 594
Volume, measurement of, 105
— change on fusion, 178
— — — vaporization, 188

Water equivalent, 148
— expansion of, 126
Watt, 19, 600
Wave length, 266
— motion, 266
— — stationary, 283, 301
— velocity, 266
Waves, capillary, 268
— electro-magnetic, 644
— — spread and length, 649
— energy of, 271
— interference of, 273
— longitudinal, 269
— pressure of, 271
— reflection and refraction, 279
— straight, 275
— water, 267
Weighing, 109
Weight, 8
Wheatstone bridge, 585
Winds, 222
Wireless telegraphs and telephones, 646
Work, 15
— absorbed elastically, 89

Young's modulus, 86

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